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Non-linear and Dynamic Optimization for Short-term Production Planning

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Non-linear and Dynamic Optimization for Short-term Production Planning

Olinjär och dynamisk optimering för kortsiktig produktionsplanering

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Abstract

The short term production planning problem is solved in two steps by integrating physical plant models into the standard approach. The first step aims at solving the discrete variables from the unit commitment sub-problem (UCP) using standard mixed integer linear models and optimization techniques. The second step focuses on the economic dispatch sub-problem (EDP) described by high-fidelity, continuous time, physics-based and non-linear models together with nonlinear optimization techniques. The output of the second step includes optimized power flows but also highly relevant variables such as supply temperature, supply flow rate, turbine by-pass valve in the cogeneration plant. The optimization is formulated as a maximization of the benefit from heat and electricity sell over a finite time-horizon.

The proposed method is validated in several test cases using experimental data from a plant in Nyköping. The optimizations demonstrate the feasibility and the high economic potential of the proposed approach when comparing with measurement data and the standard optimization techniques. The optimized planning schedules result in a balance between produced and consumed heat, give priority to low-cost boilers and maximize plant revenue. Compared to measurement data, the optimizations result in a significantly lower supply temperature, a more extensive usage of the external cooler for higher efficiency and higher electricity production, fewer starts of units as well as an appropriate use of the accumulator tank.

The high-level description of optimization problems using JModelica.org provides useful means to specify flexible optimization problems including constraints on arbitrary process variables such as heat load of the production units, supply temperature and flow rate, condenser pressure.

Sammanfattning

Det kortsiktig produktionsplaneringsproblemet är löst i två steg genom att integrera fysikaliska modeller och den nuvarande standardmetoden. Det första steget beräknar de diskreta variablerna från start/stop problemet (SSP) med hjälp av diskrettidsmodeller och linjärprogrammering. Det andra steget fokuserar på det optimala lastfördelningsproblemet (LFP) som beskrivs med fysikaliska olinjära modeller i kontinuerlig tid och löses med olinjär dynamisk optimering. Från det andra steget fås optimerad värmeproduktion för varje enhet, men även relevanta variabler såsom temperatur och flöde i framledningen och by-passventilens öppning i el- och värmekraftverket. Optimeringsproblemet är formulerat så att intäkterna från värme- och elektricitetsförsäljning är maximerad över optimeringshorizonten.

Den föreslagna metoden är validerad i flertalet testfall som använder mätdata från ett fjärrvärmesystem i Nyköping. Optimeringarna visar att den föreslagna metoden är möjlig att använda och ger en stor ekonomisk vinning jämfört med mätdata och optimeringstekniker som idag anses vara standard. De resulterande körschema från optimeringarna ger en balans mellan producerad och konsumerad värme, ger prioritet till enheter med billigast drift och maximerar intäkten. Jämfört med mätdata så visar optimeringsresultaten att temperaturen och flödet i framledningen kan sänkas, respektive höjas, användning av den externa kylaren kan ökas, färre enheter behöver köras och att ackumulatorn kan användas på ett bättre sätt.

Den högnivåbeskrivning som JModelica.org erbjuder ger flexibilietet när optimeringsproblem ska formuleras som inkluderar begränsningar på godtyckliga processvariabler såsom värmelaster för enheterna, temperatur och flöde i framledningen samt kondensortryck.

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1 Introduction

1.1 Background

1.1.1 Short-Term Production Planning

Production planning aims at finding a cost optimal scheduling of the heat and power production plants, which satisfies both the network load demand and operational constraints. Scheduling refers to the status of the production unit (on-off, a discrete variable), and the produced power (a continuous variable). The resulting optimization problem that involves both discrete and continuous variables is referred to as mixed integer non-linear problem (MINLP), for which no robust algorithm is available. It is therefore necessary to make reasonable assumptions on both the modeling and the computation approaches to get a tractable optimization problem. The problem can be described as being composed of two sub-problems:

- The Unit Commitment Problem (UCP), in which decisions are taken on whether a plant should be running or not. The main difficulty lies in the combinatorial nature of the problem.
- The Economic Dispatch Problem (EDP) in which the load decisions for all active plants are taken. The main difficulty lies in that the nature of the plant is non-linear and most certainly, the optimization problem will be non-convex. That is, the optimization problem may have many local optimums in addition to the global optimum.

Typically for the UCP, the plant models are linearized, and the resulting problem becomes a MILP (Mixed Integer Linear Program) that can solve both UCP and EDP. The EDP, on the other hand, does not contain integer decision variables, and the technically most advanced way to cast the problem is to describe it as a non-linear problem (NLP).

As an input to the production planning problem, a predicted heat load over the entire optimization horizon should be provided. This is often generated by a load prediction model that typically includes a description of the district heating network and the effect of outdoor temperature.

1.1.2 Common Approaches

Due to the extreme difficulty to solve the general MINLP problem, the main approaches do not solve simultaneously the discrete and continuous variables but rely instead on successive solutions of related MILP (ex: Outer Approximation or General Bender Decomposition) or related NLP (ex: Branch and Bound), see [1].

There are few approaches that simultaneously solve the UCP and EDP: most of them are either based on Lagrangian Relaxation or on a Mixed Integer Linear Problem (MILP). In the Lagrangian Relaxation (LR) approach, the global optimization problem is decomposed into a global master-problem and small plant-specific problems. It is an appealing methodology, especially in the case of large networks (above 100 plants). The Lagrangian Relaxation approach allows some limited nonlinearities in the problem formulation, but no network topology is usually described to preserve the central separability property of the global problem. In the MILP formulation, the same piecewise affine models are used for both UCP and EDP, allowing a simultaneous computation of the solutions. The MILP formulation results in large-scale integer programs, but thanks to major progress in the computational efficiency of the MILP solvers it is usually not an issue, see [2], chapter 9. Lately, MILP formulations seem to be preferred over LR formulations due to the increased speed and quality of MILP solvers.

A good survey of the available approaches for short-term production planning is given in [3] and [4]. Earlier studies within Värmeforsk are [5] that focus on the effect of uncertain load predictions on the optimization results, and [6] that integrated a model of the distribution network in the MILP formulation of the production planning problem.

1.1.3 Limitation of the Standard MILP Approach

The MILP formulation of the EDP/UCP problems relies on a simplified representation of the model equations. The continuous decision variables of the optimization problem are typically the energy flows whereas the influence of the supply temperature and mass flow are usually not modeled. This represents a limitation since e.g., supply temperature affects many critical parameters such as the amount of energy that can be stored in the network, the heat loss in the network and the electric efficiency of steam turbines. The MILP formulation can actually be extended to account for the influence of the supply temperature, see [6]. The proposed approach introduces additional discrete variables and results in a higher model complexity. To maintain a low complexity, it is also common to model all processes by static relationships except for the storage dynamics (heat and fuels) and the transport delays in the distribution network. The linearization process, which is a necessary and critical part of the MILP approach, is consequently a trade-off between model accuracy and model complexity.

1.1.4 Proposed Approach

The proposed approached is based on the natural decomposition of the discrete problem (UCP) from the continuous one (EDP).

1. UCP. The entire optimization problem is formulated as a MILP and solved using a MILP solver. The main result of this stage is the discrete variables, which are fixed in the second step.
2. EDP. The desired load is dispatched between the running production plants to meet all plant operational and safety constraints. The state (on/off) of the plants and the start values of continuous control signals is given by the solution of the previously solved UCP.

The aim of the second step is to optimize the non-linear plant model based on physical laws without any major simplification. The plants are described by mass and energy balances, in terms of enthalpy, mass flow rate and pressure, possibly including the non-linear standard steam tables from IAPWS/IF97. Dynamics can be included without restrictions to match the real dynamic behavior of the plants. The optimization constraints and cost function can also be expressed by means of non-linear equations when necessary.

The advantages of this approach are twofold:

1. It reduces the amount of modeling work (in particular for the simplification process of the MILP description)
2. Results in highly accurate models.

This model complexity yields however a non-linear dynamic optimization problem and requires another type of solver than MILP solvers, see [7] for an overview of the available strategies. One reliable and efficient method to solve dynamic optimization problems that is based on non-linear programming solvers is the so-called collocation method. Control signals to be optimized and model equations are parameterized by a smaller number of variables, reducing considerably the complexity of the non-linear optimization problem. The original continuous-time optimization problem is transformed into a (discrete-time) Non-linear Programming (NLP) problem that can be efficiently solved using commercial or open-source solvers. The authors have applied the collocation method for dynamic optimization of a Carbon Capture plant, see [8]. Other successful applications of this optimization technique have been reported in the literature, see [9] for a list of applications where IPOPT (Interior Point Optimizer), an open-source NLP solver, was used. To the authors' knowledge, limited work on applying large-scale NLP methods for solving the economic dispatch problem has been performed.

1.1.5 Load Prediction

The load prediction model described in the Värmeforsk project reports [10] and [11] is intended to be used to generate a load profile over the optimization horizon. The load of the district heating network is therein modeled by a dynamic black-box model working on the diurnal (day-to-day) difference of the load. The network model together with a supply temperature control strategy has been tested on different network configurations. This grid load model was designed with the purpose of grid load control, where there is a need of predicting the future customer load two hours ahead, due to transport time in the grid, to deliver the optimal supply temperature at customers. The prediction horizon was chosen to be 2 hours but it will here be investigated if it can be extended to 24 h.

1.2 Project Goal

The main project goal is to show the potential of Nonlinear Programming methods to solve the short-term production planning problem based on high fidelity physical plant models. The optimization problem is formulated as a maximization of the benefit over a given time-period (about 1-2 days), the benefit being the difference between the income (from heat and electricity sales) and the cost (from fuel purchase and operation cost).

2 Modelica and Simulink

The modeling language used for the models in the economic dispatch problem is Modelica, which is becoming widely used in industry, see [12]. Modelica has some major differences compared to one of the largest modeling environments used today, namely Simulink.

Simulink is a graphical, block-based, modeling environment where a diagram corresponding to the system is built up using blocks such as integrators, summations, gains and user specified functions. First of all, a particular disadvantage with this approach is that the direction of data is pre-defined by the user. That is, the causality of each connection is fixed. This may be convenient when modeling e.g., a control system, but not when modeling a physical system. For instance, the flow direction through a valve is influenced by the pressure before and after the valve. Secondly, another major disadvantage is that algebraic loops may not easily be solved, which requires the user to solve for the right hand side of the differential equation describing the system.

The Modelica language is based on equations instead of block connections which make acausal modeling an available option. The equations can be stated by mixing differential, algebraic and discrete equations and there is no need for the modeler to solve for the right hand side of a differential equation. Modelica is also an object-oriented language and provides structuring features such as classes, components and inheritance which give opportunity to build reusable and extendable modeling libraries. On the negative side, the object-oriented feature results often in hierarchal models which cannot be directly interfaced with numerical integration routines when simulating the model, which block based models can as the right hand side is already solved for. Additionally, acausal connections often result in differential-algebraic systems, which have to be transformed into suitable format before applying numerical integration. However, such equation manipulations and transformations are provided by many of the Modelica tools available, for instance Dymola and JModelica.org, see [13] and [14], which are used throughout this report.

An example of a Modelica model of a steam turbine used in the plant model for optimization in this report is found in Figure 1.

```
model SteamTurbine "Steam turbine base model"
  package M = IdbackenMedium;
  package SIunits = Modelica.SIunits;

  parameter Real eta_is = 0.92 "Nominal isentropic efficiency”;
  parameter Real eta_mech = 0.98 "Mechanical efficiency”;
  final parameter SIunits.Area Kt=m_flow_nom/(sqrt(p1_nom*d_nom)*sqrt(1-p2_nom/p1_nom)^2))
                                                                    "Flow area coefficient"
  parameter SIunits.Density d_nom = Medium.density_ph(p1_nom, h_nom) "Nominal density"
  parameter SIunits.AbsolutePressure p1_nom "Nominal inlet pressure"
  parameter SIunits.AbsolutePressure p2_nom "Nominal outlet pressure"
  parameter SIunits.Enthalpy h_nom "Nominal inlet enthalpy "
  parameter SIunits.MassFlowRate m_flow_nom "Nominal inlet flowrate"

  SIunits.MassFlowRate m_flow "Mass flow rate";
  SIunits.AbsolutePressure p_out "Outlet pressure";
  SIunits.SpecificEnthalpy h_in "Inlet enthalpy";
  SIunits.SpecificEnthalpy h_out "Outlet enthalpy";
  SIunits.SpecificEnthalpy h_is "Isentropic outlet enthalpy";
  SIunits.PressureRatio p_ratio "Pressure ratio, p2/p1";
  SIunits.Power P_mech "Mechanical power input";

  Interfaces.FlowPort feed "Inflow port";
  Interfaces.FlowPort drain "Outflow port";

equation

  p_ratio = drain.p/max(feed.p, 1e6) "Pressure ratio";
  h_in-h_out = eta_is*(h_in-h_is) "Computation of outlet enthalpy";
  h_is = M.isentropicEnt(drain.p, feed.p, actualStream(feed.h_outflow)) "Isentr. Enth.";
  P_mech = eta_mech*m_flow*(h_in-h_out) "Mechanical power from the steam";
  m_flow = Kt*sqrt(feed.p*M.density(feed.p, actualStream(feed.h_outflow))*
                                           sqrt(1 –p_ratio^2) "Stodola's law";
  feed.m_flow + drain.m_flow = 0 "Mass balance";


  //Boundary conditions
  feed.h_outflow = inStream(drain.h_outflow)+ h_in - h_out;
  drain.h_outflow = h_out;
  h_in = inStream(feed.h_outflow);
  m_flow = feed.m_flow;
  drain.p = p_out;

end SteamTurbine;
```

Figure 1 Modelica implementation of a steam turbine used in the heat and power plant model. The equation section contains the equations describing the physics without any manipulations for solving output variables.

Figur 1. Modelicaimplementering av en ångturbin. Avsnittet “Equation” innehåller de ekvationer som beskriver fysiken och löser inte ut utsignalerna med hjälp av symbolisk manipulering.

3 Idbäcken Plant

3.1 Overview

The district heating plant Idbäcken, providing heat to Nyköping customers, contain in total seven different heat producing units, neglecting a gas boiler of only 1.5 MW, see Figure 2. The Idbäcken plant has been the topic of interest in other research projects, see for instance [10] [11] and [15].

The main unit of Nyköping is P3, which contains a 105 MW boiler that provides steam to the co-generation of heat and power. The turbines of P3 can together produce, at maximum, approximately 35 MW of electricity, and the condensers can, at maximum, transfer approximately 75 MW of heat to the district heat water. The flue gas of P3 is used in a flue gas condenser of maximum 12 MW to pre-heat district heat return water before entering P3.

Two circulating fluidized bed units, P1 and P2, of each 35 MW used only for heat production may be used when heat demand is high. Additionally, there is an electric boiler of 14 MW and two oil based heat production units at the hospital and the residential area Brandkärr 2×12 and 3×12 MW, respectively.
There is also an accumulator tank of hot water and an external cooler Beriden (not shown in figure).

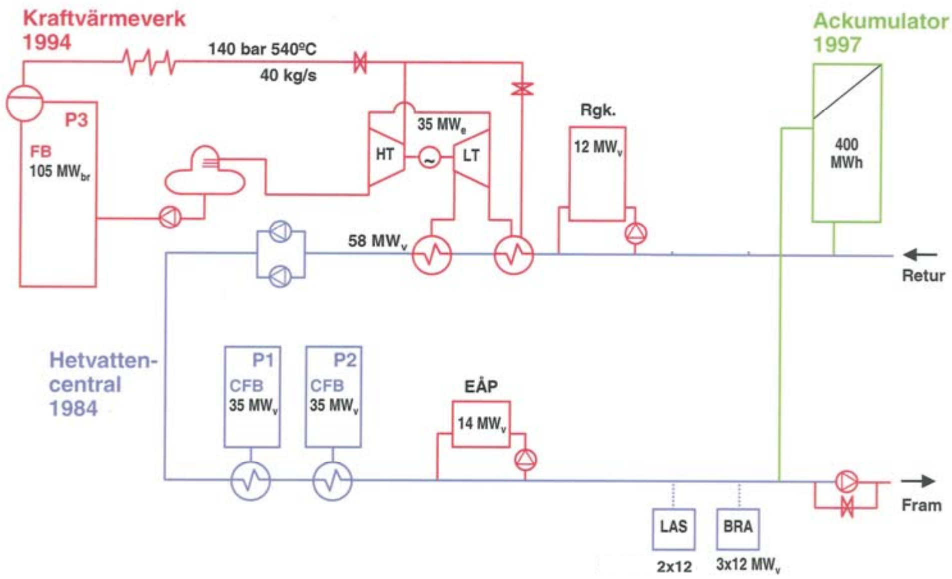


Figure 2 Explanatory sketch of the Idbäcken plant. Picture provided by Vattenfall AB.

Figur 2. Diagram over Idbäckenanläggningen. Bild från Vattenfall AB:

3.2 Measurement Data

Three different data sets from Idbäcken plant together with customer load predictions have been provided by Idbäcken and Vattenfall AB. All data sets start and end at 00:00 at night and the specific dates and features of each set can be seen in Table 1. The measurement data has mainly been used to initialize the models before optimization and compare optimization results with.

Table 1 Dates and main features of data sets provided by Idbäcken and Vattenfall AB.

Tabell 1 Datum och egenskaper av dataserier från Idbäcken och Vattenfall AB:

Date	Features
2012-10-14–2012-10-31	• P1, P2, EÅP, LAS and BRA not used • Beriden and accumulator used • Electricity production of 8-24 MW • P3 α value of 0.35-0.55 • Pump flows of 200-600 kg/s • Supply temperature of 70-91^{o}C. • Load prediction of 13-79 MW.
2012-12-01–2012-12-09	• Beriden, EÅP, LAS and BRA not used • P1 and P2 turned on/off several times • Accumulator used • Electricity production of 24-31 MW (calculated) • P3 α value missing in data. Assumed to be 0.53 which is the highest value from all data sets that P3 has during continuous operation. Electricity production of P3 calculated from assumed α and measured heat production. • Pump flows of 250-575 kg/s • Supply temperature of 83-103^{o}C. • Load prediction of 60-103 MW
2013-01-20–2013-02-08	• EÅP, LAS and BRA not used • P1 and P2 turn on/off several times • P3 turned off during short time • Beriden and accumulator used • Electricity production of 0-29 MW • P3 α value of 0-0.53 • Pump flows of 0-600 kg/s (0 kg/s when P3 turned off) • Supply temperature of 74-110^{o}C • Load prediction of 40-135 MW

4 Modeling and Optimization Overview

The main goal of the modeling and optimization effort in this report is to in the end deliver a schedule of how the different units should be run in the near future such that correct heat is delivered to customers in a way that is as economically beneficial as possible while all plant constraints are followed.

The mean to reach this goal in the work presented here, is to divide the problem into two different modeling and optimization problems as described before,

1. Unit commitment problem (UCP). The UCP use very coarse models and consider discrete time only and a linear optimization problem formulation giving a mixed-integer linear program to solve. The main result from the UCP is the optimal, possibly time-varying, status of each unit, i.e., on or off, during the optimization interval.
2. Economic dispatch problem (EDP) – Usage of complex and high fidelity physics-based models in continuous time together with non-linear dynamic optimization techniques to determine what the load of the running units should be. The results from the UCP regarding the status of each unit is utilized and considered constant. Due to the higher fidelity of models compared to the UCP, the optimization problem formulation can here be significantly more advanced in terms of constraints on physical properties of the plant.

The main workflow with the different models and optimizations performed in this work is presented below and in Figure 3.

1. Extract relevant plant measurements for model validation and model initialization.
2. Derive UCP, EDP and load prediction models. Use measurement data for support.
3. From plant measurements, initialize both UCP and EDP models such that both optimizations start in the same point as the real plant. This includes first and far most status of each unit and what load they are running on. For the more advanced EDP model, also e.g., district water temperature and flow, internal plant water flow and accumulator temperatures are initialized.
4. Initialize and run the heat load prediction model, giving a prediction trajectory of customer heat load to reach in the optimizations.
5. Using the heat load prediction and the UCP model, formulate the UCP optimization problem and solve it over the optimization horizon, e.g., 24 h. This gives the statuses of the units during the optimization interval as well as their heat load and P3 electricity load.
6. Pass the statuses of the units resulting from the UCP optimization to the EDP, where they will be considered constant.

7. Using the statuses of the units, the EDP model and the heat load prediction, formulate the EDP. Set the optimized statuses as time varying constraints on the units. Thus, in the EDP it is from the beginning pre-specified what units and when the units should be on or off. However, using the high fidelity models, the loads of the units are optimized again which are the outputs of the optimization together with optimal valve, pump and splitter control signals. Output is also the resulting revenue over the considered time horizon.

In the following sections, each of the different points in the list will be given a more in-depth view.

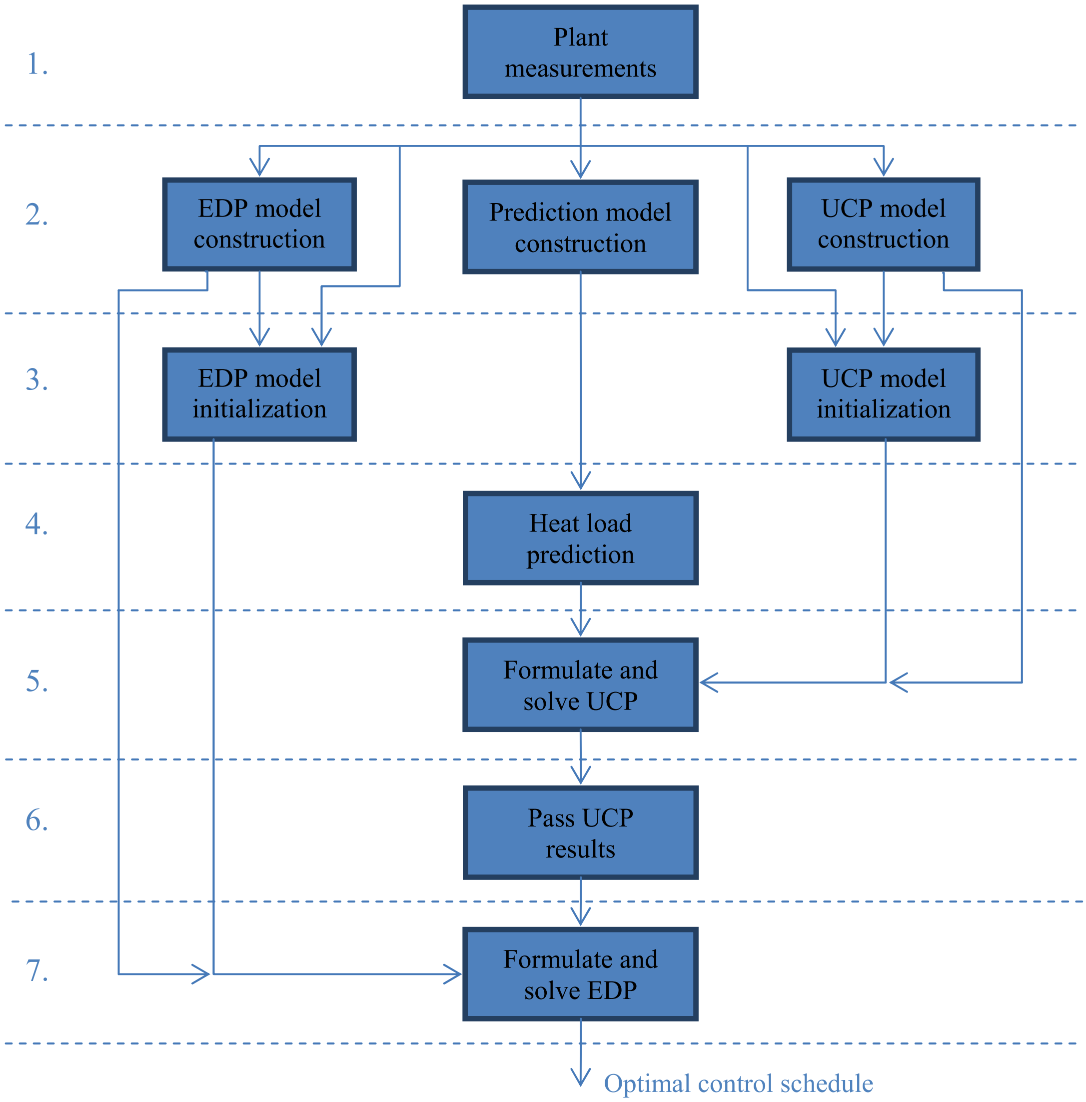


Figure 3 Overview of modeling and optimization workflow.

Figur 3. Översikt over modellerings- och optimeringsarbetsflöde

5 Plant Models

5.1 EDP Model

The continuous time model, i.e., the EDP model, used in EDP optimizations is divided into the Idbäcken plant and the district heat network of Nyköping, see Figure 4 below. Even though the different units are not located beside each other in reality, they are all considered to be part of the Idbäcken plant. The following sections will give descriptions of the two models, and their sub-models.

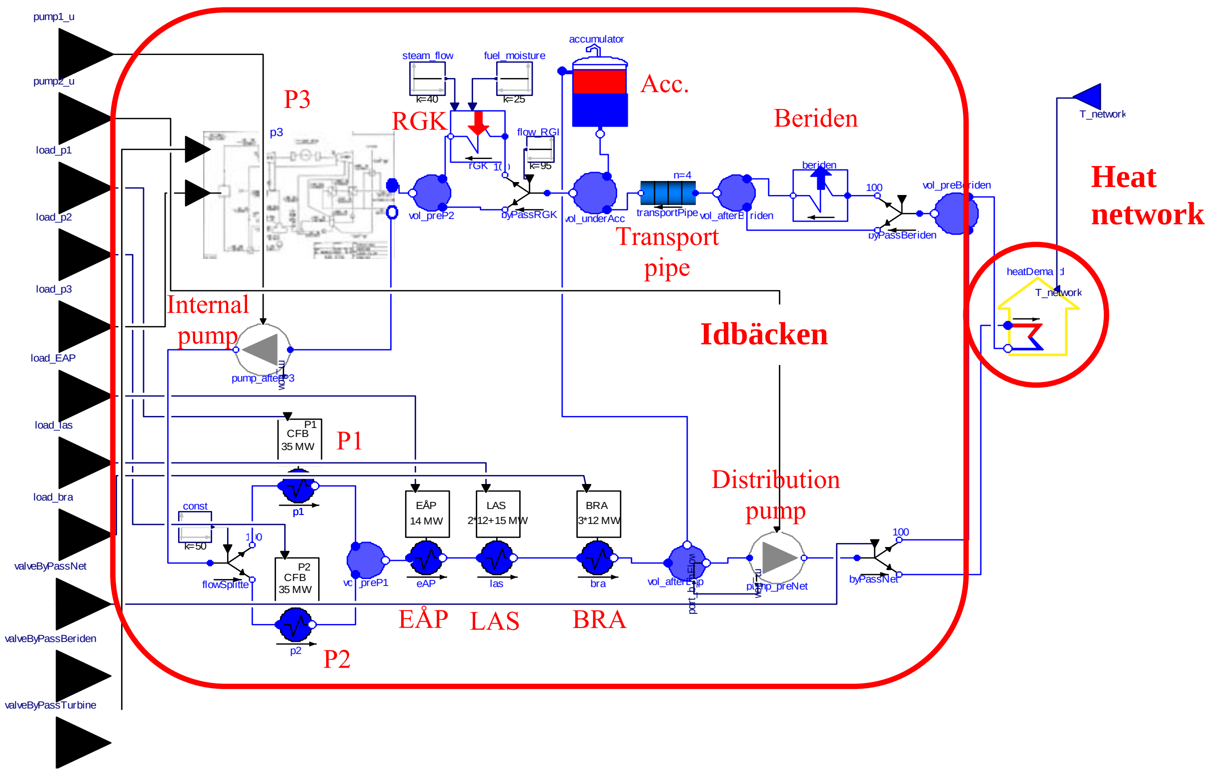
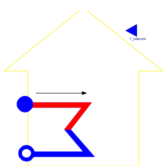


Figure 4 Overview of Nyköping heat district network Modelica model connected to the Idbäcken plant Modelica model.

Figur 4. Översikt over Modelicamodellen för fjärrvärmenätet och Ifbäckenanläggningen I Nyköping.

5.1.1 District Heat Network



The district heat network of Nyköping receives the district water from the Idbäcken plant and outputs the return water received by the Idbäcken plant. It also has an input available for setting the return water temperature. Using inlet and outlet temperatures and flow, the received heat by customers is calculated.

5.1.2 Idbäcken Plant

The Idbäcken plan model, which is the main part of the model used in the optimization, is divided into the following sub-models:

- P1 and P2 - circulating fluidized bed units of 35MW each.
- P3 - combined heat and power plant containing one high-pressure and two low-pressure turbines with a combined electricity production of approximately 35MW. It also contains two condensers that are able to transfer approximately 75MW of heat to the district water.
- Flue gas condenser (Rökgaskondensor) (RGK) of 12MW that uses the flue gas of P3 and pre-heats the return water prior P3.
- Electrical boiler (Elektrisk ångpanna) (EÅP) – electric boiler of 14MW.
- Brandkärr (BRA) – oil based boiler of 3×12MW
- Hospital (Lasarettet) (LAS) – oil based boiler of 2×12MW+15MW
- Accumulator – heat storage using 10000 m^3 water where produced excess heat can be stored and taken when needed.
- Beriden – external cooler using river water (Nyköpingsån) for cooling return or supply water.
- Transport pipe – discretized volumes approximating a 15 minutes time delay between the cooler and the plant.
- Internal and district water pumps
- Splitters for directing flows to e.g., Beriden or by-passing district heat network.

The production units' nominal maximum heat/electricity generation and efficiency from fuel to heat/electricity are summarized in the table below for convenience.

Table 2 Maximum heat/electricity and efficiency of each production unit.

Tabell 2 Maximal effekt och verkningsgrad för varje produktionsenhet

Unit	Maximum power	Efficiency η
P1	35MW	0.78
P2	35MW	0.78
P3	75MW heat 34MW electricity	0.85
EÅP	14MW	0.9
BRA	3×12MW	0.9
LAS	2×12MW	0.9

5.1.2.1 Co-generation Plant P3

The co-generation (heat and electricity production) unit P3, shown in Figure 5, is described by the following models

- One high pressure turbine and two low pressure turbines, all with bleed streams
- Two condensers
- By-pass valve for by-passing steam directly to the condensers
- Pressure loss for approximating the pressure loss of lumped bleed streams
- Control volumes

The characteristics that are important to capture with respect to the optimization is how the co-generation plant model can predict the influence of the turbine by-pass valve, the district heat network flow and temperature and the boiler load on the produced heat and electricity. It is not necessary to describe e.g., the furnace, instead, focus has been directed towards the vapor cycle. The main modeling simplifications on the vapor cycle is that it is not closed, which results in the following assumptions

- The vapor characteristics (pressure and enthalpy) at the boiler outlet are constant and the boiler mass flow rate is linearly affected by the boiler load.
- The feed water heaters that are downstream of the condensers are not modeled. All the bleed streams that normally go to the non-modeled pre-heaters are represented by a single stream connected to a constant pressure source.
- The condensate leaving the condenser is assumed to be at saturation pressure.

The following subsections will give a more detailed view of the components found in P3.

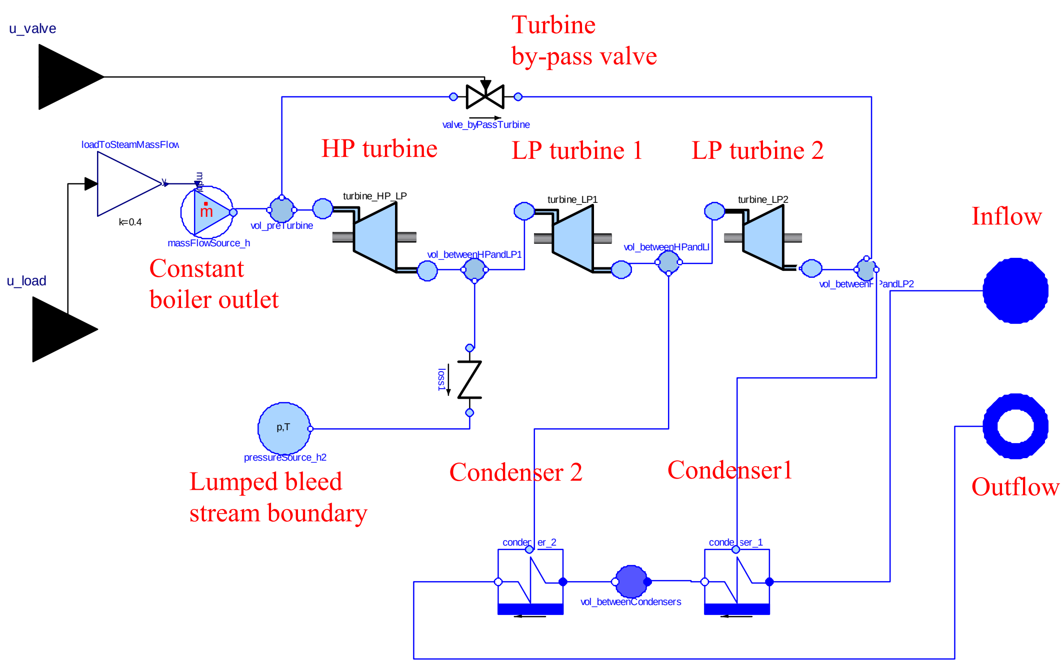
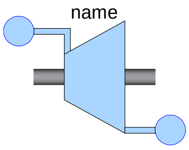


Figure 5   Overview of P3 Modelica implementation.

Figur 5.   Översikt over Modelicaimplementeringen för panna 3.

5.1.2.1.1 Turbine



Physics based model defined by an isentropic efficiency to calculate the outlet enthalpy and turbine work. The mechanical power generated from the steam is calculated using a mechanical efficiency and the pressure drop is related to the flow rate using Stodola's law. The electric power is calculated from the mechanical power using an efficiency parameter (generator losses).

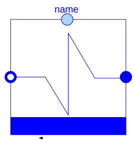
The different efficiency parameters are found in Table 3 and are equal for all turbines.

Table 3 Efficiency parameters and default values for turbine model.

Tabell 3 Verkningsgrad för ångturbinen

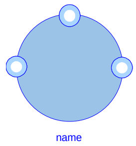
Efficiency type	Value
Isentropic	0.92
Mechanical	0.87
Electrical	0.84

5.1.2.1.2 Condenser



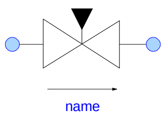
The heat flow rate transferred to the district water is driven by the temperature difference between the incoming water and the saturation temperature in the condenser. This heat flow rate is further used to compute the condensation rate that drives the bleeding flow from the turbine.

5.1.2.1.3 Control Volume



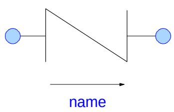
The control volume is a straightforward implementation of dynamic mass and energy balances expressed using pressure and enthalpy as states. Temperature is computed using pressure and enthalpy. The model requires partial derivatives of density with respect to enthalpy and pressure, see Section 5.1.3. The control volume is not a component with a physical equivalent in the plant, it is a component for holding physical balances and numerics.

5.1.2.1.4 Valve



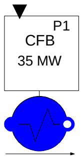
The mass flow through the valve is computed using the pressure difference, the valve opening and data from a nominal point. A standard quadratic equation relates mass flow and pressure drop.

5.1.2.1.5 Pressure Loss



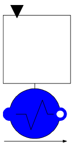
The mass flow through the pressure loss model is calculated using the pressure difference and a nominal pressure loss using a quadratic loss.

5.1.2.2 Circulating Fluidized Bed Units P1 and P2



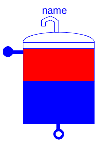
The model is not physics-based and heat transferred to the water is calculated by first-order filtering of the load and using parameters for efficiency and maximum heat transfer.

5.1.2.3 Electric Boiler EÅP and Oil Driven Boilers LAS and BRA



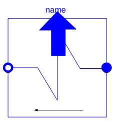
The model is not physics-based and heat transferred to the water is calculated by the load and using parameters for efficiency and maximum heat transfer.

5.1.2.4 Accumulator



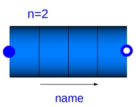
The accumulator is modeled using a finite-volume approximation that neglects buoyancy effects, i.e. no mixing is assumed when the accumulator is not charging or discharging. The accumulator is charged and discharged from the top and bottom. Return water enters from the bottom. Heat loss has been neglected.

5.1.2.5 Beriden



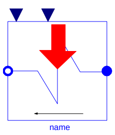
The cooler is modeled as a lumped epsilon-NTU heat exchanger where heat transfer is driven by the difference between the inlet temperatures. It is assumed that the minimal heat capacity flow is always on the district side flow.

5.1.2.6 Transport Pipe



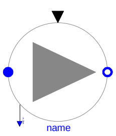
It is a finite volume implementation of a pipe with control volumes in series. The nominal number of discretization segments is 4 and the pipe diameter and length are parameters.

5.1.2.7 Flue Gas Condenser



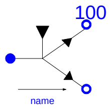
The model is not physics-based and is an ideal heat source which produces constant heat as long as P3 is running.

5.1.2.8 Pump



The pumps are ideally modeled, delivering a specific mass flow rate depending linearly on the control input, i.e., pump speed.

5.1.2.9 Splitter/Valve



The splitter/valve is ideally modeled and splits the incoming flow into two flows which sizes depend on the control signal to the splitter.

5.1.3 Media Models

Two different types of media models are used in the model package:

1. Simple water media – a water media with constant specific heat capacity, density and pressure and simple functions for specific enthalpy and temperature.
2. Advanced water media – a water media with polynomial functions approximating IF97 reference functions.

5.1.3.1 Simple Water Media Model

The simple water medium, which is used in all components outside of P3 and also on the water side of the condensers inside of P3, uses constant specific heat capacity, density and pressure as in Table 4. The specific enthalpy h and temperature T is calculated as

$$h = cp_{const}(T - T_0)$$
$$T = T_0 + h/cp_{const}$$

Table 4 Values of constants in simple water medium model

Tabell 4 Konstanta koefficienter för den enkla vattenmodellen

Constant	**Description**	**Value**
cp_{const}	Constant specific heat capacity at constant pressure	4184 kJ/kg
d_{const}	Constant density	995.586 kg/m^3
p_{const}	Constant pressure	1 MPa
T_0	Zero enthalpy temperature	273.15 K

5.1.3.2 Advanced Water Media Model

The media functions in the advanced water media package are polynomial approximations of the IF97 reference medium functions, and use mainly enthalpy h and pressure p as independent variables.

Table 5 gives an overview of which functions that are available in the advanced water media package. The functions are approximated within the one-phase regions, i.e., liquid and vapor regions, within the two regions

1. $100 \leq h \leq 1900$ kJ/kg, $0.1 \leq p \leq 200$ bar (liquid)
2. $2300 \leq h \leq 3800$ kJ/kg, $0.1 \leq p \leq 200$ bar (vapor).

The functions values within the two-phase region for $0.1 \leq p \leq 200$ bar are found by interpolation.

The functions have a maximum absolute relative error of 5%. The error is mainly in the temperature function for low pressures (<1 bar) and high enthalpies ($2300 \leq h \leq 3800$ kJ/kg). The functions are consistent at the phase boundaries. For instance, the temperature functions in the liquid and vapor regions, *Tliq(p,h)* and *Tvap(p, h)*, give the saturation temperature at the phase boundary as they have the structure

$$T = T_{sat} + dT,$$

where T_{sat} is the calculated saturation temperature and dT is a polynomial in enthalpy h and pressure p which is equal to 0 at the boundary. The functions *pbub(h)* and *hbub(p)* have specific structures such that they are analytically invertible between each other, i.e., *pbub(hbub(p)) = p*.

Examples of polynomial approximations of dew enthalpy, invertible bubble enthalpy and vapor temperature, and related approximation errors are found in Figure 6-Figure 8.

Table 5 Functions available in the water poly medium package.

Tabell 5 Tillgängliga funktioner i den polynom vattenmodellen

Function name	**Description**
Tsat(p)	Saturation temperature as function of pressure
hdew(p)	Dew enthalpy as a function of pressure
ddew(p)	Dew density as function of pressure
pbub(h)	Bubble pressure as function of enthalpy (invertible with *dbub(p)*)
Tvap(p,h)	Vapor temperature as function of pressure and enthalpy (valid for vapor only)
vvap(p,h)	Specific volume as function of pressure and enthalpy (valid for vapor only)
specificEntropy(p,h)	Specific entropy as function of pressure and enthalpy
isentropicEnthalpy(s,po)	Isentropic enthalpy as function of entropy and output pressure
isentropicEntalpy(po, p, h)	Isentropic enthalpy as function of output pressure, input pressure and enthalpy
dbub(p)	Bubble density as function of pressure (invertible with *pbub(h)*)
Tliq(p,h)	Liquid temperature as function of pressure and enthalpy (valid for liquid only)
dliq(p,h)	Liquid density as function of pressure and enthalpy (valid for liquid only)
x(p,h)	Vapor quality as function of pressure and enthalpy
rho(p,h)	Density as function of pressure and enthalpy
T(p,h)	Temperature as function of pressure and enthalpy

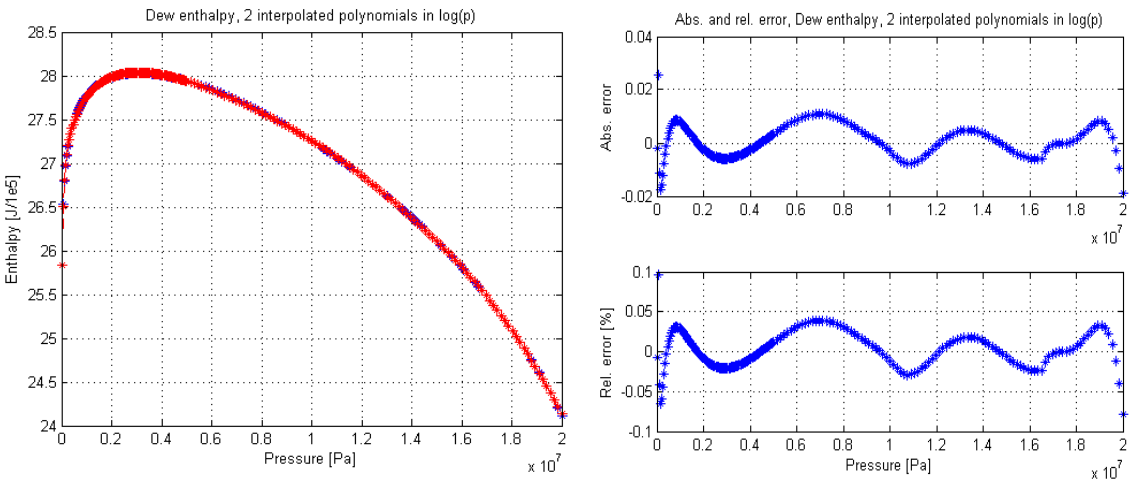


Figure 6 Left: Dew enthalpy from IF97 (blue) and polynomial approximation (red). Right: Absolute and relative error of polynomial approximation.

Figur 6. Vänster: dagg-entalpi enligt IF97 (blå) och polynomanpassning (röd). Höger: Absolut och relativt fel av den polynomanpassningen.

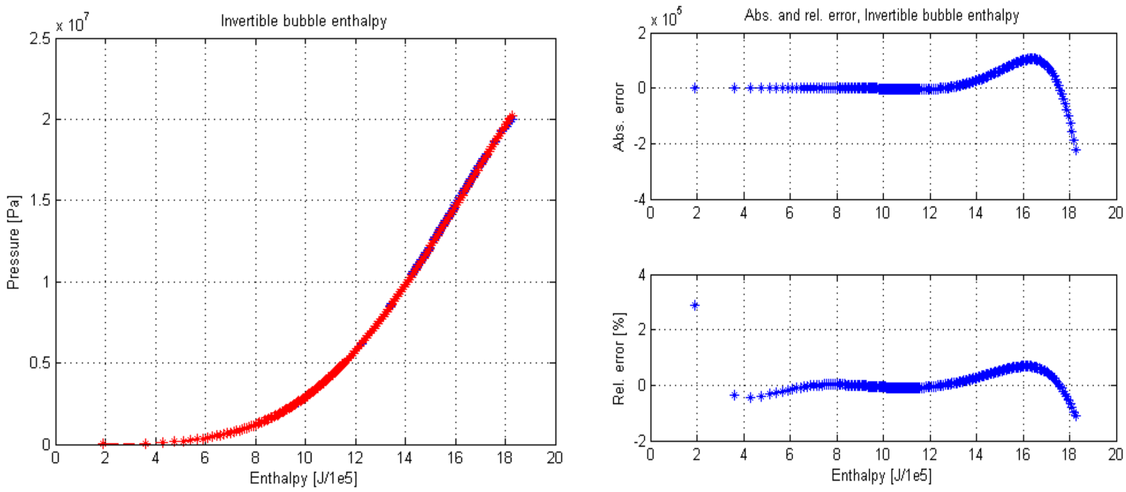


Figure 7 Left: Invertible bubble enthalpy from IF97 (blue) and polynomial approximation (red). Right: Absolute and relative error of polynomial approximation.

Figur 7. Vänster: Inverterbar function för bubble-entalpi från IF97 (blå) och polynomanpassning (röd). Höger: Absolut och relativt fel av polynomanpassningen.

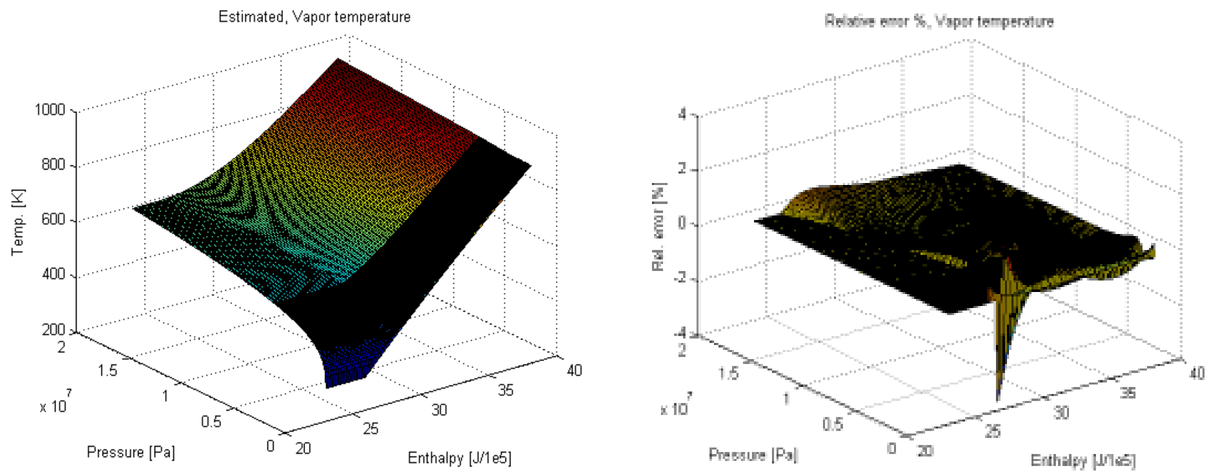


Figure 8   Left: Approximated vapor temperature. Right: Relative error of approximation.

Figur 8.   Vänster: Approximation av ångtemperaturen. Höger: Relativt fel av approximationen.

5.1.4 Model Calibration

The model calibration has mainly been concerned for the co-generation plant P3.

5.1.4.1 Calibration of Co-generation Plant P3

The combined heat and power plant P3 has been validated using design data at different loads: 100%, 75%, 50% and 30%. For the different loads, the following settings were applied:

- Closed turbine by-pass valve
- Constant temperature on return water
- Return water mass flow rate varied.

The model does not contain many calibration parameters as it is mainly based on physical sub-models. The main calibration parameters set manually are:

- Turbine stage:
  - Nominal value of pressure, density and mass flow rate at one nominal point
  - Constant mechanical and isentropic efficiency
- Lumped turbine bleeds
  - Pressure loss coefficient
  - Value of pressure source

Model results compared to design data after calibration can be found in
Table 6. The greatest relative error is at low operating load and is approximately 8%.

Table 6 Design data, electricity production $P_{el,}$ heat Q and α value, compared to simulation results for P3 for different operating conditions.

Tabell 6 Designdata, el-produktion $P_{el,}$, värmeproduktion Q och α-värdet, jämfört med simuleringar av panna 3 vid olika laster.

Op. load	Variable	Design data	Dymola model	Rel. error
100%	P_{el} [MW]	34.8	34.4	-1.1%
	Q [MW]	58.6	60.1	2.6%
	α value	0.60	0.59	-1.7%
75%	P_{el} [MW]	26.1	25.1	-3.8%
	Q [MW]	45.8	45.3	-1.1%
	α value	0.55	0.57	3.6%
50%	P_{el} [MW]	16.7	16.1	-3.6%
	Q [MW]	32.7	31.3	-4.3%
	α value	0.52	0.51	-1.9%
30%	P_{el} [MW]	8.3	9.0	8.4%
	Q [MW]	22.0	21.9	-0.5%
	α value	0.41	0.38	-7.3%

5.2 UCP Model

The discrete time plant model, i.e., the UCP model, is very coarse compared to the EDP model. All heat producing units i, except for P3, are modeled only by the produced heat Q_i and its minimum and maximum value. The influence of plant actuators such as pumps, valves and splitters or the influences of variables such as temperature and mass flows are not described in the chosen simple formulation. The modeling of P3 concerns both heat and electricity production, requiring a slightly more advanced model as described in Section 5.2.1 below. The fuel consumption U_i of a pure heat unit is calculated using the efficiency parameters η_i in Table 2 as

$$U_i = \frac{Q_i}{\eta_i}$$

and for P3 it is calculated as

$$U_3 = \frac{Q_{P3} + P_{el}}{\eta_3}.$$

The Beriden cooler may not be modeled in a simple way as it is temperature dependent and is therefore left out, while the accumulator energy E_{acc} is given by a simple integrator equation as

$$E_{acc}[t] = E_{acc}[t-1] - hQ_{acc}[t-1]$$

where $Q_{acc}[t]$ is the energy flow from the accumulator, h is the sample time and t is the time index.

5.2.1 Co-generation Plant P3

The EDP model of P3 has two control variables: the fuel load and the turbine by-pass valve control signal. This modeling is too complex for the UCP, where the decision variables should be generated heat Q_{P3} and electric power P_{el}. Thus, there is a need for a mapping of the boundary conditions and control variables to Q_{P3} and P_{el}. Using the physics based EDP model of P3 and varying the two control inputs and the condenser water flow temperature within the ranges found in Table 7, a feasible region in the Q_{P3}-P_{el}-plane could be constructed, see Figure 9 below. Note that the slope of the data in the upper left plot is the α-value at a fixed by-pass valve of 50%. The resulting feasible region can be described using four inequalities of the form

$$P_{el} \geq (Q_{P3} - X_o)\frac{Y_1 - X_1}{Y_0 - X_0} + X_1$$

where the points (X_0, Y_0) and (X_1, Y_1) are extreme points, i.e., corner points, of the polytope.

The co-generation plant is in summary described by a region in the Q_{P3}-P_{el}-plane. The UCP optimization should optimize Q_{P3} and P_{el} with the constraint that they should be in the feasible region described by the inequalities.

Table 7 Control variables and temperature ranges for P3 mapping.

Tabell 7 Reglervariabler och temperaturområde för P3 avbildningen.

	Minimum value	**Maximum value**
Valve opening (%)	0	100
Fuel load (%)	20	100
Condenser water flow temp. (°C)	37	50

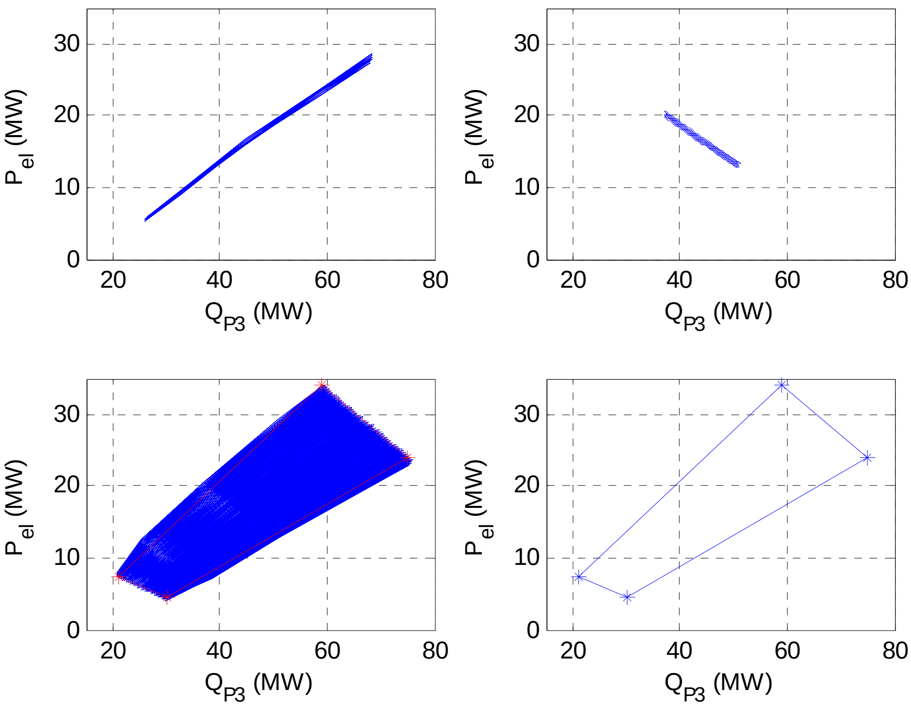


Figure 9 Feasible region in the Q_{P3}-P_{el}-plane for P3. Upper left: Varying fuel load and condenser water temperature with fixed by-pass valve at 50%.Upper right: Varying by-pass valve and condenser water temperature with fixed fuel load at 60MW. Lower left: Varying fuel load, by-pass valve and condenser water temperature and the selected feasible region. Lower right: Feasible region selected for P3.

Figur 9. Panna 3 giltighetsområde i Q_{P3}-P_{el}-planet. Övre vänstra figuren: halvt öppen by-pass ventil, varierande bränsleflöde och kondensortemperatur. Övre högra figuren: konstant bränsleflöde (60MW) och varierande öppningsgrad för BP-ventilen och varierande kondensortemperatur. Nedre vänstra fig.: Varierande bränsleflöde, BP-ventil och temperatur i kondensorn samt den utvalda giltighetsregionen (röd). Nedre högra fig: giltighetsområde för panna 3.

5.3 Heat Load Prediction Model

The heat to be produced and sent to customers is supposed to be predicted using a heat load prediction model provided by Vattenfall AB, see [10] and [11]. The predictions corresponding to the measurement data available have been produced by Vattenfall AB and are 24 h long. However, the predictions, when compared to the true outcome of the customer heat load, were not accurate enough to be used when evaluating the optimization framework using measurements of e.g., unit loads. Figure 10 shows predictions and the measured customer heat load and that the deviation between them varies between 0-30MW. Instead, the real measured customer heat load, corrected regarding transport delay to customers, will be used as predictions of required heat load at the plant at a certain time. Throughout the report, these time series will however be referred to as heat load predictions.

The load prediction model was designed with the purpose of supply temperature control, where there is a need of predicting the future customer load 2 hours ahead, due to transport time in the grid, to deliver the optimal supply temperature at customers. The model is difference based and estimates normal changes in load and outdoor temperature. It predicts future changes, modified with the current temperature prognosis. The starting point for the 24 hour prediction is the latest measured load and outdoor temperature, which works well for the changes in the nearest 2 hours (for which it was developed), but it turns out to not be accurate enough for a 24 hour prediction horizon. For a 24 hour prediction some modifications would improve the results. Suggested improvements would be to put less weight on the latest measurements as starting point and more weight on averages for the last period as the prediction horizon becomes longer.

The predictions are time series with 15 min sampling time. For optimization, it is preferable that all equations are at least -continuous. To reach this preference for the heat load predictions, polynomial approximations are performed of the time series. A prediction time series and a polynomial approximation are shown in Figure 11. The polynomial approximation is used in both the UCP and EDP optimizations.

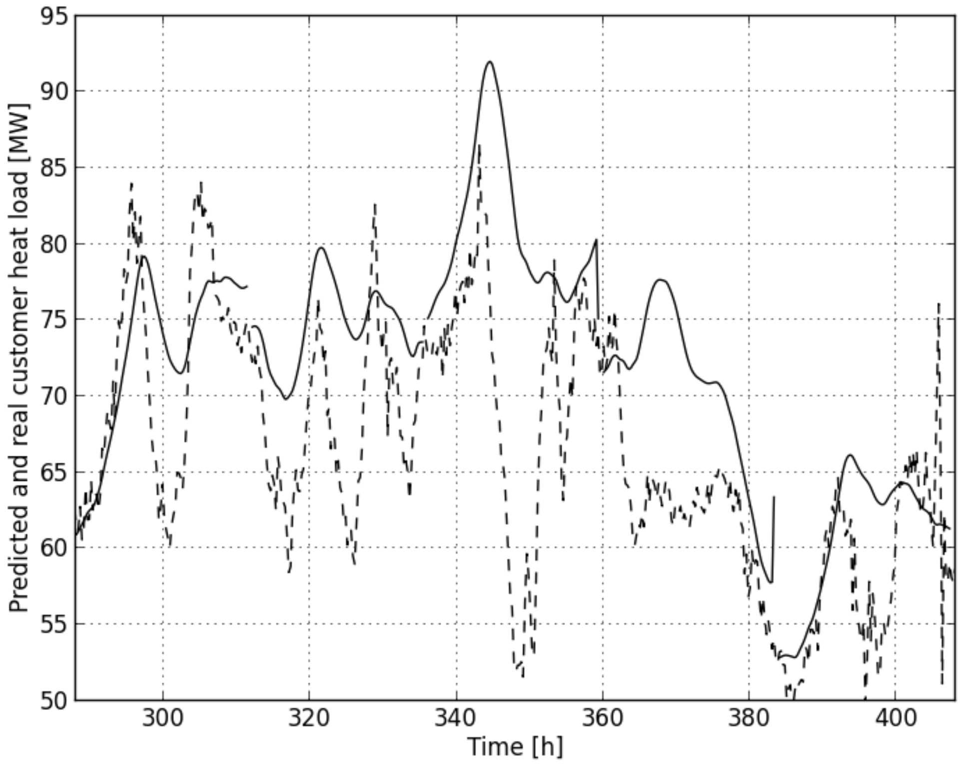


Figure 10 Predictions (solid) and measured (dashed) customer heat load. The predicted customer heat load is calculated every 24th hour, starting at 288 h (12 days) into data set 2013-01-20–2013-02-08.

Figur 10. Predikterad (hel) och uppmätt (sterckad) kundlast. Den predikterade lasten beräknas var 24de timme från t=288h (12 dagar) till dataserien 2013-01-20–2013-02-08.

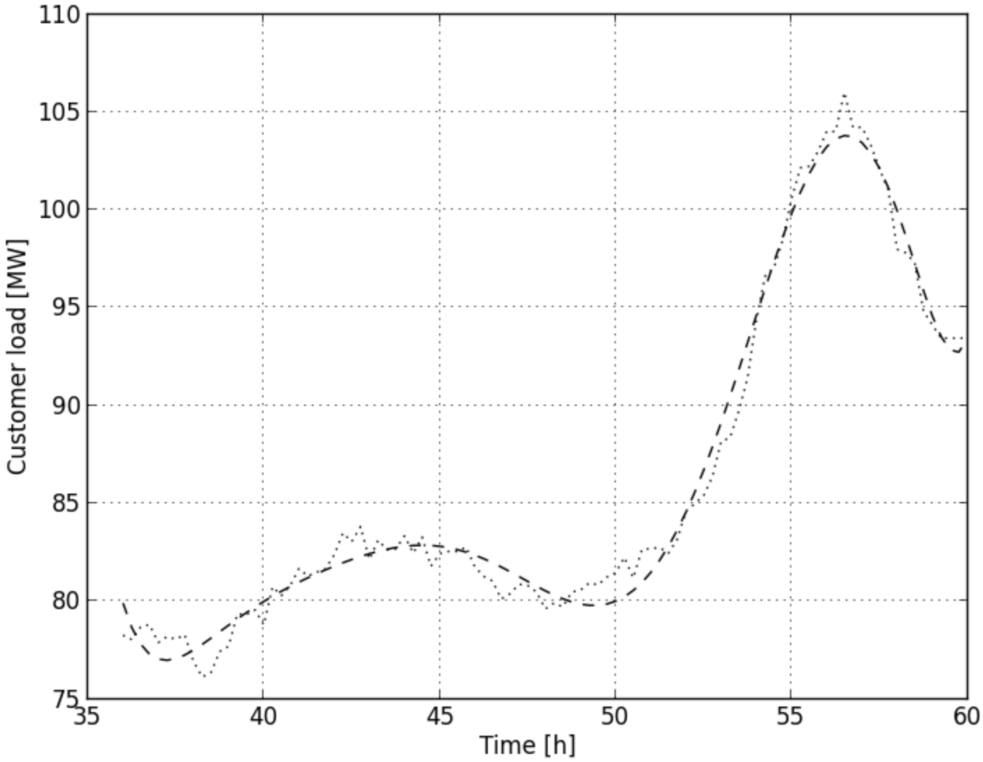


Figure 11 Example of customer heat load prediction (dotted) and polynomial approximation (dashed).

Figur 11. Exampel på kundlastprediktion (prickad) och polynomapproximation (streckad).

5.4 Prices

The UCP and EDP optimizations will regard plant economics but only the fuel prices and sell prices of heat and electricity will be considered together with start-up costs. Thus, costs that are neglected are for instance maintenance and general operating costs such as pumping costs. The fuel and sell prices are based on information from Idbäcken plant and Vattenfall AB, but are normalized with electricity sell price. The start-up cost of P1 has been estimated from operation history and oil price while the start-up costs for the other units have manually been set with P1 start-up cost as reference. The start-up costs are normalized with P3 start-up cost. Table 8 shows the prices and start-up costs used in the UCP and EDP optimizations.

Table 8: Fuel prices for the different units and sell prices for electricity and heat, all normalized with the electricity price. Start-up costs for the different units normalized with P3 start-up cost.

Tabell 8. Bränslepris för pannorna samt salupris för el och värme. Alla priser är normaliserade med el-priset. Uppstartskostnaderna är normaliserade med uppstartkostnaden för panna 3

Unit	Fuel price
P1	0.35
P2	0.35
P3	0.20
EÅP	0.67
LAS	0.83
BRA	0.83

Type	Sell price
Electricity	1
Heat	0.67

Unit	Startup-cost
P1	0.5
P2	0.5
P3	1
EÅP	0.1
LAS	0.1
BRA	0.1

6 Optimization Formulations

6.1 UCP Optimization Formulation

In this section, the formulation of the unit commitment optimization problem is given an overview. The resulting optimization problem can be formulated as a mixed integer linear program (MILP) in order to solve for the discrete variables that are inputs to the EDP. A more detailed treatment of the optimization formulation can be found in [16].

6.1.1 Degrees of Freedom

The variables that can be used in the UCP optimization are either continuous, meaning that they can take any value with user specified bounds, or binary, meaning that they can only take the values 0 or 1. In either case, none of the types are continuous in time, and all models, constraints and cost functions are discretized in time with a sampling period of h. The time *t* will thus here indicate sample time.

The main degrees of freedom for each heat producing unit i in the UCP optimization are

- $Q_i[t]$ – heat produced by the unit.
- $\Delta Q_i[t]$ – change per sample in heat produced by the unit. Used for constraining the change per sample.
- $P_{el}[t]$ – electricity produced by the unit. Only applies to P3.
- $\Delta P_{el}[t]$ – change per sample in electricity produced by the unit. Only applies to P3. Used for constraining the change per sample.
- $status_i[t]$ – status (on/off) of each unit. 1 when on, 0 otherwise.

The heat and electricity produced by each unit *i,* and their change per sample, are continuous variables while the status variables are binary variables. Each of these variables is initialized using measurement data at an optimization.

For handling of starting and stopping ramps of the units, formulated as constraints, additional decision variables are needed for each heat producing unit *i*. These binary variables are

- $start_i[t]$– variable indicating the time point for when a unit is beginning its starting sequence by being equal to 1 at that sample, 0 otherwise.
- $stop_i[t]$– variable indicating the time point for when a unit is beginning its stopping sequence by being equal to 1 at that sample, 0 otherwise.
- $starting_i[t]$– variable indicating if a unit at the moment is following a starting sequence by being equal to 1, 0 otherwise
- $stopping_i[t]$– variable indicating if a unit at the moment is following a stopping sequence by being equal to 1, 0 otherwise.
- $overlap_i[t]$ – variable used for setting constraint so that a unit is not starting and stopping at the same time.

The accumulator, modeled as a simple integrator in Section 5.2, has the flow $Q_{acc}[t]$ as a decision variable.

6.1.2 Constraints

The main constraints that are used in the UCP formulation are given below. Full descriptions of all constraints needed are found in [16].

The capacity constraints of each running heat producing unit, except for P3 that is constrained by it operating region, are formulated as

$$Q_{i,min} \leq Q_i[t] \leq Q_{i,max}$$

where $Q_{i,min}$ and $Q_{i,max}$ are minimum and maximum heat production of the unit, respectively. For the flue gas condenser, the upper and lower limits are the same as long as P3 is running, i.e., the flue gas condenser is modeled such that it cannot change it heat production.

Maximum and minimum change of the heat produced for each unit is formulated as

$$\Delta Q_{i,min} \leq \Delta Q_i[t] \leq \Delta Q_{i,max}$$

where $\Delta Q_{i,min}$ and $\Delta Q_{i,max}$ are minimum and maximum change per sample.

The start and stop of large solid fuel boilers is time-consuming and it is therefore critical to model the delay between the start/stop decision and the time the boiler is running at minimum/zero load. All units that are starting or stopping are required to follow a pre-defined trajectory $Q_{i,start}[t]$ or $Q_{i,stop}[t]$, respectively. The constraints are formulated as

$$Q_i[t] = Q_{i,start}[t], \quad t \in [t_{start}, t_{start} + t_{startdelay}]$$
$$Q_i[t] = Q_{i,stop}[t], \quad t \in [t_{stop}, t_{stop} + t_{stopdelay}]$$

where $t_{startdelay}$ and $t_{stopdelay}$ are the lengths of start and stop sequences in time. The details on how the timing variables $start_i[t]$, $stop_i[t]$, $starting_i[t]$ and $stopping_i[t]$ relate to the starting and stopping sequence equations above can be found in [16].

The accumulator storage capacity and heat flow are constrained with minimum and maximum values as

$$E_{acc,min} \leq E_{acc}[t] \leq E_{acc,max}$$
$$Q_{acc,min} \leq Q_{acc}[t] \leq Q_{acc,max}.$$

Note, however, that the limits are neither temperature nor flow dependent. Also, the accumulator energy is given an end point constraint of

$$E_{acc}(t_f) \geq E_{acc}(t_0),$$

meaning that the energy at the optimization end time t_f should be greater than or equal to the energy at the optimization start time t_0. This constraint prevents the accumulator to be at minimum energy at optimization end which would be the case otherwise as this energy is seen as free by the optimization, i.e., there is no production cost of it. This type of constraint is also reasonable as there exists periodic elements in the heat demand and that a buffer of energy at the end is beneficial if unexpected customer loads occur.

Fulfilling the customer heat demand up to a certain degree can be formulated by setting a constraint on the heat delivered. Denoting the predicted heat demand with Q_d, a straightforward constraint is

$$-Q_{dev.max} \leq Q[t] - Q_d[t] \leq Q_{dev.max}$$

where Q is the total heat produced by the units and $Q_{dev.max}$ is the maximum deviation of produced heat from the predicted heat demand that is acceptable. In the optimizations performed, the maximum deviation is set to 1 MW.

6.1.3 Cost Function

The cost function expresses the economics of the plant regarding fuel prices and sell prices of heat and electricity. Additionally, also the price of starting a unit must be added as a startup may use additional expensive fuel and no or limited heat flow is provided to the district water. The revenue $R[t]$ at a particular time instant is thus

$$R[t] = P_{el}[t]p_{el} + Q[t]p_Q - \sum_{\substack{i=P1,P2,P3,\\ BRA,LAS,EÅP}} (U_i[t]p_i - start_i[t]s_i),$$

where p_{el} and p_Q are sell prices for electricity and heat, respectively, p_i is the fuel price for unit *i* and s_i is the startup price for unit *i*. The prices used are shown inTable 8 on page 27. Thus, the cost function to minimize during optimization is the cumulative revenue, defined as

$$J = -h \sum_{t=t_0}^{t_f} R[t]$$

where t_0 and t_f are start and end time of the optimization interval. The length of the optimization interval considered are 24 h, which is as long as the prediction time series provided by the heat load prediction model.

6.1.4 Optimization with PuLP

The scripting language of Python together with the package PuLP is used when formulating the resulting MILP of the UCP. PuLP is a light weight package that allows modelers to easily express math programming problems, including mixed integer linear programs, see [17]. It uses a high level modeling language and has been built to interface with separate solvers such as GLPK, COIN, CPLEX, CBC and GUROBI. In the work presented in this report, the solver CBC has been used, see [18].

6.2 EDP Optimization Formulation

The continuous time optimization formulation for solving the EDP is presented in the following sections.

6.2.1 Degrees of Freedom

The EDP model inputs are the following:

- U_{P1}, P1 fuel load
- U_{P2}, P2 fuel load
- U_{P3}, P3 fuel load
- U_{P3v}, P3 by-pass valve position
- U_{LAS}, Hospital (Lasarettet) (LAS) fuel load
- U_{EAP}, Electrical boiler (Elektrisk ångpanna) (EÅP) fuel load
- U_{BRA}, Brandkärr (BRA) fuel load
- U_{pump1}, Speed of pump taking water from P3 (internal pump)
- U_{pump2}, Speed of pump taking water from BRA outlet and accumulator top (distribution pump)
- U_{split1}, Flow splitter ratio to split flow between going to customer and by-passing customer (net by-pass splitter)
- U_{split2}, Flow splitter ratio to split flow between going to Beriden cooler and by-passing Beriden cooler (Beriden by-pass splitter)

In the optimizations however, the decision variables used by the optimization routine will be the derivatives of the above inputs. Thus, an equation on the form

$$U_j(t) = \int_t \dot{U}_j(t)dt$$

is introduced for each input. This extension makes it easy to set minimum and maximum constraints on the input signal derivatives in the optimizations. In Modelica code, for a general input u with minimum and maximum values u_{min} and u_{max} and minimum and maximum derivative values $\dot{u}_{min}$ and $\dot{u}_{max}$, an example is the following:

```
model inputExample
   parameter Real u_min = -1;
   parameter Real u_max = 1;
   parameter Real u_der_min = -1;
   parameter Real u_der_max = 1;
   input Real u_der(min = u_der_min, max = u_der_max);
   Real u(min = u_min, max = u_max);
equation
   der(u) = u_der;
end inputExample.
```

Another possibility is to utilize the `constraint`-section in the optimization model as shown in the example in Section 8.2. However, it is more efficient to set minimum and maximum values of the variables as above as this effectively cuts off the search space for the optimization algorithm.

6.2.2 Control Input Test Cases

Before formulating any cost function regarding plant economy all inputs have been tested in optimization by performing optimal change of input from one value to another using the following quadratic cost function

$$J = \int_{t_0}^{t_f} \left(q_U \left(U_j - U_{j,ref}\right)^2 + q_{\dot{U}} \dot{U}_j^{\,2}\right)\, dt.$$

This has given insight on e.g., variable and model equation scaling, but it has also been important so see that all inputs, as the model is currently structured, can be used in optimization.

Note that a cost on the derivative (optimization/decision variable) is used. If such a cost was not present, the optimization problem would not be well behaved as, practically, the input could move arbitrarily without any additional cost. In fact, the Jacobian of the cost function would be singular for the decision variable $\dot{U}_J$. Even though the optimization problem considered here is performed on non-linear models, one can refer to the linear quadratic optimization theory, which states that the weight in the cost function on the input should be positive definite, i.e., in this case $q_{\dot{U}} > 0$.

6.2.3 *Optimization Problem*

A quadratic cost function as shown in the previous section can be hard to design, i.e., to choose the variables to be used in the cost function and the corresponding weights, from a perspective of plant economics. The goal of the optimization, both in the UCP and EDP, is to produce enough heat to follow the customer heat load over time and at the same time do it as economically beneficial as possible. The economics of the plant are simplified in the EDP to consider only fuel costs and incomes from selling heat and electricity to customers. The stopping and starting of units have already been decided upon in the UCP problem and considered constant in the EDP problem. Hence, the starting costs do not need to be added in the cost function used in the EDP and the revenue at a time instant can thus be formulated as

$$R(t) = P_{el}(t)p_{el} + Q(t)p_Q - \sum_{\substack{i=P1,P2,P3,\\ BRA,LAS,EÅP}} U_i(t)p_i,$$

A minor cost on the input derivatives must be used, as mentioned in the previous section. This cost at a certain time instant is formulated as

$$\dot{W}(t) = \sum_{\substack{j=All\ model\\ inputs}} q_{\dot{U}_j} \dot{U}_j(t).$$

where $q_{\dot{U}\,j}$ is the weight for derivative $\dot{U}_j$.

The cost function to be minimized, considering the cumulative revenue over the optimization horizon, can thus be formulated as

$$J = \int_{t_0}^{t_f} (\dot{W}(t) - R(t))\, dt$$

where the optimization interval is $[t_0, t_f]$ and 24 h long. This is as long as the optimization horizon in the UCP and also as the prediction time series provided by the heat load prediction model. Without the extra term of the input derivatives, this cost function is the continuous time counter part of the cost function in the UCP except for the start-up costs.

For fulfillment of the heat demand from the customers, the same type constraint is used as in the UCP. That is, the constraint

$$-Q_{dev.max} \leq Q(t) - Q_d(t) \leq Q_{dev.max}$$

is introduced where Q now is the heat delivered to the customers. The maximum deviation is as in the UCP set to 1 MW. Note that there is a minor difference compared to the UCP where Q was the total heat produced at a certain time instant. With the higher fidelity model of the EDP, it is possible to use the heat delivered to the customers.

Without the optimization procedure, the customer temperature control curve is pre-determined to an almost straight line between 74.5°C and 110°C when the outdoor temperature varies. However, in the optimization performed only minimum and maximum values of 74.5°C and 110°C, respectively, are set. Also the flow to the customers, essentially the distribution pump flow, is given minimum and maximum values. These are 0 kg/s and 550 kg/s. The flow and the temperature are selected by the optimizer such that the correct heat flow to customers is given. Optimizations have been performed with maximum flow rate set significantly higher, which yielded a very low customer temperature showing that the optimizer is trying to reduce the produced heat, i.e., the spent fuel.

The input derivatives, i.e., the decision variables for the optimizer, are given minimum and maximum values. A maximum change of ±2%/min. has been set for all input derivatives relative to the maximum value of the input to the model. These limits can be set both from a physical perspective but also from a numerical perspective such that highly changing control signals are avoided. In the optimization results presented in this report, the derivative constraints are far from being active.

The accumulator energy end-point constraint introduced in the UCP for not allowing the accumulator to be emptied of energy, i.e.,

$$E_{acc}(t_f) \geq E_{acc}(t_0),$$

is also introduced in the EDP.

The statuses of the units optimized in the UCP are transferred into the EDP formulation as inequality constraints. How this transferring is performed is covered in detail in Section 6.3.

If the differential-algebraic equations of the model are expressed as $0 = F(\dot{x}, x, w, U)$, with x, w and U as states, algebraic variables and inputs, respectively, then the optimization problem can generally be expressed as

$$\min_U \; J = \int_{t_0}^{t_f} \left(\dot{W} - R\right) dt$$

$$\text{s.t. } 0 = F(\dot{x}, x, w, U)$$

$$U = \int_t \dot{U} dt$$
$$0 \leq g(x, w, U, t)$$

where a constraint function $g(x, w, U, t)$ formulates all inequality constraints.

As can be seen, the cost function does not purely consider plant economics as there is a cost on the input derivatives as well. However, in all optimizations, the ratio $\dot{W}/R$ is less than 0.1, and in most cases also less than 0.05.

6.3 Integration of UCP and EDP

The result of the discrete time optimization is the statuses of the units, either 0 (off) or 1 (on), the heat production of each unit and electricity production of P3. The main result to be transferred to the dynamic optimization problem is however only the variables considered discrete in the dynamic optimization problem, i.e., the statuses, which at this moment cannot be handled efficiently if free in the dynamic optimization problem. These variables will be considered constant in the EDP and not to be optimized in the dynamic optimization problem, i.e., they are pre-determined.

The status of each unit determines how much heat, and electricity if P3 is considered, a unit can produce and if the unit is in start-up or shut-down mode and must follow a certain pre-determined trajectory. The produced heat is in the dynamic optimization problem a continuous variable and cannot easily be set to follow a certain pre-determined trajectory. Instead, time varying minimum and maximum values may be set.

An example of status result from discrete time optimization and how time varying minimum and maximum constraints are set on the heat production of P2 is shown in Figure 12. During shut-down of P2, produced heat should follow a trajectory that first is 15MW and then go steep towards 0 MW. During start-up, the produced heat should go steep towards 15MW and then be at 15MW during a specified time. When started, a minimum production of heat is 15 MW, while an upper limit is set by the maximum capacity of the unit. This start and stop procedure has been found in measurement data from P2. The time varying minimum and maximum constraints used in the dynamic optimization problem are set by subtracting and adding 1 MW, respectively, to the desired start-up and shut-down trajectories, see lower plot in Figure 12. When the status is 0, the maximum is 1MW while the minimum is -1 MW which effectively is 0 MW as the produced heat cannot be negative. When the status is 1, the minimum is 14 MW, which is the minimum load of the unit when on, and the maximum is set well above the maximum set by the unit model itself. In the figure, also the result of the dynamic optimization is shown and it is seen that the produced heat of P2 is within its minimum and maximum values the whole time.

The time varying constraints are implemented using time tables in the Modelica optimization model. Linear interpolation is performed between the entries in the table to provide constraint values at all time-points. This type of interpolation is not C^2-continuous, but since time varying constraints are only set on the units that have changing status during the optimization interval, this is not a major problem. If, however, more time varying constraints are to be implemented, one should consider a -continuous interpolation implementation.

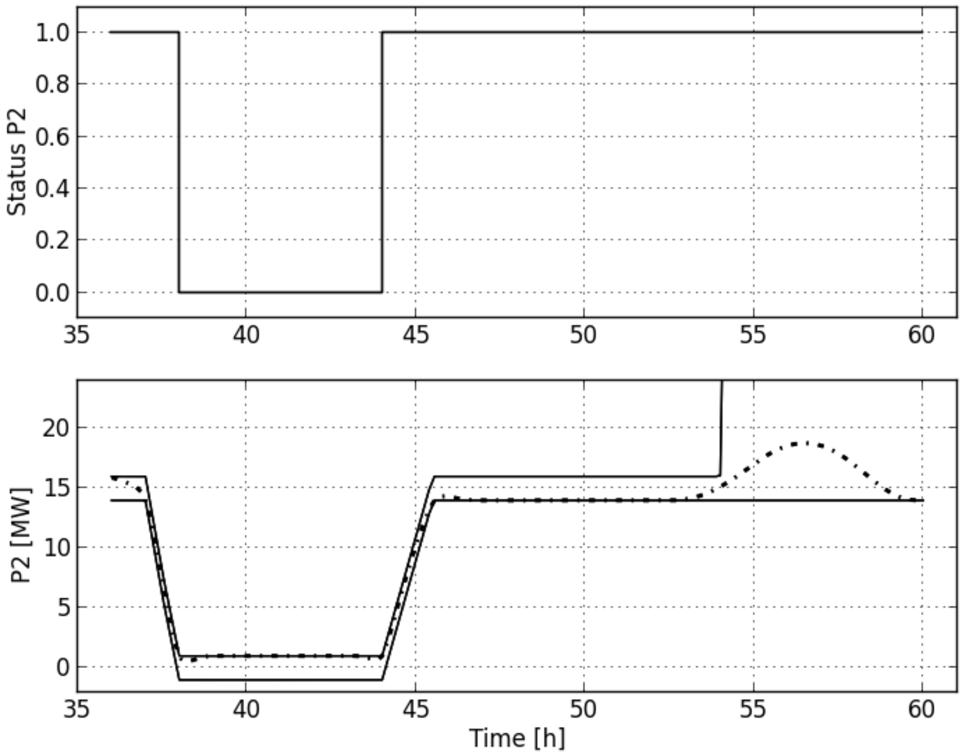


Figure 12 Top: Example of status change of P2. Bottom: Constraints on and resulting heat flow of P2 in a continuous time optimization. The load of P2 is limited to 15 MW when on.

Figur 12. Övre: Exampel på förändring i tillstånd för panna 2. Nedre: bivillkor och resulterande värmeflöde för P2 i den kontinuerliga optimeringen. När panna 2 är i drift, lasten är under-begränsad till 15 MW.

7 Model Initialization Using Measurement Data

Using measurement data provided by Idbäcken and Vattenfall AB, both the EDP and UCP models are initialized before optimization is performed. The EDP model uses more data than the UCP model due to higher fidelity and the additional dynamics. Below is a summary of which variables are initialized and how they are initialized.

7.1 UCP Model

The UCP model, with its crude production unit models, only contains the status and produced heat and electricity of each production unit. These variables can be directly initialized from measurement data. If the measurement data show that the produced heat of a unit is less than 0.1 MW, it is assumed that the produced heat of that unit is 0 and that its initial status is 0 (off). Otherwise, the status is set to 1 (on) and the produced heat/electricity is initialized to measurement data value. This procedure is performed such that negative values in measurement data of produced heat are ignored and a certain threshold is applied.

Using temperature measurements and a fixed reference temperature, the accumulator initial energy is computed and set.

7.2 EDP Model

The EDP model is richer than the UCP model and more data can therefore be used when initializing it.

Similar to the UCP model initialization, it is assumed that when measurement data shows a heat production less than 0.1 MW, the unit is assumed off and the heat produced in the model is set to 0 MW. However, the continuous time model does not directly use the produced heat as input at optimization as it depends on how each unit is run. Instead, the heat of each unit is set from measurement data and from that a non-linear equation system is constructed concerning the heat of each unit and the fuel load to each unit. Thus, when the equation system is solved, the result is the fuel loads of the units that give the specified heats. For P3, as it also produces electricity and has the turbine by-pass valve, the produced heat and electricity is set from measurement data and the result from the equation system is the load and the turbine by-pass valve position. See Figure 9 on page 24 that shows how fuel load and by-pass valve position relates to produced heat and electricity.

The Beriden by-pass splitter, providing flow to Beriden when open, and the net by-pass splitter, by-passing the heat district net when open, are both assumed closed at initialization. The by-pass splitters are very fast compared to the unit dynamics which makes any introduced errors by these assumptions minor.

The heat delivered to the customers depends on both the flow to the customer and the temperature of the water to the customers. Heat (as prediction), flow and the temperature are available in the measurement data. However, they are not always consistent due to measurement fluctuations, heat losses, prediction errors, etc. As the net by-pass valve is assumed closed at initialization, the customer flow is the same as the distribution pump flow, which is selected for initialization as this can be used to calculate the pump control signal as well. Using the predicted customer heat as well when initializing, the temperature of the flow is calculated. Hence, when analyzing the initialization, the customer flow and provided heat will align with measurement and prediction data, while the temperature may differ slightly.

As noted above, the distribution pump flow can be initialized directly from measurement data. However, the internal pump flow cannot be initialized directly from data, as this flow will together with the distribution pump flow determine how much flow will go to the accumulator. That is, they will together affect the provided heat to customers. As the customer heat is already specified, the internal pump flow must be a free variable at the initialization and will be determined such that correct heat flow is sent to the accumulator and to the customers. Hence, when analyzing the initialization, the customer flow will align with measurement data as noted before, but the internal flow may differ slightly. The flows of the pumps are not the control inputs at the optimization; instead it is the pump speed. Hence, for each pump there is an equation system created which involves pump speed and pump flow that needs to be solved at initialization.

The accumulator, frequently used to store and provide heat, is in the model simplified by discretization to two volumes of equal sizes and different temperatures. From measurement data, top and bottom temperatures are used for initialization of the two volumes.

The return pipe, which is discretized using four equal size volumes with different temperatures, is initialized using the measured temperature after Beriden for all volumes. As the Beriden by-pass valve is closed, this temperature is used for return water as well. From suggestions of Vattenfall AB, the return water temperature is held constant to this temperature during the optimization performed after the initialization.

All equation systems that are created at the initialization, i.e., the calculation of loads, pump speeds, and valve/splitter positions from heat and electricity production and flows, are all solved simultaneously using an FMU. Initial equations expressing the desired conditions above are constructed and the FMU is then initialized resulting in the solution for the inputs to use at optimization start time.

8 Simulation and Optimization with JModelica.org

Several different environments exist for simulation of Modelica models and platforms for optimization of these models are emerging as well. JModelica.org is an open-source platform for both simulation and optimization of Modelica models and an overview of it is given below, see also [14].

8.1 JModelica.org

The Modelica language has been developed with the aim of simulation and simulation tools have become very sophisticated during the last decades. Now, focus is also directed towards using the models for optimization. Support has started to emerge in various forms, e.g., model integration tools that interface several design tools for analysis, simulation and optimization, modeling tools with limited optimization add-ons, and numerical optimization packages. Using state of the art numerical solvers is not straight forward as numerical packages often require high level of model details, including derivatives and sparsity structures. Additionally, users are often required to be skilled since the original optimization problem often as to be transcribed in some way.

Dynamic optimization is an iterative process. To try different solution methods, it is beneficial if the optimization problem is stated in a high-level fashion, directly associated with the mathematical formulation, with the option of selecting the solution approach. One software package that supports this is JModelica.org, an open-source platform that is well integrated with Modelica models. JModelica.org uses Optimica, which is additional language constructs to the Modelica language for formulation of optimization problems. This makes it possible to formulate optimization problems in a high-level fashion structured as a Modelica model. JModelica.org bridges the gap between simulation models expressed in a high-level language and state of the art optimization algorithms.

JModelica.org contains, among others, a collocation based optimization method. The non-linear program (NLP) resulting from the discretization of the differential-algebraic equation (DAE) is solved using the state of the art NLP solver Interior Point Optimizer (IPOPT), see [19].

8.2 JModelica.org Code Example

To get insight into how an optimization problem is formulated in Optimica code, a small example is given. Consider the optimization problem

$$\min_u \int_0^1 (x^2 + u^2) dt$$
$$\text{s.t.} \, \dot{x} = -x + u$$
$$-1 \le u \le 1$$
$$x(0) = 0.5$$

The problem can be formulated in a Modelica model that is augmented with Optimica constructs as Figure 13 below. The Optimica specific constructs are the new class type `optimization` and class attributes `objective`, `startTime` and `finalTime`, which are directly linked to the optimization cost function. Also the section `constraint` is specific for Optimica, making it possible to set constraints between variables in the model. Note that minimum and maximum constraints on variables can be set directly when a variable is declared or in the constraint section. The constraint section may also contain significantly more advanced constraints involving any variables in the model.

Note that it is easy to construct the Modelica model in a tool such as Dymola and then create an optimization formulation by extending the Modelica model and add the Optimica specific constructs. This is particularly useful when working with large models and has been used in the work presented in this report. The small example above is written in this format and shown in Figure 14 below.

```
optimization SimpleOptimization(objective=cost(finalTime),
                                startTime=0,
                                finalTime=1)
  Real cost(start = 0, fixed=true);
  Real x;
  input Real u(min = -1);
initial equation
  x = 0.5;
equation
  der(x) = -x +u;
  der(cost) = x^2 + u^2;
constraints
  u <= 1;
end SimpleOptimization;
```

Figure 13 Code for simple JModelica.org optimization example.

Figur 13. Kod för ett enkelt optimeringsexampel i JModelica.org.

```
model SimpleModel
  Real x;
  input Real u(min = -1);
initial equation
  x = 0.5;
equation
  der(x) = -x +u;
end SimpleOptimization;


optimization SimpleOptimization(objective=cost(finalTime),
                                startTime=0,
                                finalTime=1)
  extends SimpleModel;
  Real cost(start = 0, fixed=true);
equation
  der(cost) = x^2 + u^2;
constraints
  u <= 1;
end SimpleOptimization;
```

Figure 14 Code for simple JModelica.org optimization example divided into two models.

Figur 14. Kod för ett enkelt optimeringsexampel I JModelica.org, delat I 2 modeller.

8.3 General Optimization Formulation

Over the last decades, dynamic optimization methods have received significant attention in both academia and industry. It is used both for on-line tasks such as e.g., model predictive control and state estimation, and also for off-line tasks such as trajectory and parameter optimization. The work presented in this report is in the latter category, more specifically, off-line trajectory optimization.

The optimization problem on the time interval $[t_0, t_f]$ may in general be formulated as

$$\begin{aligned} \min_{u(t)} \quad & J\left(x(t_f)\right) \\ s.t. \quad & F\left(\dot{x}(t), x(t), w(t), u(t)\right) = 0 \\ & F_0\left(\dot{x}(t_0), x(t_0), w(t_0), u(t_0)\right) = 0 \\ & C_{ineq}(x(t), x(t), w(t), u(t)) \leq 0 \\ & C_{end}(x(t), x(t_f), w(t_f), u(t_f)) \leq 0 \\ & x(t_0) = x_0, \end{aligned}$$

where x, w, and u are states, algebraic, and control variables, respectively, and x_0 is the initial state. The cost function J to be minimized is scalar, F is the differential-algebraic equation (DAE), essentially the Modelica model considered, and F_0 represent the DAE augmented with additional initial conditions and C_{ineq} and C_{end} are path and end point inequality constraints.

The two main approaches for solving an optimization problem as above are sequential and simultaneous methods, see [20] and [21], where the method of collocation used here is contained in the latter category.

8.4 Collocation

In a collocation method, the optimization problem is fully discretized by approximating states, algebraic variables and control variables by polynomials, resulting in one large non-linear program (NLP). The collocation scheme implemented in JModelica.org and used here is presented in detail in Appendix 0. The resulting NLP from the general optimization formulation in the previous section can be formulated as

$$\begin{aligned} \min_{\bar{x}} \quad & f(\bar{x}) \\ s.t. \quad & h(\bar{x}) = 0 \\ & g(\bar{x}) \leq 0 \end{aligned}$$

where $f(\bar{x})$ is the cost function, $h(\bar{x})$ and $g(\bar{x})$ are equality and inequality constraints, respectively, and $\bar{x}$ is the decision variable. In the work presented here, the NLP is solved using the large-scale solver IPOPT, see [19].

8.5 JModelica.org Workflow

Simulating and optimizing Modelica models are in this project performed in two different environments.

1. Dymola: Plant modeling, simulation, verification, testing, result inspection
2. Python: Scripting for automating compilation, simulation and optimization using JModelica.org and result inspection

Note that simulation and result inspection can be made in both Dymola and JModelica.org

The work flow on the models for optimization is the following:

1. Build model of system to be used in optimization using Dymola.
   a. Perform verification and testing
   b. Set nominal, min and max values for optimization
2. Formulate optimization problem using Optimica constructs
   a. Extend models built in Dymola
3. Simulate an initial trajectory model for initial guess in optimization
4. Set optimization options: number of collocation points, optimization interval length, initial trajectories, etc.
5. Compile optimization model and perform optimization
6. Simulate with resulting input trajectories to verify solution and analyze the optimization result.

8.6 Preparing Models for Optimization

Simulation and simulation tools are more mature than optimization frameworks and tools and can handle more complicated models in terms of functions, variable magnitude spread etc. The model to be optimized must therefore be prepared with this in mind, and in the following some important notions considered for the EDP model are described.

8.6.1 Discontinuous Derivatives

Gradient based optimization routines require twice continuously differentiable functions to function at their best. Functions that contain discontinuous derivatives are common in modeling when models are to be used for simulation only. Such functions are for instance

- *min()*
- *max()*
- *abs()*
- *sqrt()*

Twice continuously differentiable approximations to the above functions have been implemented and used throughout the Nyköping district heat model.

8.6.2 Scaling of Variables

For good numerical properties of the optimization problem, e.g., Jacobian scaling, all variables in the optimization problem should be in the same order of magnitude. In the model, there are for instance pressures of 15 MPa, temperatures of 350 K and pressure ratios of 0.02. Before the model is used by the optimization routine it is scaled by setting nominal values of all variables, i.e., an attribute describing the normal magnitude of the variable. In the Modelica language, this can be made by using the `nominal`-attribute. For convenience, a units-package has been constructed for the Nyköping model where the nominal value is set on all variables with the same unit by only changing the unit declaration. For instance, the unit declaration of pressure is extended by a nominal value of 10^5 by the construction

```
AbsolutePressure =
          Modelica.SIunits.AbsolutePressure(nominal = 1e5);
```

8.6.3 Minimum and Maximum Values of Variables

By actively setting minimum and maximum values of all variables effectively reduces the search space for the optimization algorithm. For instance, a pressure should not be able to have a negative value and a flow might have a maximum or minimum value. Minimum and maximum values can be set in the units package as well, again by extension of the units declaration,

```
AbsolutePressure = Modelica.SIunits.AbsolutePressure(
                                             nominal = 1e5,
                                             min = 0,
                                             max = 200e5);
```

8.6.4 Input Normalization

For simple comparison of the input signals to the plant, i.e., pump speed signals and unit fuel loads, they have all been normalized to be in 0-100%. Additionally, to avoid numerical problems with e.g., zero flow through a valve or a heat production of 0 W in a unit, an offset have been introduced such that 0% on the input signal gives 1% open valve or 1% heat production of the maximum heat production. Linear increase is then applied such that 100% on the input signal gives 100% open valve or 100% of the maximum heat production. The introduced error is small, but helps the optimization significantly.

9 Optimization Examples

9.1 Optimization Settings and Results Presentations

The following sections will show six different optimization cases. The cases are based on the measurement data from Idbäcken plant in Nyköping provided by Idbäcken and Vattenfall AB. All optimizations utilize the problem formulations in Section 6.

All optimizations, both UCP and EDP, are performed using an optimization horizon of 24 h, as this is the length of the provided customer heat load predictions. The UCP sampling time is 0.5 h while the number of elements in the collocation scheme is 72, i.e., the length of each element is 20 minutes.

For each optimization case in the forthcoming sections, explanations of the used decision variables, main features of the result and the economic benefit of optimization will be given together with a short clarification of how the UCP and EDP optimizations suggest the plant to be operated and, if any, the differences between them.

The UCP decision variables in each optimization are found in Section 6.1.1, while the decision variables for the EDP optimization vary and depend on the UCP results. For instance, if a unit is decided to be off during the whole optimization horizon, it is not included in the EDP as a decision variable, but instead set constant off.

The economic benefit of using optimization is calculated by comparing the cumulative revenue from the EDP optimization to the cumulative revenue using the measurement data using the specific fuel, heat and electricity prices in Table 8. That is, the EDP cost function is evaluated using the measurement data. The stored energy in the accumulator is not taken into account in this comparison as the temperature reference level used in measurements seems to be time varying.

The economic benefits shown are evaluated with perfect load predictions as input to the optimization. In reality there will always be some prediction errors. The expected consequences to load prediction errors, when running the plants according to the optimization plan, is that the customers will change their demand and the supply water flow will change from plan. To maintain the supply water temperature according to the optimisation result, the accumulator will balance the difference required. If the optimisation takes into account the range of likely prediction errors the optimisation result will not be allowed to give a plan on the very end of capacity, a margin must be planned for both in heat storage of the accumulator and in supply water flow.

For each optimization case, nine plots will show time traces of both measurement data and UCP and EDP optimization results when applicable. A summary of what the nine different plots will show are found in
Table 9. Note that in the optimization results the electric boiler EÅP and the oil driven boilers LAS and BRA are not shown. The optimizations when using the provided data resulted in no usage of these three producers, i.e., their status where always off, producing no heat in the six optimization cases.

Table 9 Summary of what plots contain for the different optimization cases.

Tabell 9 Sammanfattning för den data som visas i optimeringsfallen.

No	Plot description	Meas.	UCP	EDP
1	Customer heat load	Yes	Yes	Yes
2	Status (on/off) of each producer	No	Yes	No
3	P1, P2, P3, and RGK heat production	Yes	Yes	Yes
4	Internal plant water flow, customer heat district water flow and temperature	Yes	No	Yes
5	Accumulator energy and temperatures.	Yes	Yes (energy only)	Yes
6	Beriden cooling power	Yes	No	Yes
7	P3 heat and electricity production and α value.	Yes	Yes	Yes
8	Valve positions	No	No	Yes
9	Revenue per time unit and cumulative revenue	Yes	No	Yes

9.2 Computational Statistics

For all optimization cases that have been run, the UCP optimization formulation in Section 6.2 results in a MILP of approximately 4000 decision variables and 7000 constraints. The sampling time considered is 30 minutes and the optimization horizon is 24 h. With the CBC MILP solver, see [18], the solution time is less than 25 s for all optimization cases.

The resulting NLP, after discretization of the EDP optimization problem using 72 elements each of a length 20 minutes, is solved using IPOPT v3.10.0 running with the linear solver MA27, see [19] and [22]. The NLP contains approximately 130 000 variables for each of the considered optimization cases, which can be considered a small to medium sized problem for IPOPT. Much larger problems have been solved using the JModelica.org framework and IPOPT, see [23] and references therein. The solution time is approximately 2-5 minutes and depends on e.g., initial guesses number of decision variables and number of active constraints.

9.3 Case 1: Low Customer Demand and Beriden Cooling for Maximum Electricity Production I.

Data and figures

The measurement data used is 2013-02-30 00:00 - 2013-02-31 00:00 and the measurement data and optimization results can be found in Figure 15-Figure 23.

Decision variables

- UCP: See Section 6.2.1
- EDP: Derivatives of
  - P3 fuel load
  - Turbine by-pass valve position
  - District network by-pass valve position
  - Beriden by-pass valve position
  - Internal pump speed

- o Distribution pump speed

Main features
Electricity production increase of P3 is made possible by significant cooling by Beriden and the heat delivery to customers may be made by decreasing supply temperature and increased supply flow compared to measurement data. No unit is started or stopped during the optimization. An increase of revenue is possible compared to measurement data.

Economic benefit
At the end of the optimization interval, the cumulative revenue when using optimization is about 10% higher than the cumulative revenue calculated from measurement data. The main contributors to this increase are:

1. Increased usage of Beriden, i.e., cooling of the return water such that a higher production of heat and, more significantly, electricity, is possible in P3.
2. A decreased supply temperature and increased supply mass flow rate.

Clarification of UCP plant operation
The customer heat load prediction is low, which P3 together with the flue gas condenser and the accumulator alone should be able to handle. This is also the result of the UCP optimization where all statuses are 0 except for P3 and the flue gas condenser. P3 is producing constant electricity and heat. The minor variations in customer heat load are taken care of charging and discharging the accumulator instead. Note that the heat produced in P3 is less than in the measurement data, which is due to that Beriden is not included in the UCP model and cannot provide cooling. This is a major drawback of the UCP method as the cooling of Beriden cannot be utilized to decrease inflow temperature to P3 and increase electricity production accordingly.

The increase of produced heat and electricity in P3 during the very last samples is an effect of finite horizon of the optimization. The inputs at these time points do not affect the outputs during the optimization horizon due to the delay of one sample from input to output and the dynamics of the accumulator. A simple remedy to this is to increase the optimization horizon slightly and only use the first 24 h. However, as the optimization is to be re-performed, for instance, every 6th hour the effect is neglected.

Clarification of EDP plant operation
The EDP optimization directly notices that an increase in beneficial electricity production can be made by utilizing the cooling of Beriden more than what is done in the measurement data. This results in that the Beriden by-pass valve is opened, and is in fact open during the whole optimization interval. This gives the operation of P3 the opportunity to increase its heat load, but more importantly, the electricity production is maximized as this has the highest sell price. This is performed by increasing the fuel load and shutting the turbine by-pass valve, i.e., more steam goes through the turbines instead of the condensers in P3.

Another major difference compared to the measurement data is the mass flow rate and temperature of the heat district water. Even though the heat production of P3 is increased it is kept as low as possible yielding the customer water temperature to be at the lower limit of 74.5°C. To meet the customer load demand, the distribution pump mass flow rate is instead increased.

The accumulator is not used extensively, only minor charging and discharging is performed.

The net by-pass valve makes a minor increase at the very last hour. This is due to the finite horizon of the optimization problem and that the effect of the inputs at this time point do not affect the outputs during the optimization horizon due to plant dynamics. This may be neglected as the optimization is to be re-performed, for instance, every 6th hour, thus only using the 6 first hours of optimization result as run schedule.

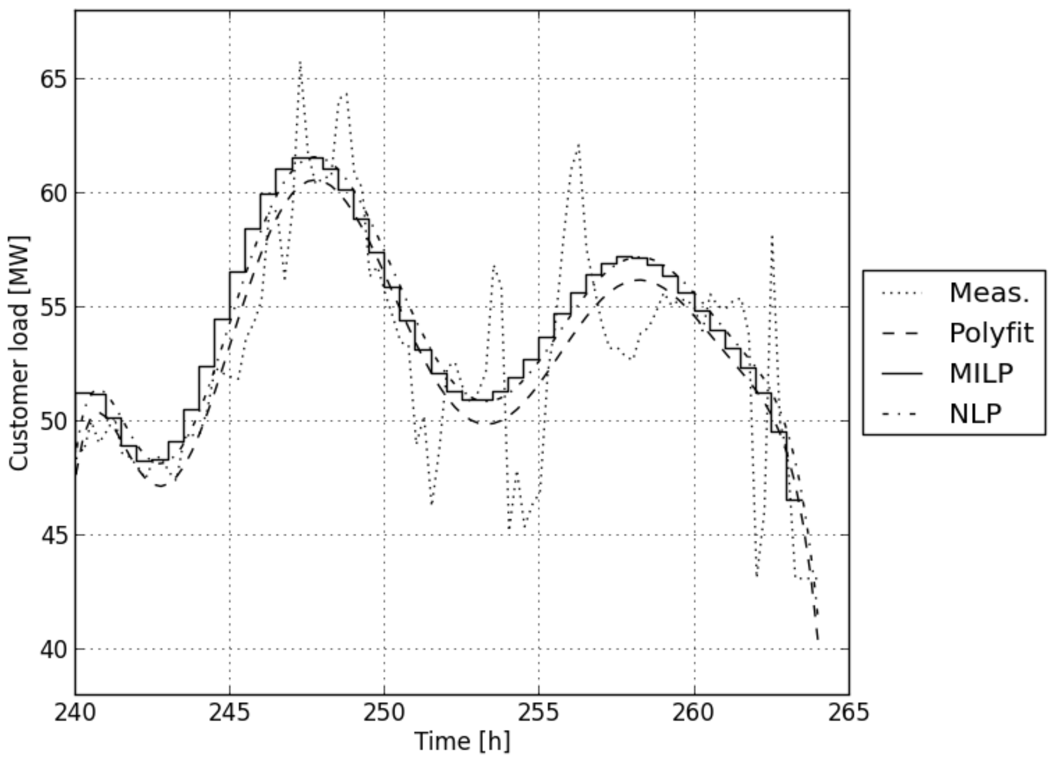


Figure 15 Case 1 predicted and optimized customer heat load. The heat load demand is followed within specification of ±1MW.

Figur 15. Fall 1. Predikterad och optimerad värmelast. Lasten följs enligt specifikationerna med ±1MW.

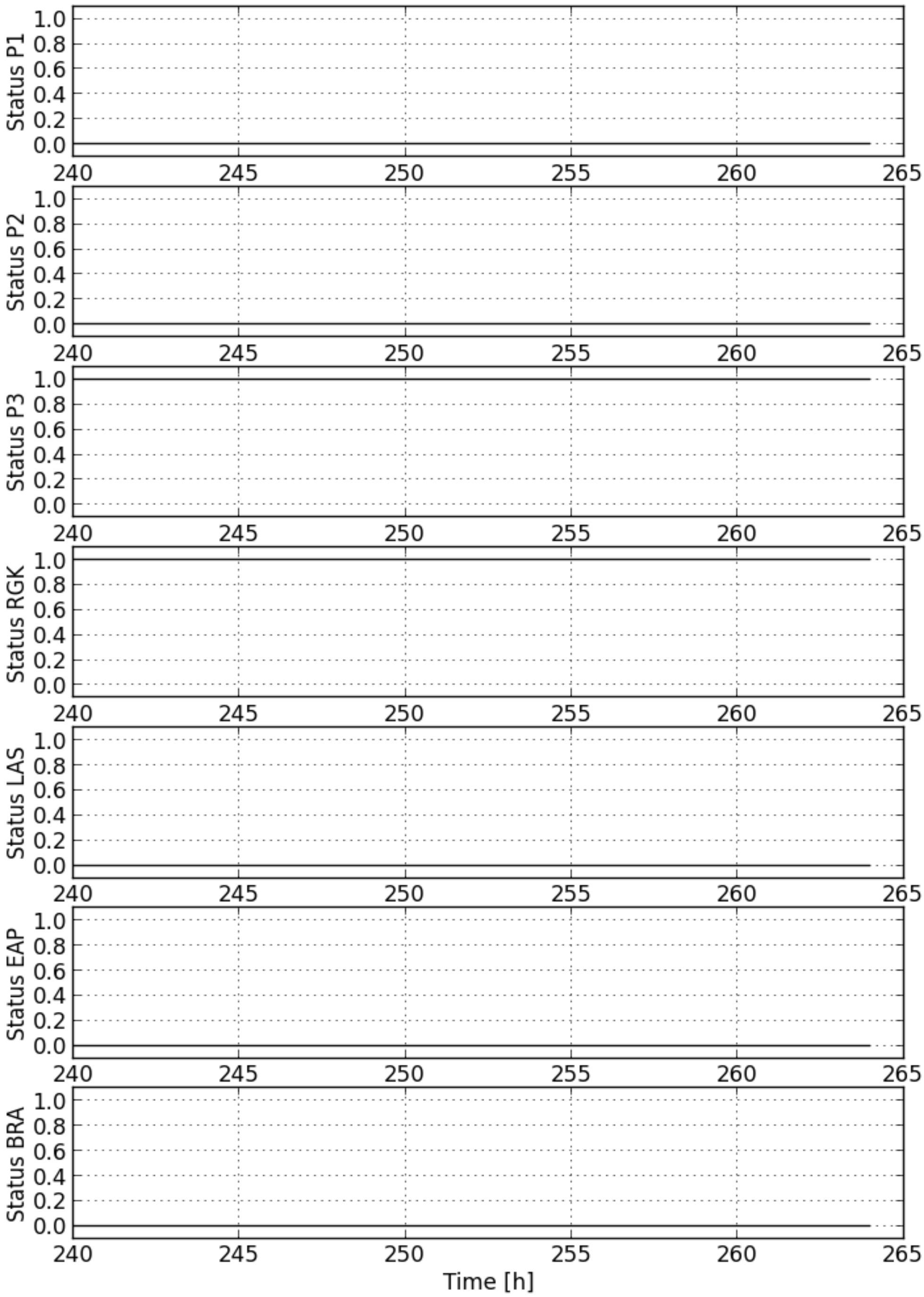


Figure 16 Case 1 optimized statuses of producers in the UCP. No unit changes its status.

Figur 16. Fall 1. Optimerade tillstånden i körplansproblemet. Ingen tillståndsförändring.

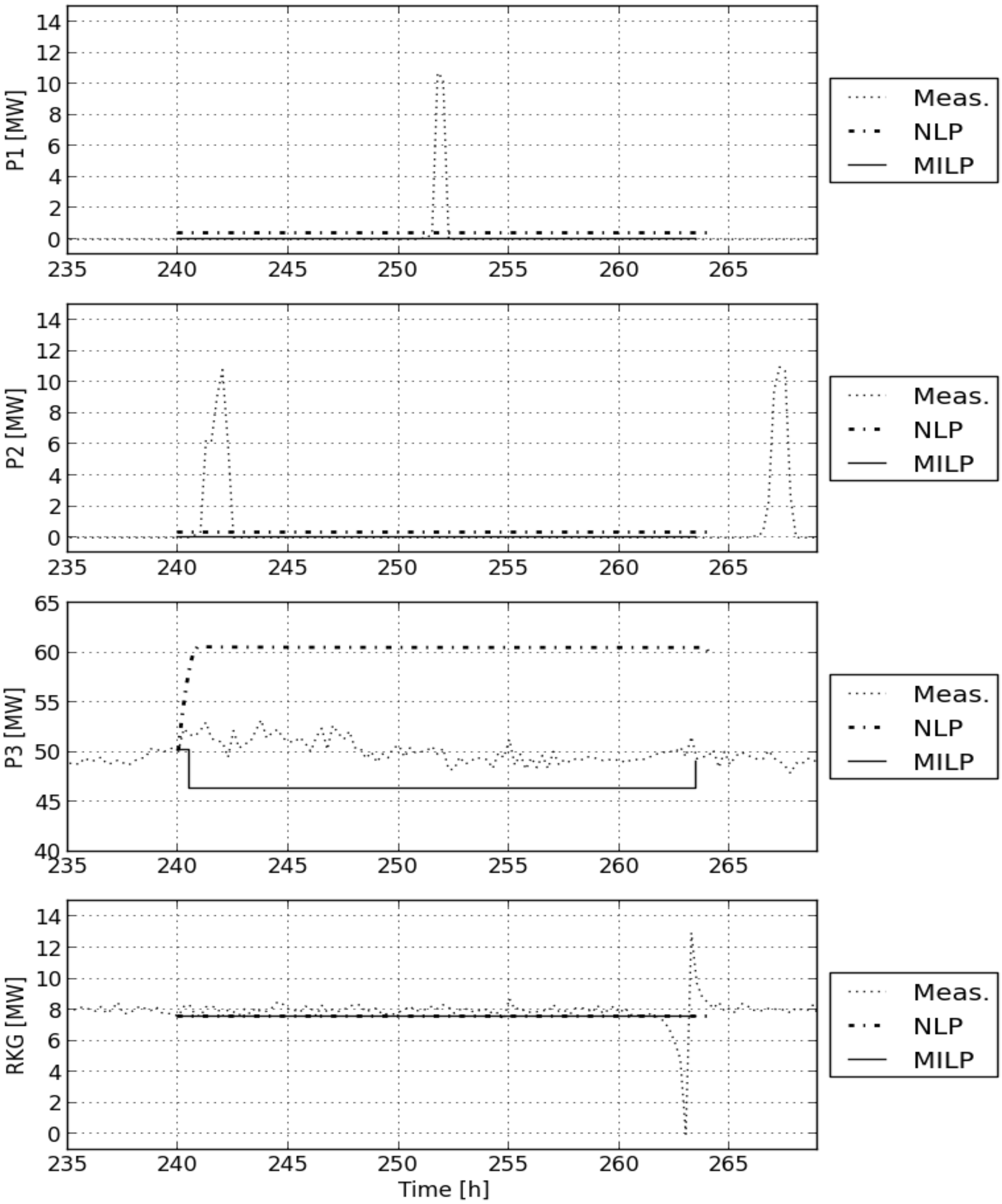


Figure 17 Case 1 measured and optimized heat production at P1, P2, P3 and RGK. P3 heat production is increased in the EDP solution and decreased in the UCP solution as the UCP model lacks a model of the Beriden cooler.

Figur 17. Fall 1. Uppmätt och optimerad värmeproduktion för P1, P2, P3 och RGK. P3's last ökar i EDP-lösningen medan den minskar i UCP-lösningen eftersom UCP-problemet skanar modell för kylaren Beriden.

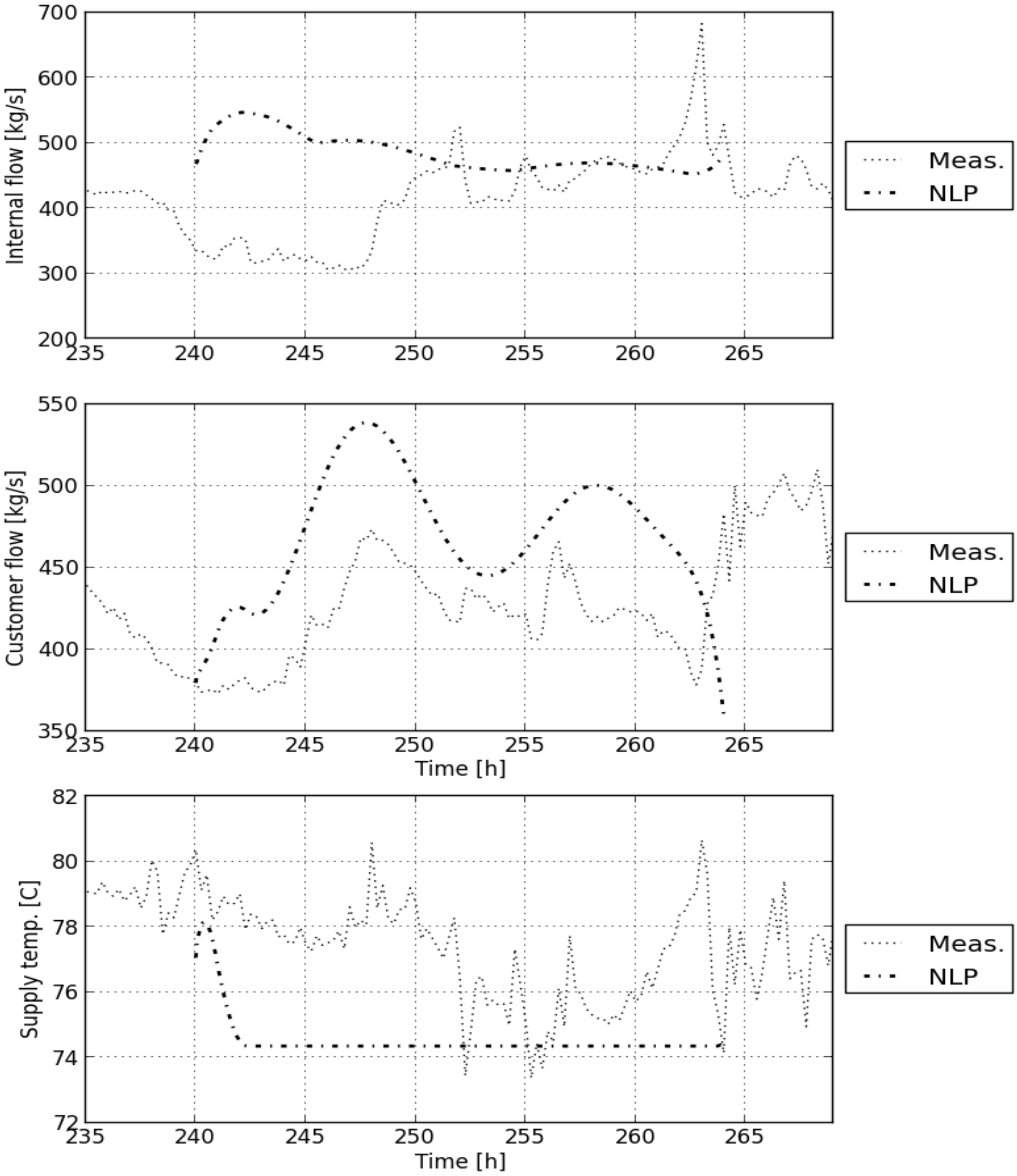


Figure 18 Case 1 measured and optimized internal and customer flow and supply temperature. The supply temperature is at the lower limit and the customer flow is increased compared to measurement data.

Figur 18. Fall 1. Uppmätta och optimerade flöden och framledningstemperatur. Framledningstemperaturen ligger på sin min-gräns och distributionspumpen levererar ett högre flöde jämfört med mätdata.

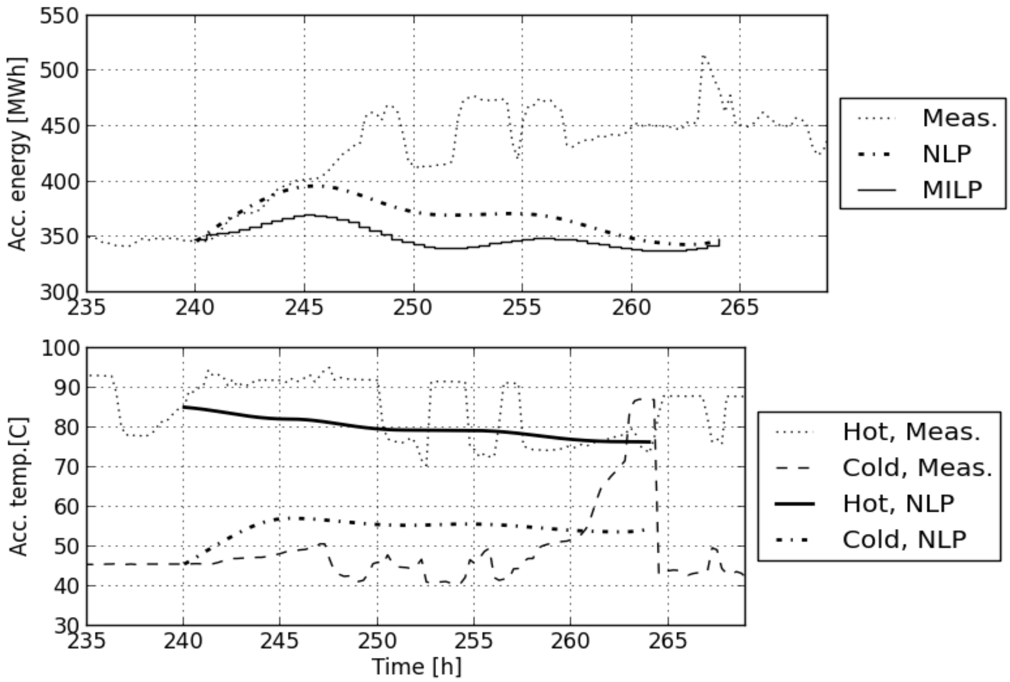


Figure 19 Case 1 measured and optimized accumulator energy and top (hot) and bottom (cold) accumulator temperatures.

Figur 19. Fall 1. Uppmätt och optimerad ackummulatorenergi och temperatur.

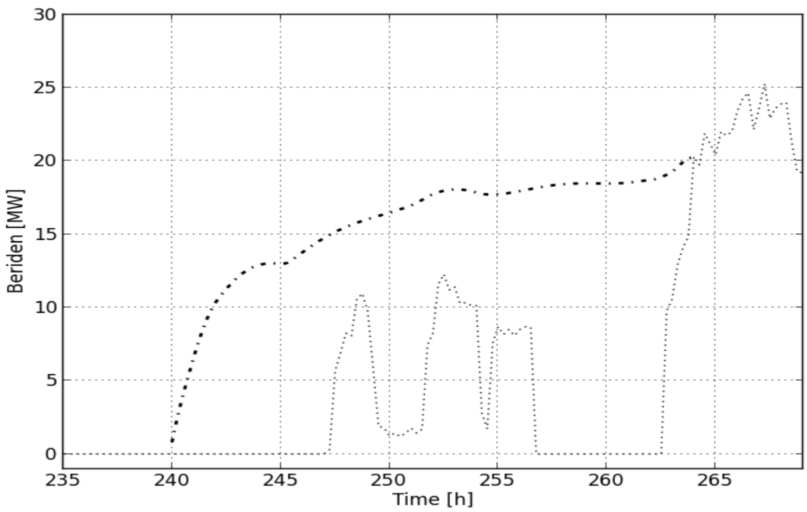


Figure 20 Case 1 measured and optimized Beriden cooling. The EDP solution shows that more cooling that what measurement data shows is beneficial.

Figur 20. Fall 1. Uppmätt och optimerad Beriden kylning. EDP-lösningen leder till en högre kyleffekt jämfört med mätdata.

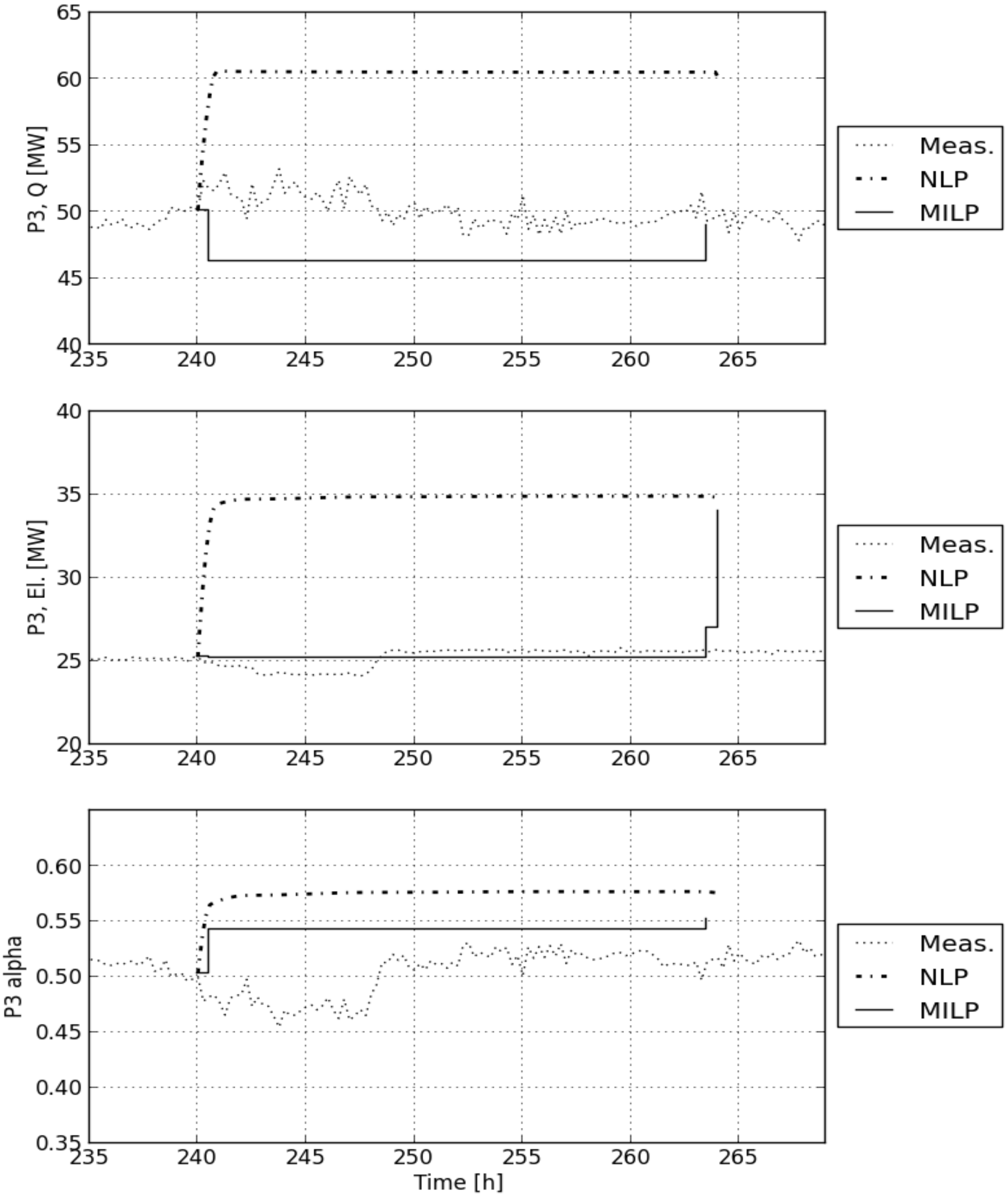


Figure 21 Case 1 measured and optimized P3 heat and electricity production and α value. Electricity production is maximized in the EDP solution, which cannot be made in the UCP due to the lack of Beriden cooler model in the UCP model.

Figur 21. Fall 1. Uppmätta och optimerade värme och el-produktion för P3, samt alpha-värden. EDP-lösningen resulterar i en högre el-produktion tack vara den externa kylaren.

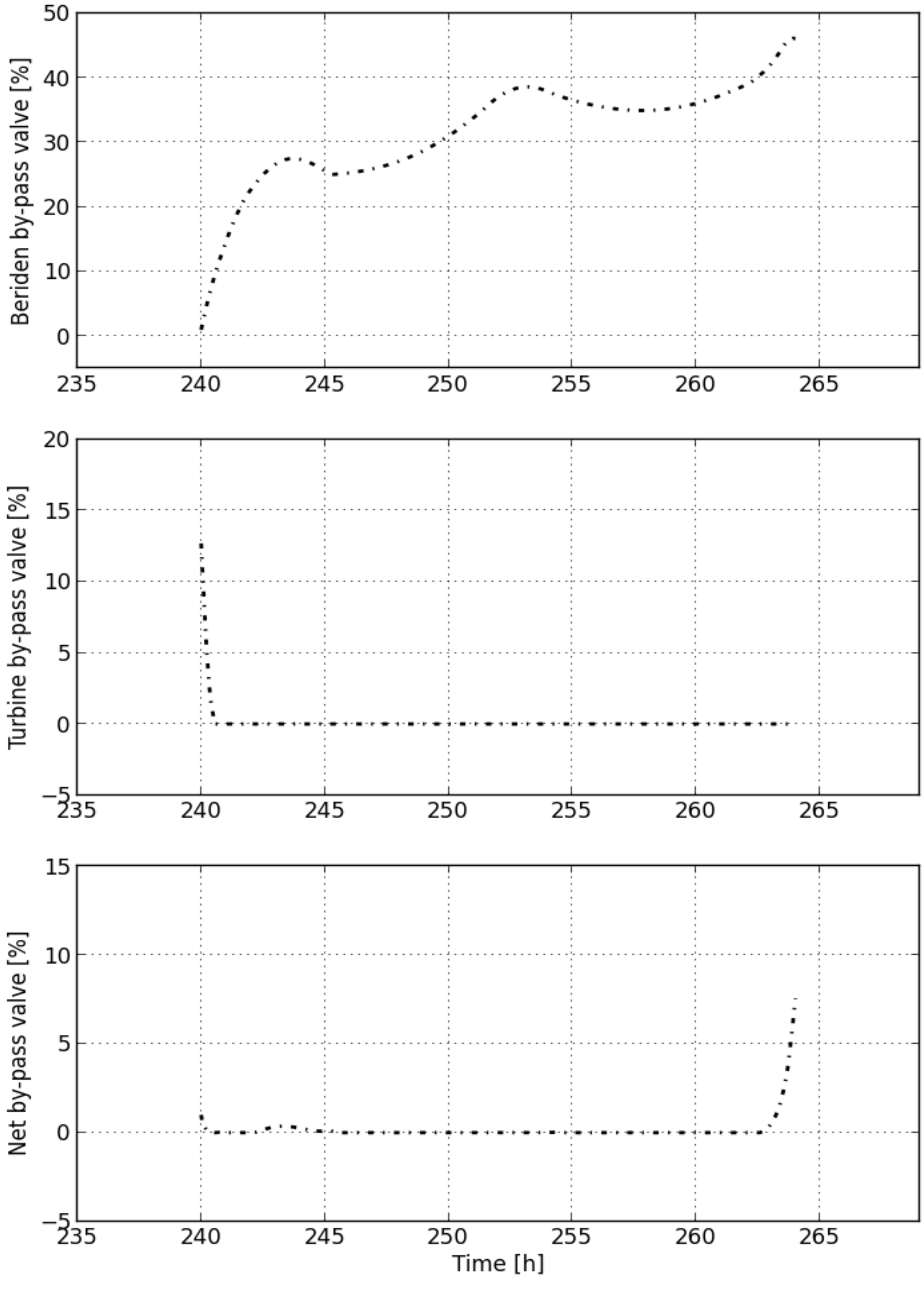


Figure 22 Case 1 optimized by-pass valve positions. The Beriden by-pass valve is opened, providing cooling to increase P3 electricity production.

Figur 22. Fall 1. Optimerad by-pass ventil. Beridens BP-ventil öppnar för att öka elproduktionen i P3.

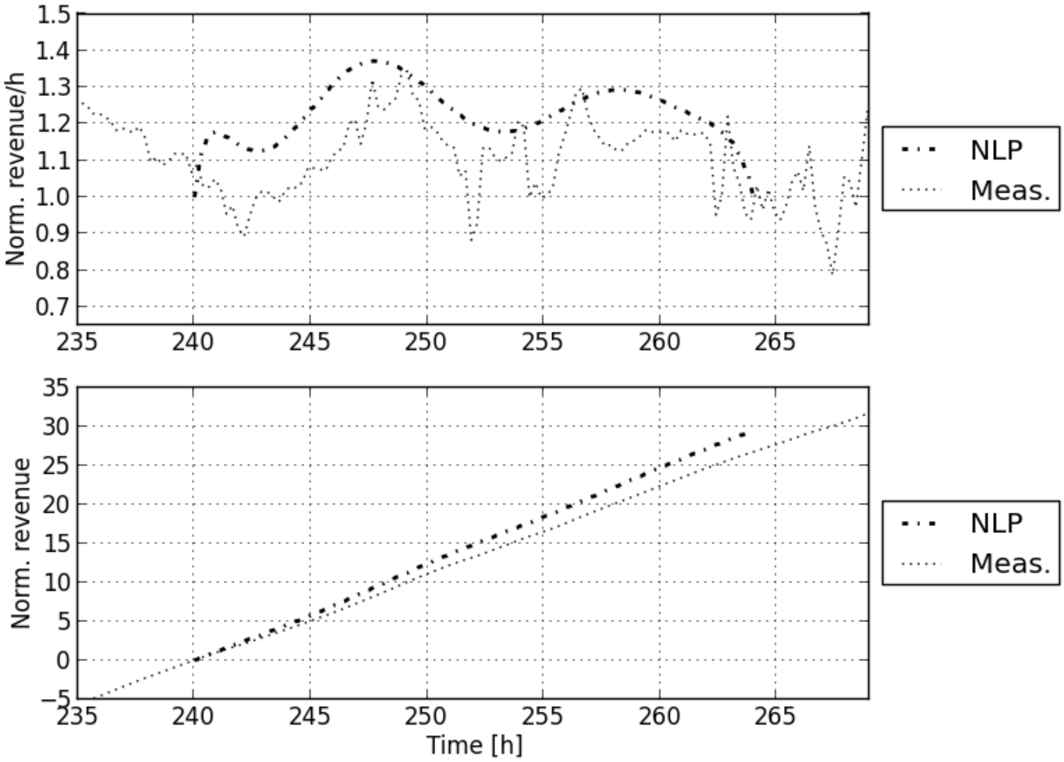


Figure 23 Case 1 measured and optimized normalized revenue/h and cumulative normalized revenue. The revenue/h is normalized such that the optimized solution starts at 1. The optimization shows a possibility of revenue increase.

Figur 23. Fall 1. Uppmätt och optimerad momentan och ackummulerad vinst. Vinsten per timme är normaliserad för att börja på 1. Optimeringen visar en potentiell vinst.

9.4 Case 2: Low Customer Demand and Beriden Cooling for Maximum Electricity Production II.

Data and figures

The measurement data used is 2012-10-25 07:00 - 2012-10-26 07:00 and the measurement data and optimization results can be found in Figure 24-Figure 32.

Decision variables

- UCP: See Section 6.2.1
- EDP: Derivatives of
    - P3 fuel load
    - Turbine by-pass valve position
    - District network by-pass valve position
    - Beriden by-pass valve position
    - Internal pump speed
    - Distribution pump speed

Main features
Maximization of electricity production in P3 is made possible by usage of Beriden and the customer heat demand can be fulfilled by decreasing supply temperature and increasing supply flow compared to measurement data. No unit is started or stopped. Compared to measurement data, a revenue increase is possible.

Economic benefit
The cumulative revenue when using optimization is about 12% greater than calculated from measurement data. This is, as in Case 1, mainly due to

1. Increased usage of Beriden giving the possibility to produce more electricity at P3.
2. A decreased supply temperature and increased supply mass flow rate.

Clarification of UCP plant operation
The low customer heat load prediction is handled by P3, the flue gas condenser and the accumulator only. The electricity production is increased in the beginning, but the production of heat in P3 prevents it to increase further due to the lack of cooling of the unmodeled Beriden. This drawback is the same as noted in Case 1.

The increase of produced heat and electricity in P3 during the very last samples is the same effect as seen in Case 1.

Clarification of EDP plant operation
The Beriden by-pass valve is opened directly at optimization start, yielding that the important, highly priced, electricity production of P3 is at maximum. This is done by increasing P3 fuel load and shutting the turbine by-pass valve, letting as much steam as possible go through the turbines. The additional heat produced in P3 is used for charging the accumulator or is cooled off by Beriden.

Also, as in Case 1, compared to the measurement data, the customer water temperature is as low as possible, i.e., at its lower limit of 74.5^{o}C, and the distribution pump mass flow rate is increased to meet the customer heat demand.

The accumulator is not used extensively. However, a minor charge can be seen in the beginning of the optimization interval and a minor discharge can be seen at the end of the optimization interval where the customer load prediction increases to approximately 60 MW.

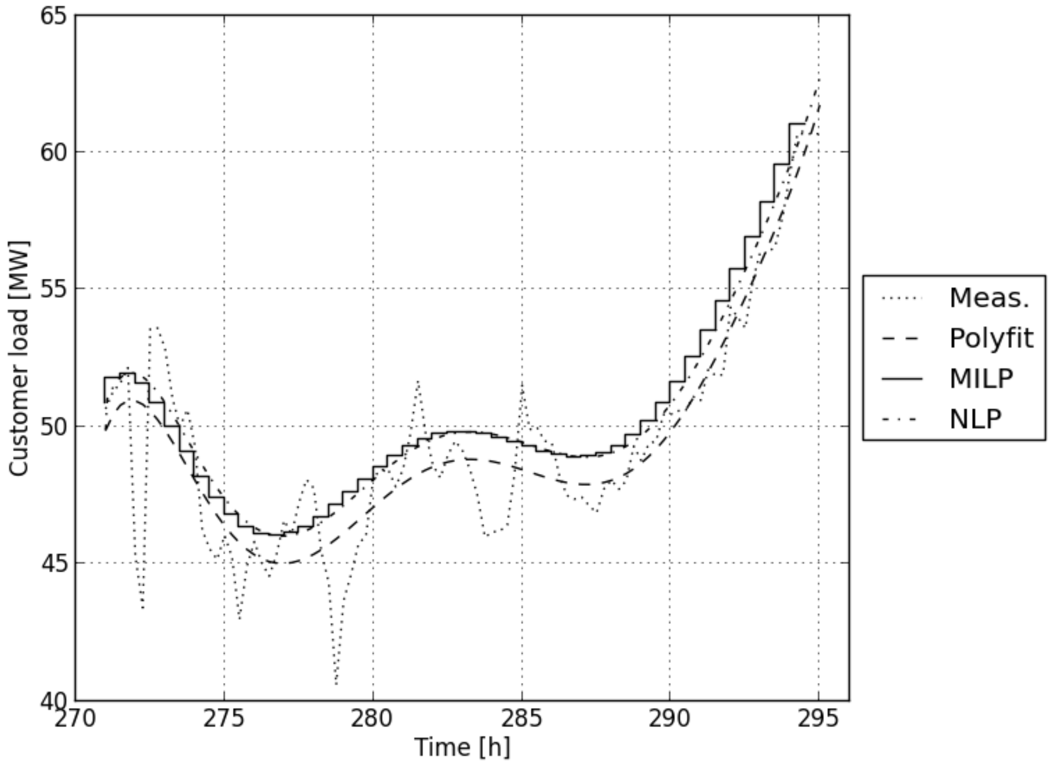


Figure 24 Case 2 measured and optimized customer heat load. The heat load demand is followed within specification of ±1MW.

Figur 24. Fall 2. Uppmätt och optimerad värmelast. Produktionen följer kundlasten enligt specifikationerna (±1MW).

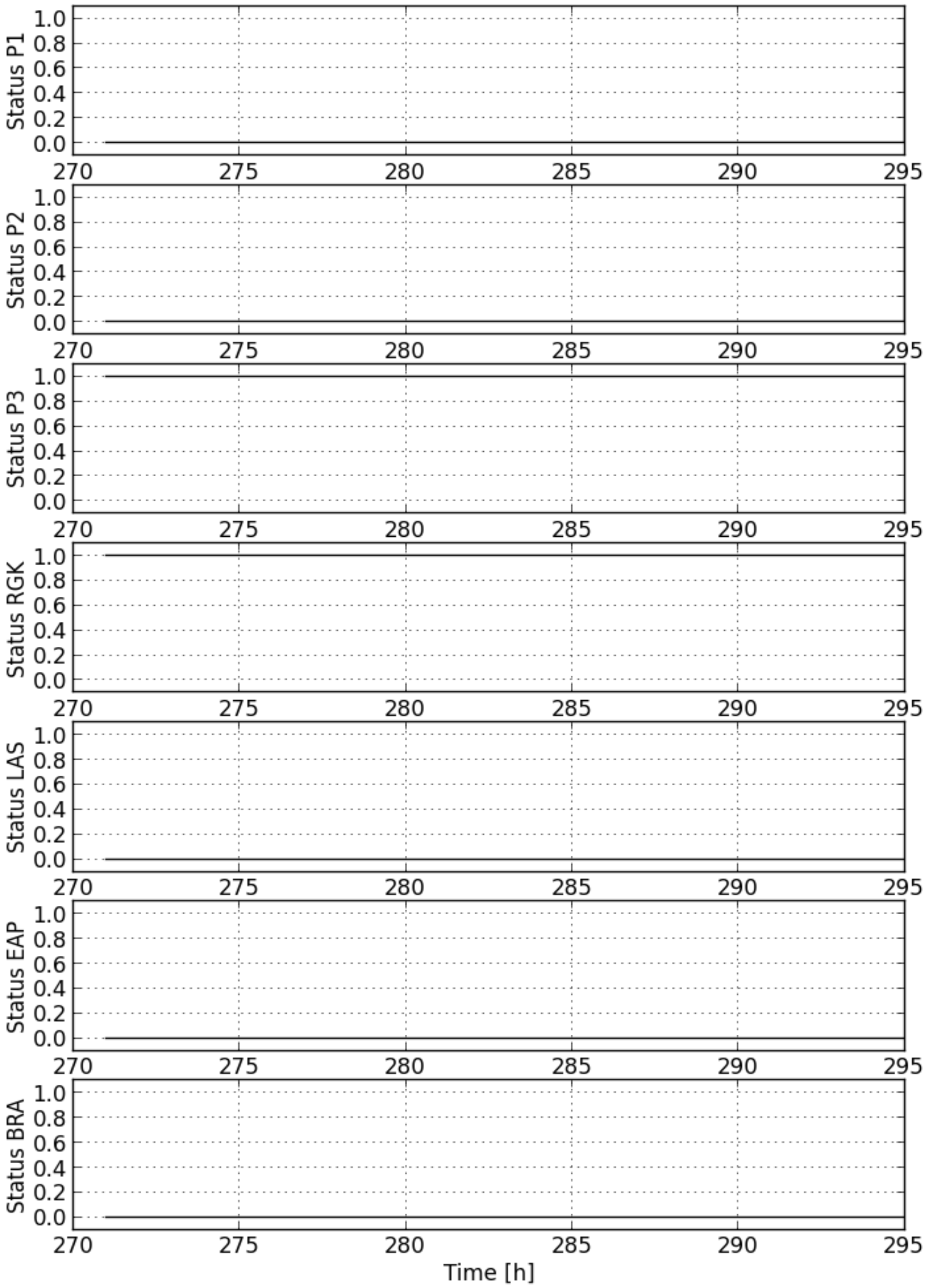


Figure 25 Case 2 optimized statuses of producers in the UCP. No unit is changing its status.

Figur 25. Fall 2. Optimerade tillstånden I UCP-problemet. Ingen förändring i producenternas status.

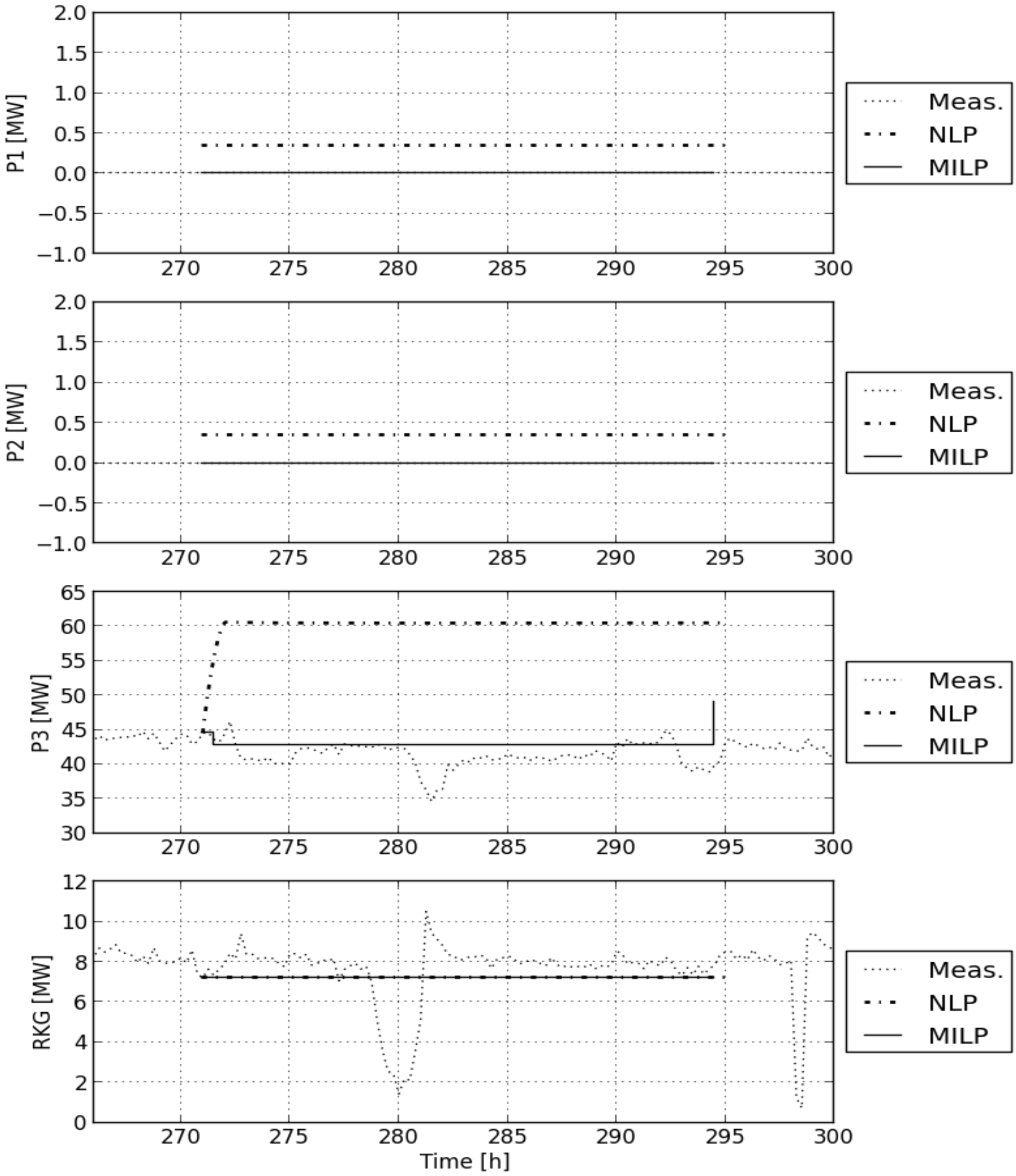


Figure 26 Case 2 measured and optimized heat production at P1, P2, P3 and RGK. P3 heat production is increased in the EDP solution, which is not possible in the UCP due to the lack of Beriden cooler model in the UCP model.

Figur 26. Fall 2. Uppmätt och optimerad värmeproduktion för P1, P2, P3 och RGK. P3's last ökar i EDP-lösningen medan den minskar i UCP-lösningen eftersom UCP-problemet skanar modell för kylaren Beriden.

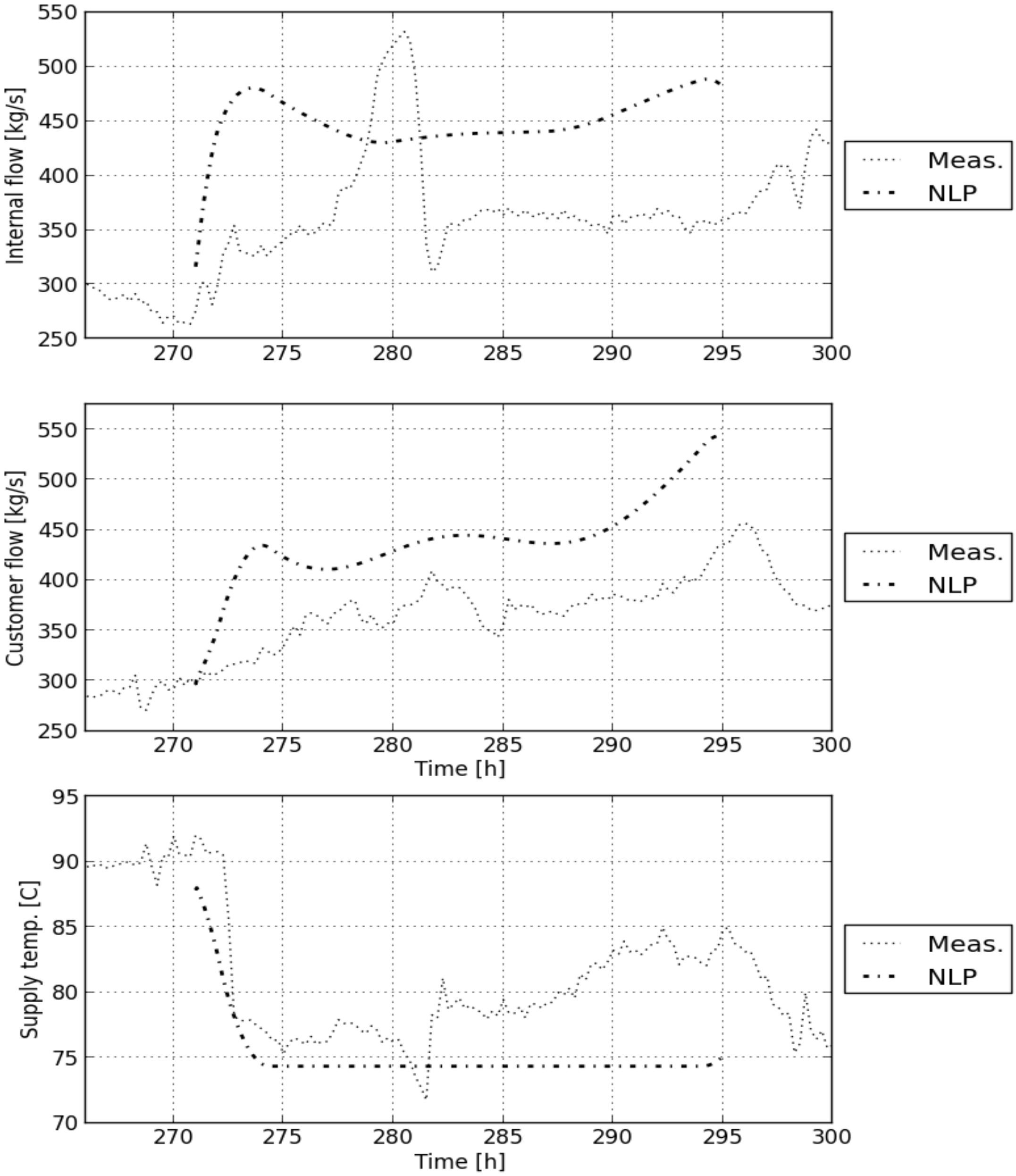


Figure 27 Case 2 measured and optimized internal and customer flow and supply temperature. Supply temperature is at lower limit and the customer flow is higher than measurement data.

Figur 27. Uppmätta och optimerade flöden samt temperatur i framledningen. Den optimerade framledningstemperaturen ligger vid sin min-gräns medan det optimerade flödet är högre än i mätdata.

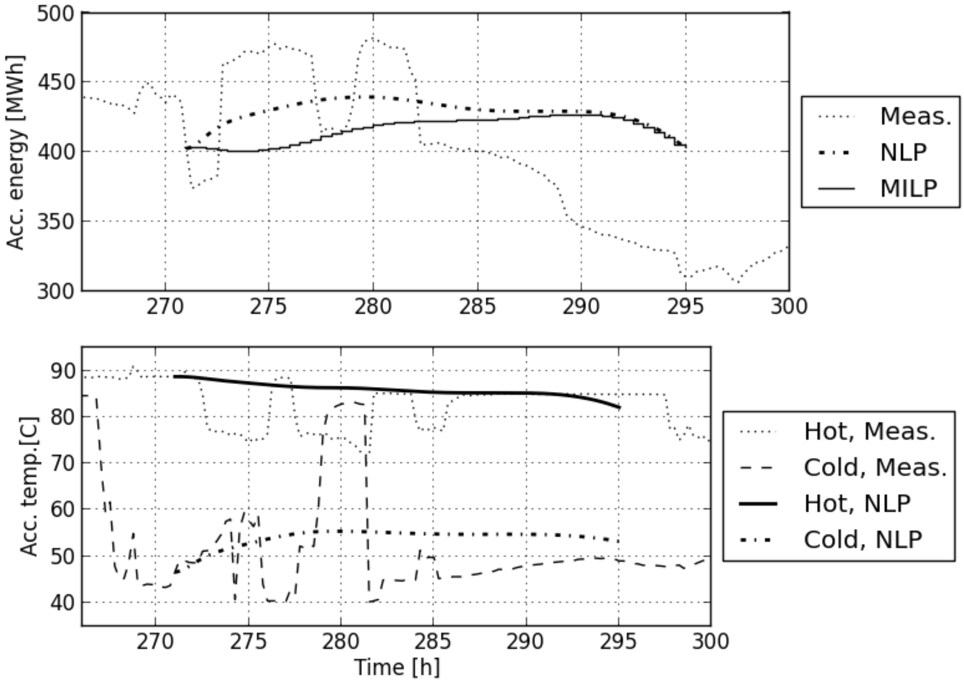


Figure 28 Case 2 measured and optimized accumulator energy and top (hot) and bottom (cold) accumulator temperatures.

Figur 28. Fall 2. Uppmätta och optimerade energy och temperature i ackummulatorn.

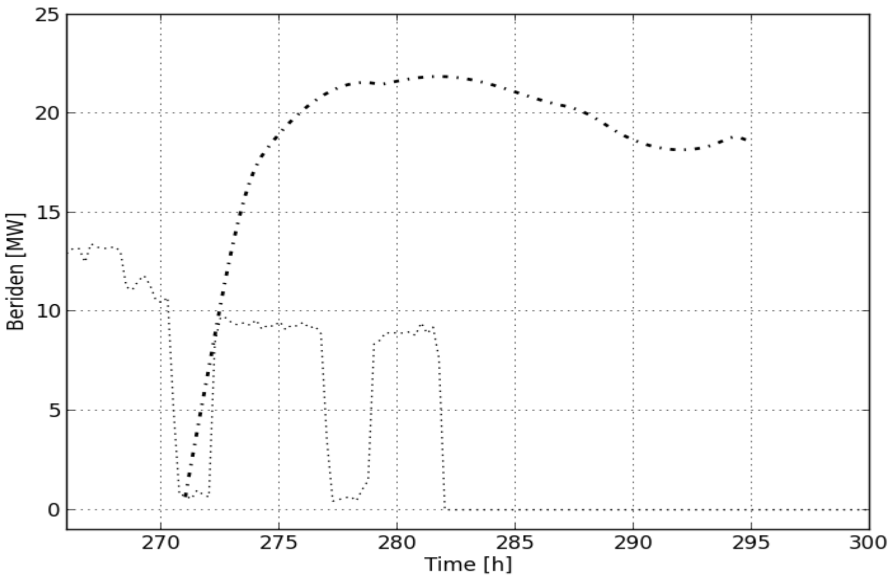


Figure 29 Case 2 measured and optimized Beriden cooling. EDP solution shows that increased usage of Beriden is beneficial.

Figur 29. Fall 2. Uppmätt och optimerad kyleffekt. EDP-lösningen ökar Beridens kyleffekt.

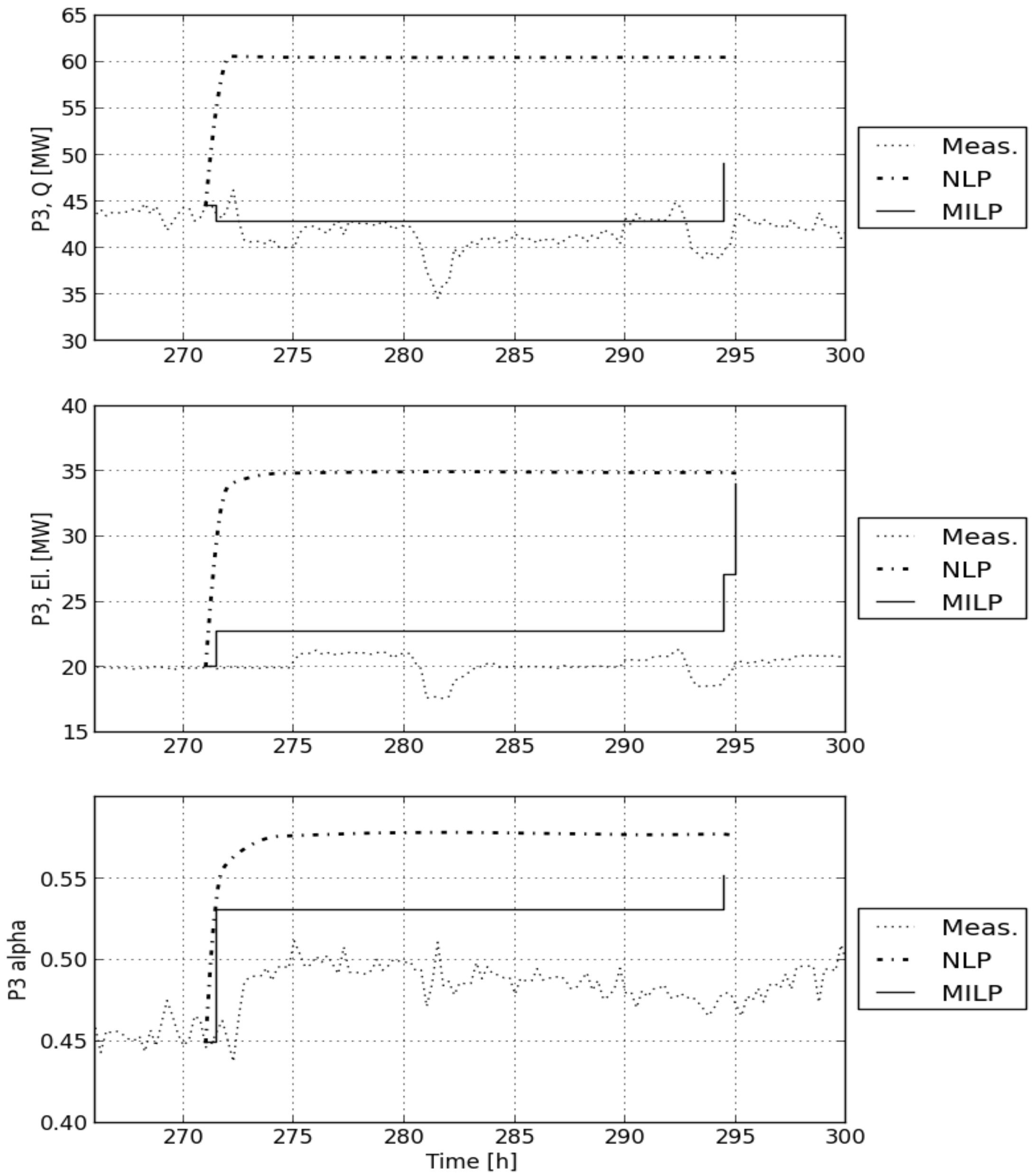


Figure 30 Case 2 measured and optimized P3 heat and electricity production and α value. The EDP solution maximized electricity production, which cannot be made in UCP as the associated produced heat is higher than the customer load and the UCP model is lacking a Beriden cooler model.

Figur 30. Fall 2. Uppmätta och optimerade värme och el-produktion för P3, samt alpha-värden. EDP-lösningen resulterar i en högre el-produktion tack vara den externa kylaren.

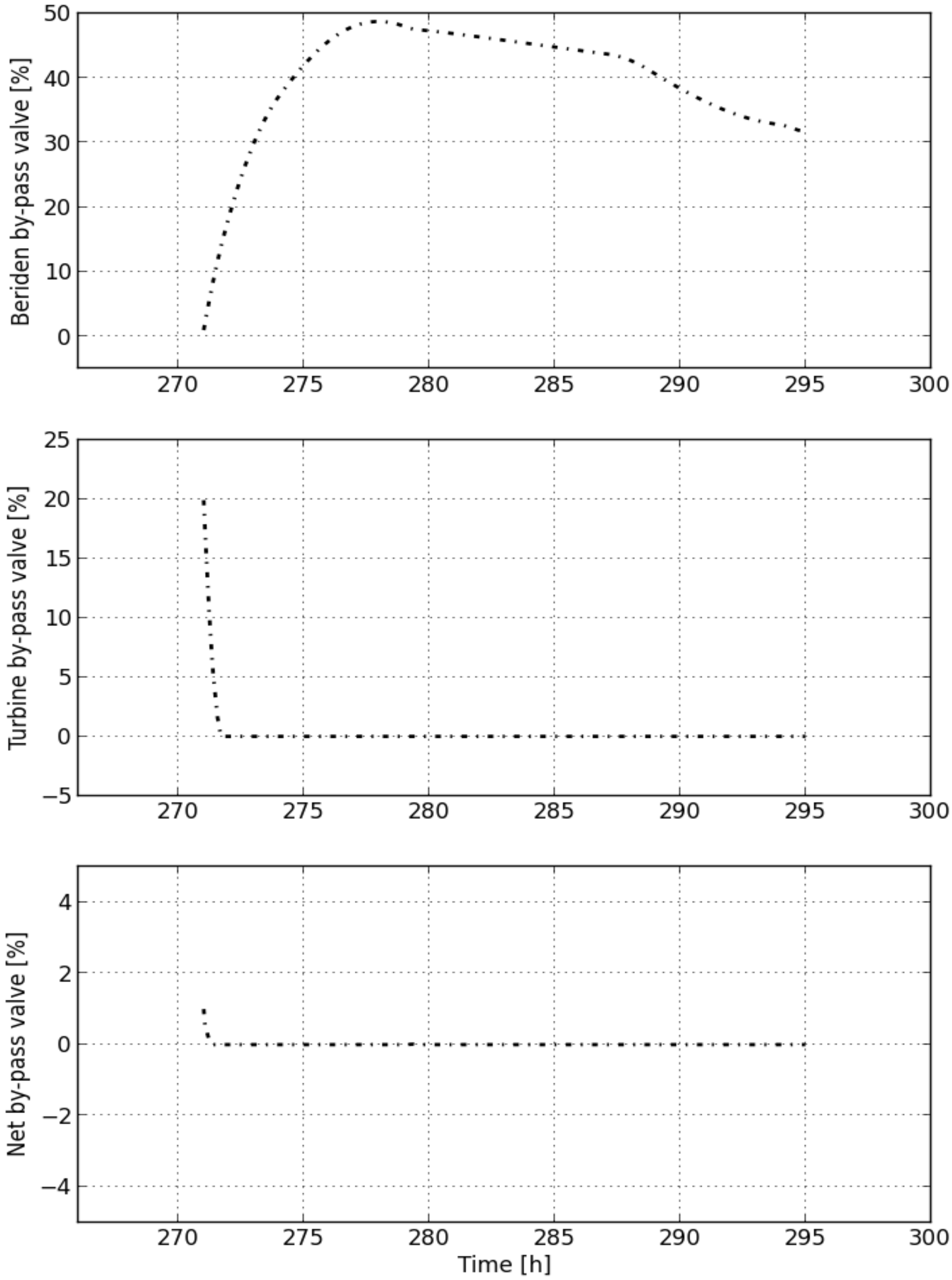


Figure 31 Case 2 optimized by-pass valve positions. Beriden by-pass valve is opened to provide Beriden cooling.

Figur 31. Fall 2. Optimerade BP-ventiler. Beridens BP-ventil öppnar för en ökad kyleffekt.

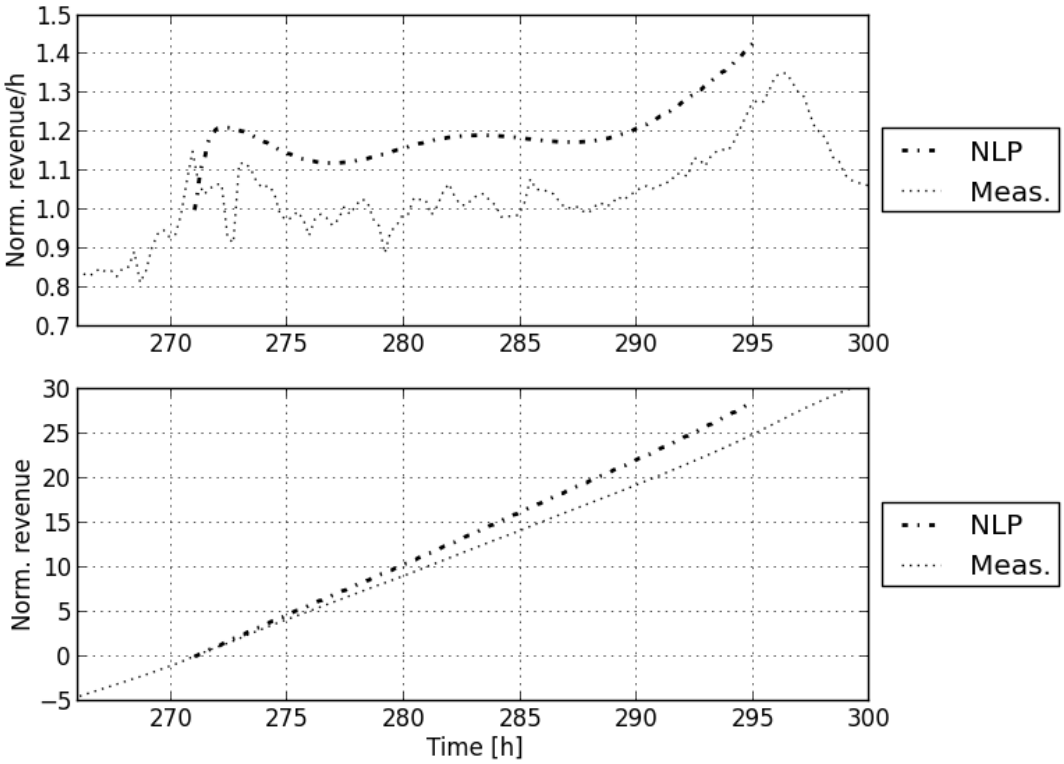


Figure 32 Case 2 measured and optimized normalized revenue/h and cumulative normalized revenue. The revenue/h is normalized such that the optimized solution starts at 1. The optimization shows a possibility of revenue increase.

Figur 32. Fall 2. Uppmätt och optimerad momentan och ackummulerad vinst. Vinsten per timme är normaliserad för att börja på 1. Optimeringen visar en potentiell vinst.

9.5 Case 3: Customer load with one peak and usage of accumulator

Data and figures

The measurement data used is 2012-01-13 12:00 - 2012-01-14 12:00 and the measurement data and optimization results can be found in Figure 33-Figure 41.

Decision variables

- UCP: See Section 6.2.1
- EDP: Derivatives of
- Continuous time: Derivatives of
    - P2 fuel load
    - P3 fuel load
    - Turbine by-pass valve position
    - District network by-pass valve position
    - Beriden by-pass valve position

- Internal pump speed
- Distribution pump speed

Main features

The electricity production of P3 is maximized and, contrary to what measurement data shows, no extra unit has to be started to fulfill the customer heat load demand. The accumulator is used and the customer supply temperature and flow may be decreased and increased, respectively, compared to measurement data to fulfill customer demands. A revenue increase, also compared to measurement data, is possible.

Economic benefit

The optimized cumulative revenue is at the optimization interval end approximately 8% higher than the cumulative revenue calculated using measurement data. The main reasons for this are threefold:

1. Increased electricity production in P3 made possible by not starting P1 to produce heat as well.
2. A decreased supply temperature and increased supply mass flow rate.
3. No start of P1, yielding no start cost.

Clarification of UCP plant operation

Compared to measurement data, the UCP result shows that the customer heat demand can be met by only using P2, P3, flue gas condenser and the accumulator. No starts or stops of the units are performed. P3 increases its electricity production to maximum, which also gives increased heat production. The remaining heat needed to meet the customer demand is taken care by P2.

The customer heat load peak at the end of the optimization interval is taken care of discharging the accumulator rapidly, which has been loaded for the major part of the optimization interval. Compared to measurement data, the accumulator is used significantly more.

The UCP optimization may give rise to multiple solutions. Either the heat is produced prior any heat load peak and stored in the accumulator and then used when a heat load peak is present, or the heat is produced when the heat load peak is present. In the current optimization formulation, no action has been taken to make the optimization choose one of the strategies over the other.

Clarification of EDP plant operation

Compared to the UCP solution, the P3 heat production is increased slightly more, which yields that the P2 heat production does not need to be increased initially and is instead close to its minimum load value of 14 MW when on. P3 is at the beginning run with fully closed turbine by-pass valve, and it is remained closed during the whole optimization interval. The electricity production is maximized, as this has the highest sell price, by increasing the P3 fuel load.

The heat production is held low, i.e., the supply temperature is held low. However, the temperature is not on its lower bound. This is because the customer heat load demand would then require a higher distribution mass flow rate which is not possible as the flow is maximized at 550 kg/s during almost the entire optimization interval. Hence, compared to Case 1 and 2, it is the distribution flow, not the supply temperature, that is limiting.

Prior the high peak in customer heat demand, the accumulator is charged slightly, which can be seen both in an energy increase in the accumulator, but also by the minor decrease in customer flow rate but sustained internal flow rate. The accumulator is discharged when the high customer peak comes such that the accumulator energy end constraint is active. However, the largest part of the customer peak is taken care of P2, which increases its heat production at the end, resulting in an increased supply temperature. Compared to the UCP solution, the accumulator is less active and the load change of P2 is greater.

Beriden is not used at all, which is because the customer heat load demand is so high that P3 can produce the heat resulting from maximum electricity production without producing too much heat.

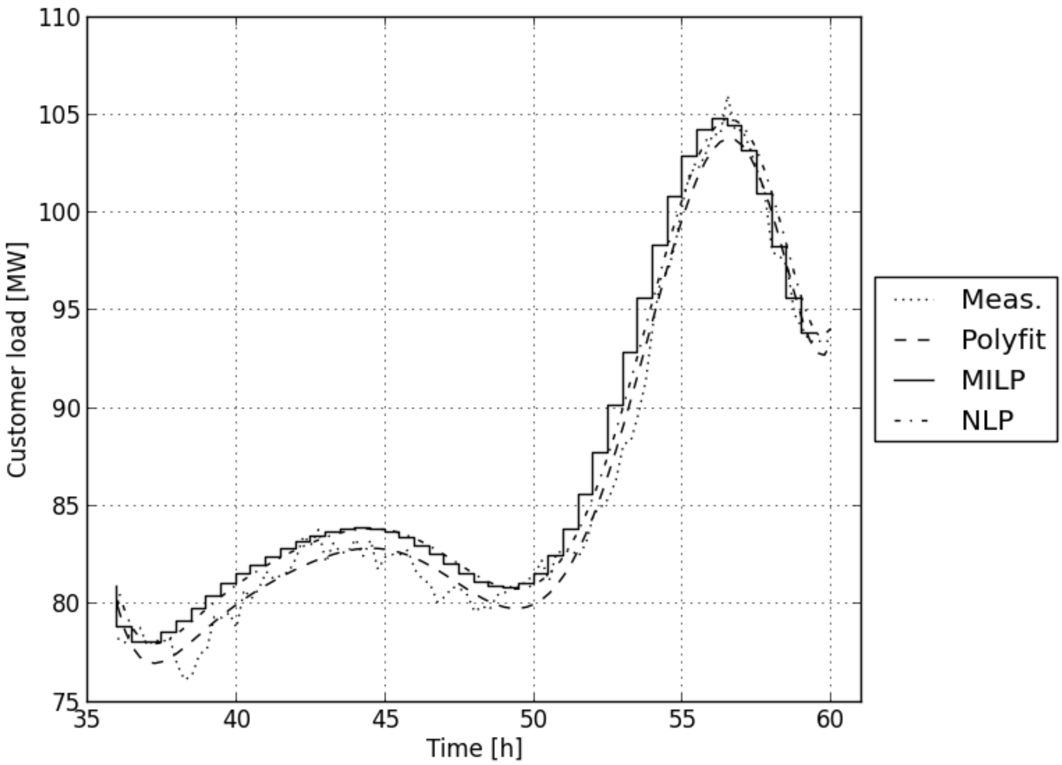


Figure 33 Case 3 measured and optimized customer heat load. The heat load demand is followed within specification of ±1MW.

Figur 33. Fall 3. Uppmätt och optimerad värmelast hos kunderna. Väremproduktionen följer lasten enligt specifikationerna (±1MW).

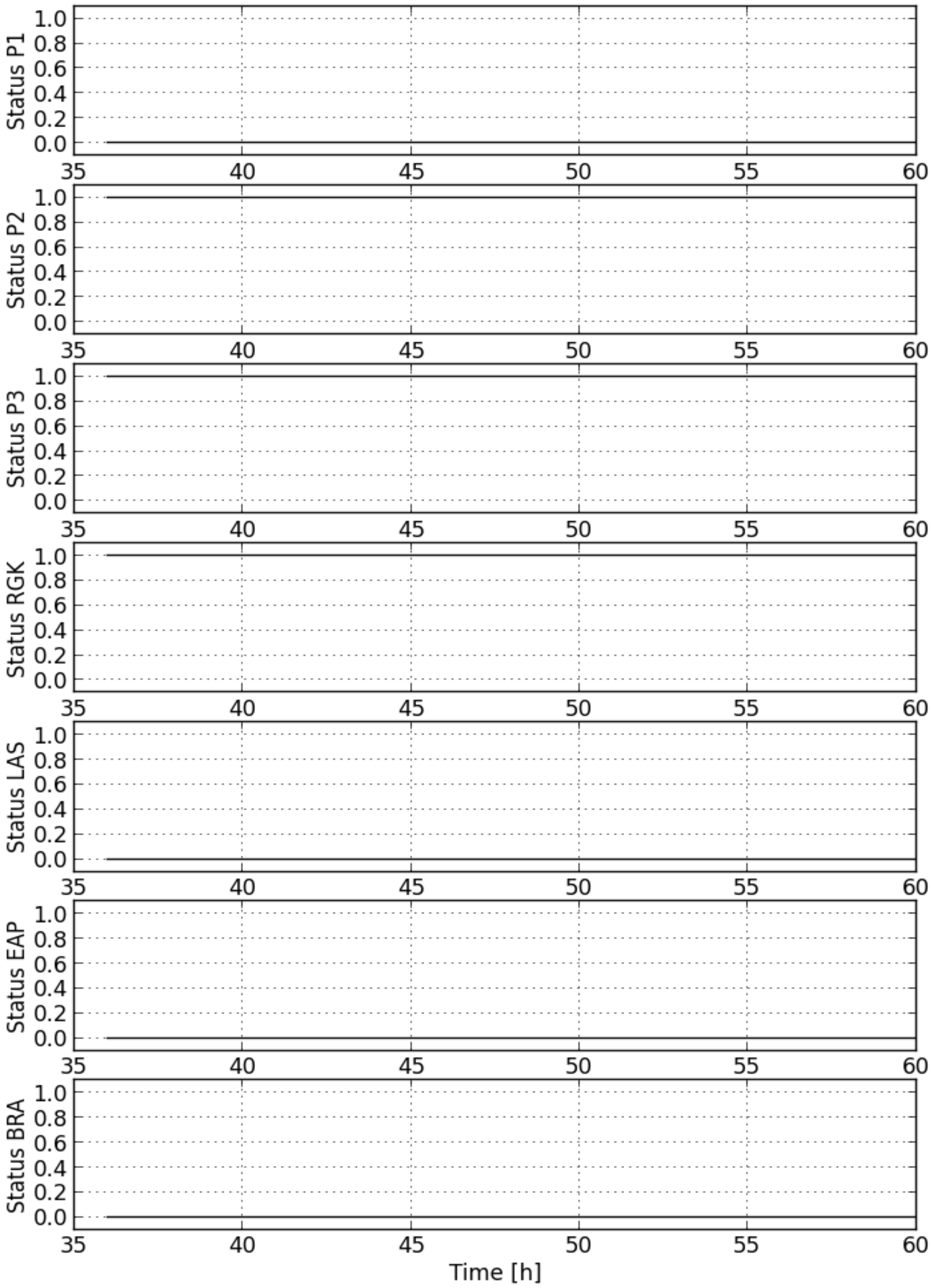


Figure 34 Case 3 optimized statuses of producers in the UCP. No unit changes its status.

Figur 34. Fall 3. Optimerade tillstånden I UCP-problemet. Ingen tillståndsförändring.

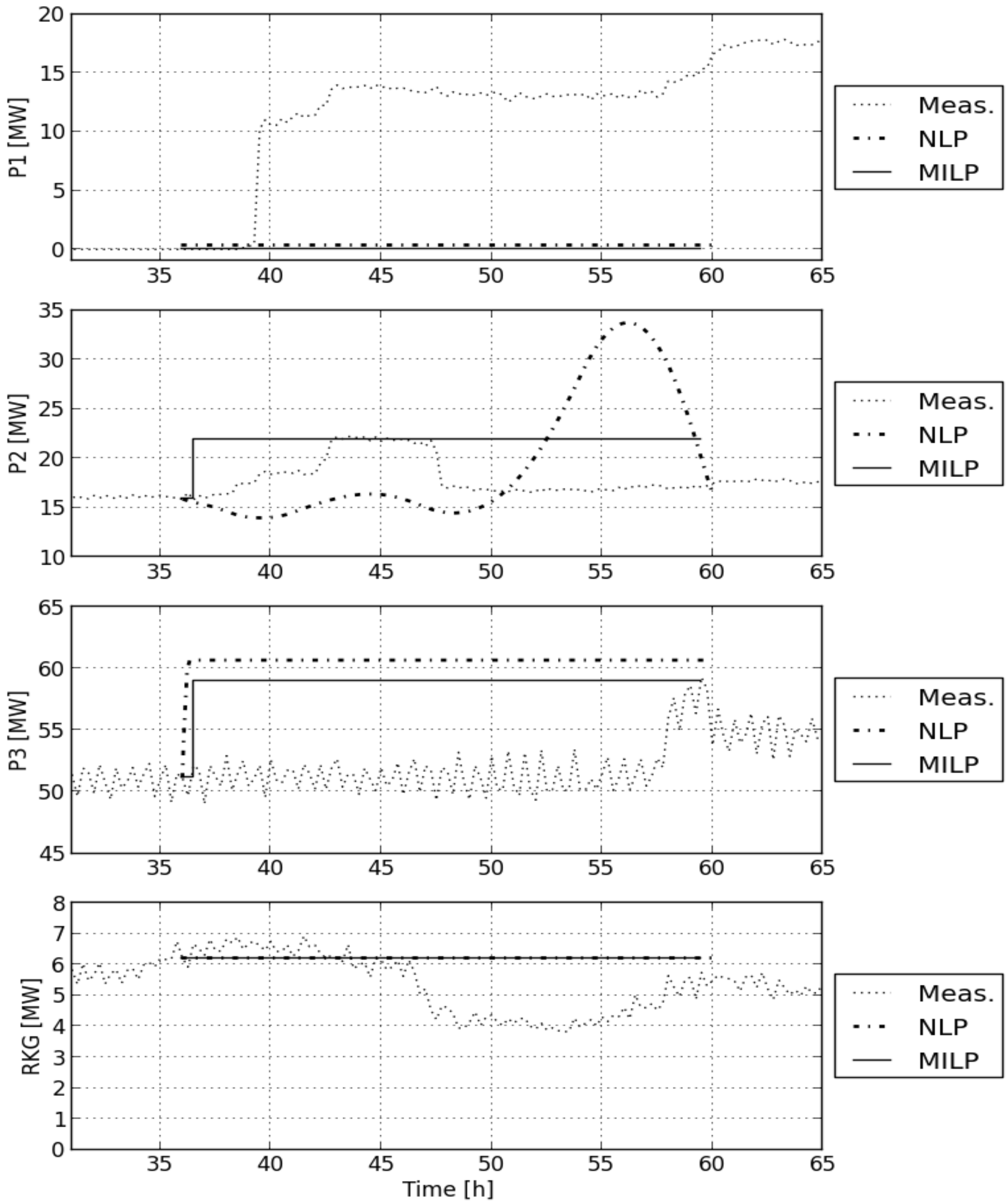


Figure 35 Case 3 measured and optimized heat production at P1, P2, P3 and RGK. P1 is not started, compared to measurement data, and both P2 and P3 increases their heat production.

Figur 35. Fall 3. Uppmätt och optimerad värmeproduktion för P1, P2, P3 och RGK. P1 startar inte som i mätdata, P2 och P3 ökar i last.

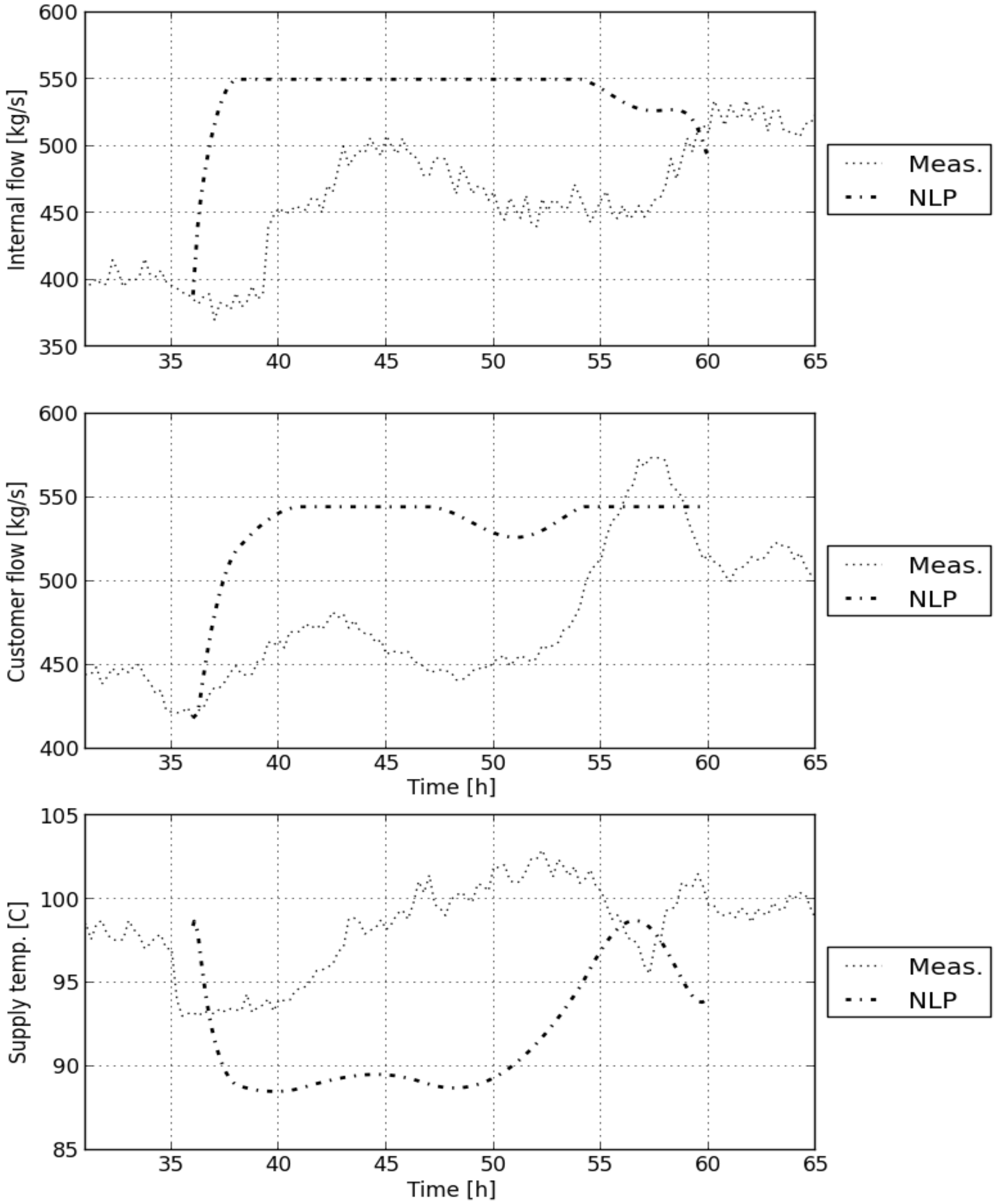


Figure 36 Case 3 measured and optimized internal and customer flow and supply temperature. Supply temperature is decreased and customer flow increased compared to measurement data.

Figur 36. Fall 3. Uppmätta och optimerade flöden och temperature i framledningen. Flödet är nära sin max-gräns och temperatur minskar.

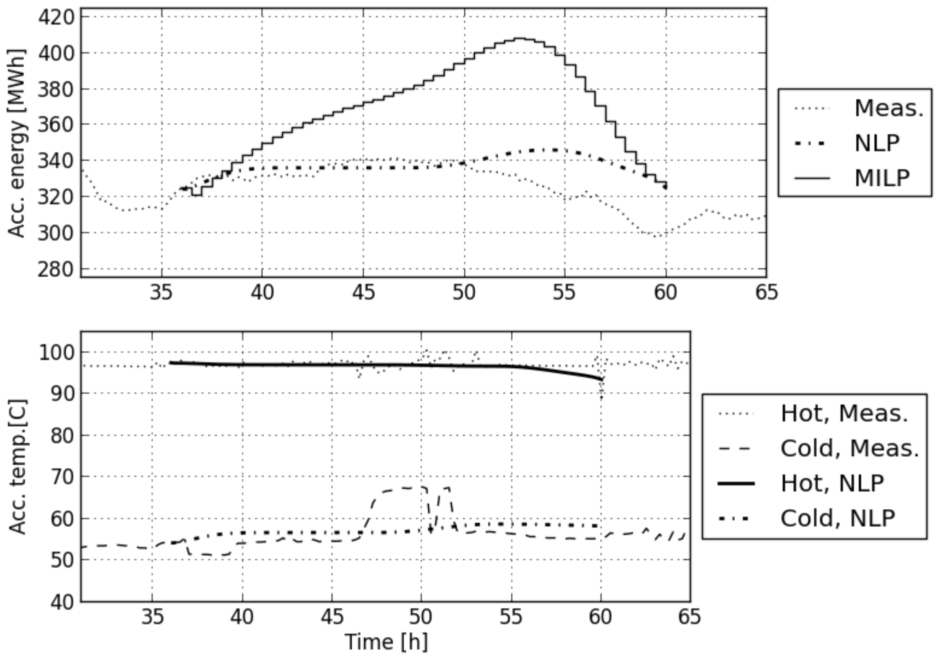


Figure 37 Case 3 measured and optimized accumulator energy and top (hot) and bottom (cold) accumulator temperatures. The accumulator is significantly more used in the UCP than in the EDP result.

Figur 37. Fall 3. Uppmätta och optimerade energi samt temperatur i ackumulatorn.

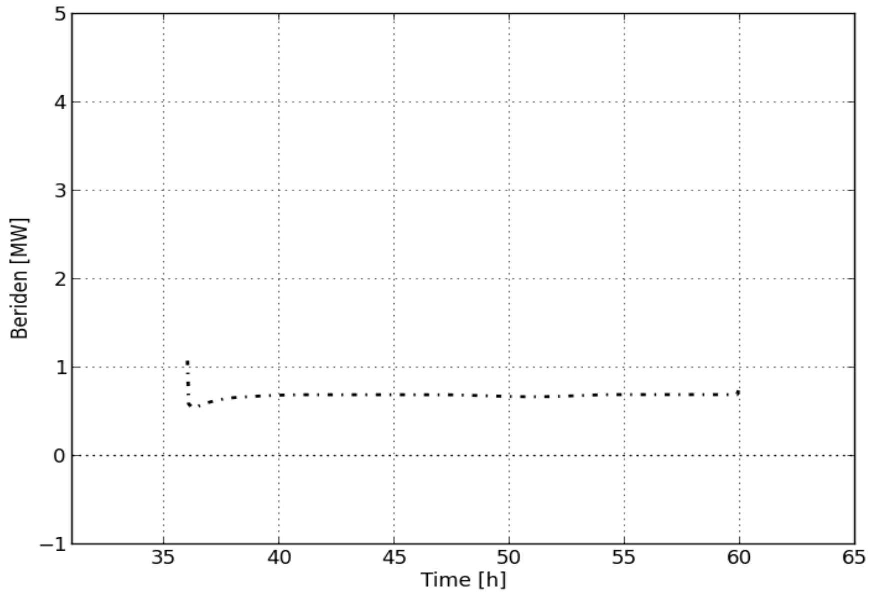


Figure 38 Case 3 measured and optimized Beriden cooling. No cooling by Beriden is performed.

Figur 38. Fall 3. Uppmätt och optimerad kylleffekt från Beriden. Ingen kylning sker.

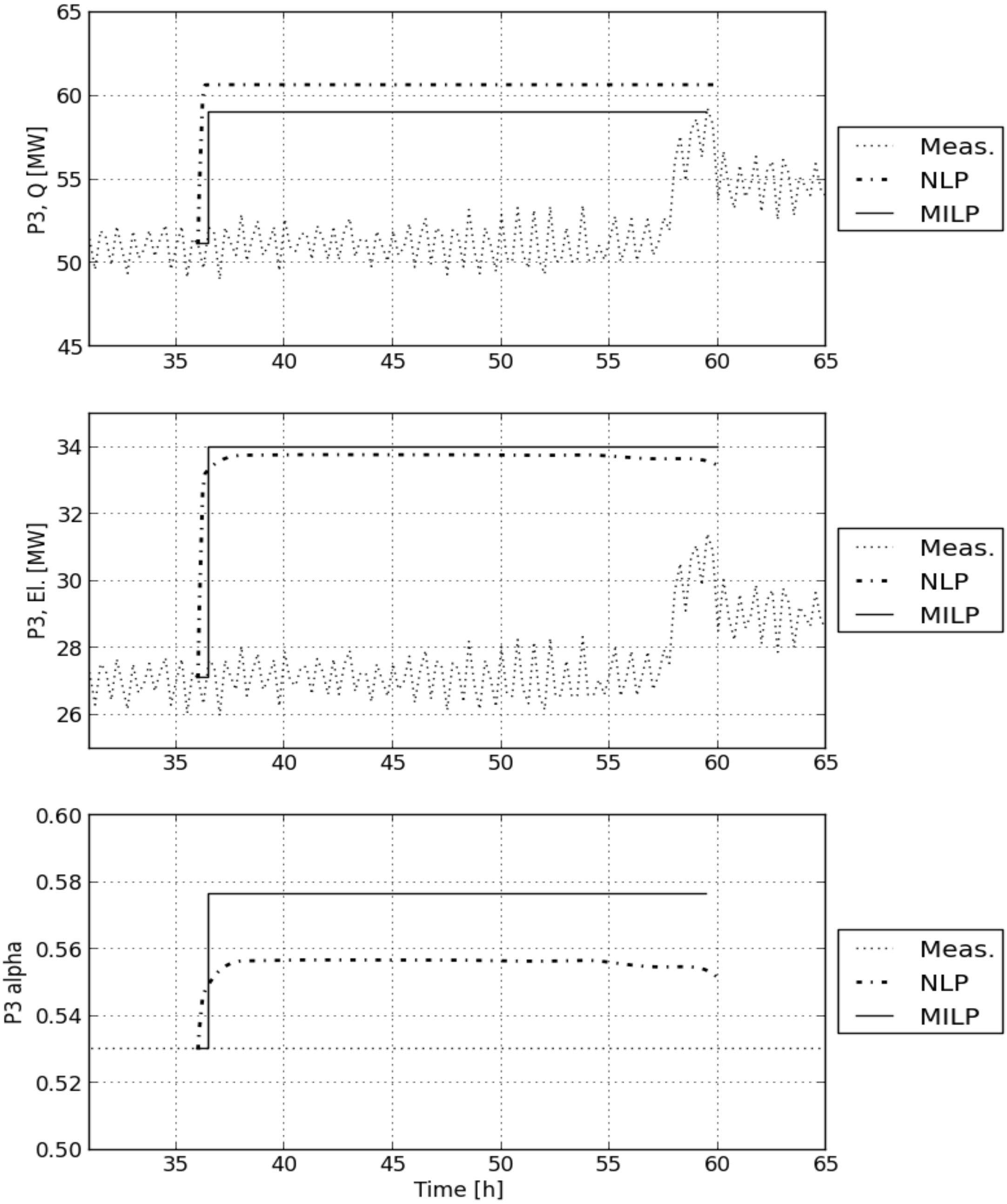


Figure 39 Case 3 measured and optimized P3 heat and electricity production and α value. Both EDP and UCP result in maximized electricity production and increased heat production.

Figur 39. Fall 3. Uppmätt och optimerad värme- och el-produktion samt alpha-värde för P3. Både EDP och UCP-lösningarna maximerar el och värmeproduktionen,

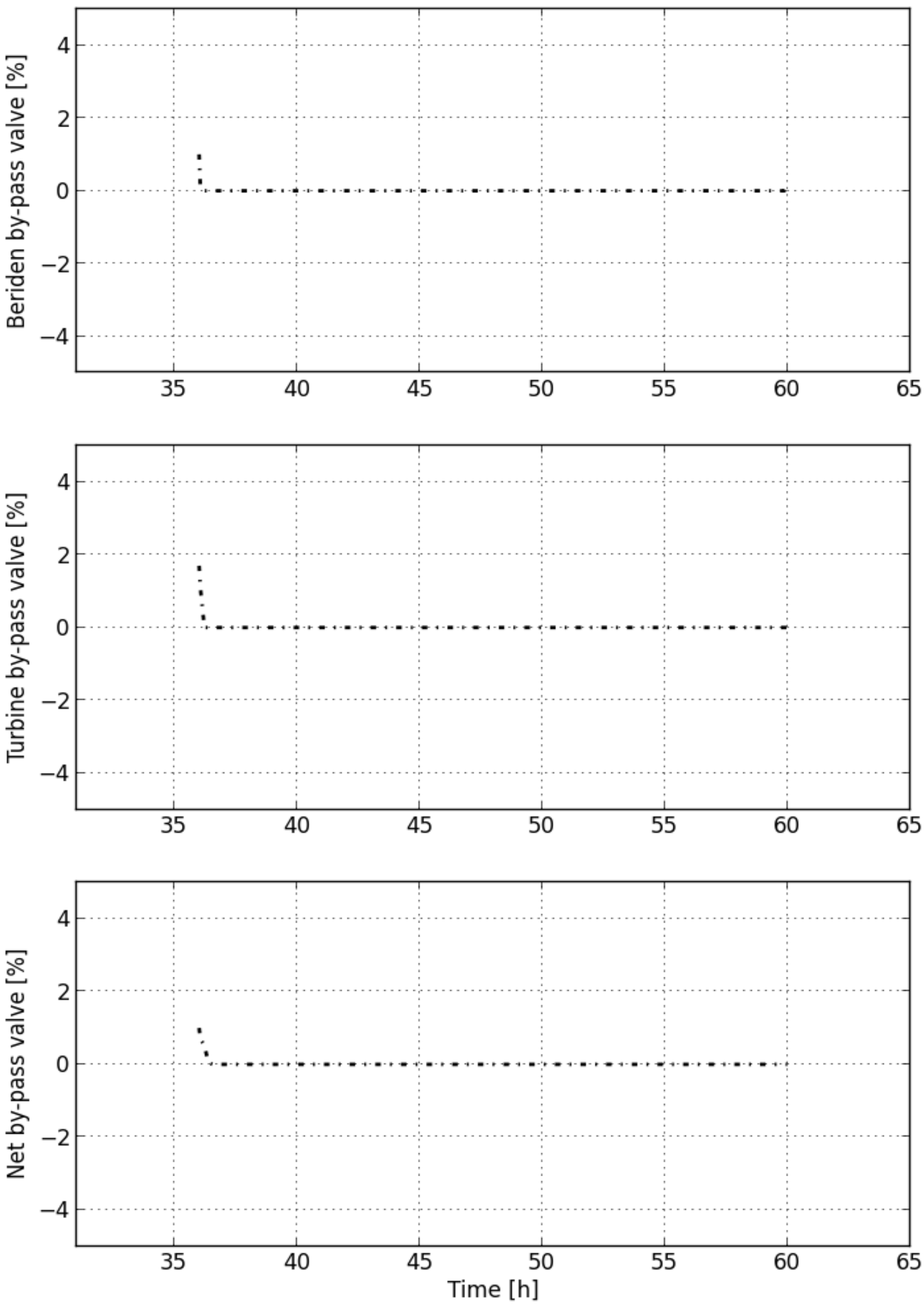


Figure 40 Case 3 optimized by-pass valve positions. All valves are closed.

Figur 40. Fall 3. Optimerade lägen för BP-ventilerna. Alla ventiler är stängda.

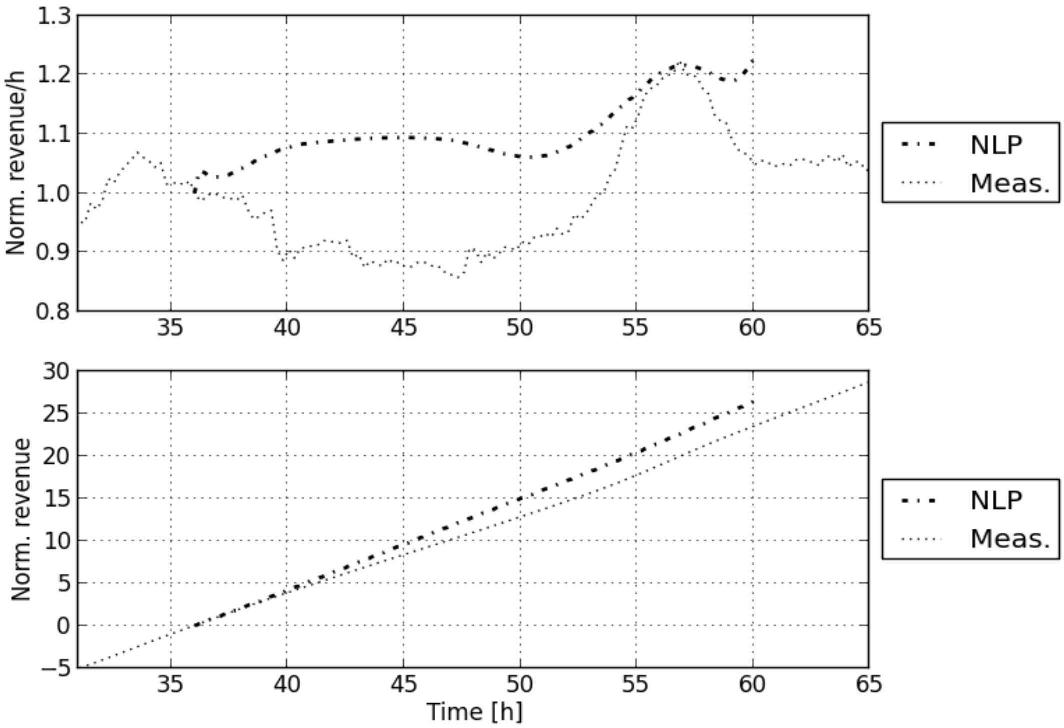


Figure 41 Case 3 measured and optimized normalized revenue/h and cumulative normalized revenue. The revenue/h is normalized such that the optimized solution starts at 1. The optimization shows a possibility of revenue increase.

Figur 41. Fall 3. Uppmätta och optimerade vinster (momentan och ackummulerad). Optimeringen visar en potentiell vinst.

9.6 Case 4: Customer Load with Two Peaks, Usage of Accumulator and Start-up of P2.

Data and figures

The measurement data used is 2013-01-22 22:00 - 2012-01-23 12:00 and the measurement data and optimization results can be found in Figure 42-Figure 50.

Decision variables

- UCP: See Section 6.2.1
- EDP: Derivatives of
  - P2 fuel load
  - P3 fuel load
  - Turbine by-pass valve position
  - District network by-pass valve position
  - Beriden by-pass valve position
  - Internal pump speed
  - Distribution pump speed

Main features

The electricity production in P3 is maximized and P2 is started to handle the customer load demand. The demand can, comparing to measurement data, be fulfilled by decreasing supply temperature and increasing the supply flow. The accumulator is used to handle customer load peak before P2 is started and compared to measurement data a revenue increase is possible.

Economic benefit

Comparing the cumulative revenue for the optimized result and the measurement, the optimized result gives approximately 27% higher revenue. The main reasons for this are:

1. Significantly higher electricity production in P3 by having closed turbine by-pass valve during the whole optimization horizon.
2. Decreased supply temperature and increase supply flow rate.
3. No start of P1, yielding no start cost

The economic benefit might in this case be overestimated as the reasons why P3 is run with such low electricity production in measurement data are unknown.

Clarification of UCP plant operation

In the measurement data, none of P1 or P2 is running at the beginning. As the heat load prediction is higher than what is possible to meet with only P3, flue gas condenser and accumulator, the UCP results in a direct start of P2. In the measurement data, however, P3 increases its heat production and decreases its electricity. Most probably, the turbine by-pass valve is fully open. Later on, both P1 and P2 are started in the measurement data and P3 production of heat and electricity go back to normal levels again. The start of P2 is beginning at the very first sample in the UCP result and the heat follows the start sequence of 10 h which first is a 1 h ramp up to 15 MW and then constant at 15 MW for 9 h. However, the heat produced is not increased from the minimum load until after staying at the minimum load for 12 h.

As the first peak of the customer load demand comes before P2 has been able to start up, the accumulator is used and its energy level is significantly decreased during the first half of the optimization interval. During the second half, when P2 has started, it is again loaded and slightly provides heat during the second heat load peak. Note that there may be several solutions regarding how P2 is increased from minimum load and how the accumulator is run, similar to Case 3.

The electricity production in P3 is directly increased at the first sample, as the electricity has the highest sell price.

Clarification of EDP plant operation

As the UCP result determined a start-up of P2, the heat produced in P2 follows the start-up sequence in the beginning, deviating at maximum 1 MW. The fuel load and turbine by-pass valve is increased and decreased, respectively, to make P3 maximize its electricity production. Even though the heat production of P3 is increased as well in the beginning, it cannot together with the start-up of P2 and the flue gas condenser meet the first customer load peak. The accumulator needs to be used, which is seen both in the accumulator energy, but also in the pump flows as the internal pump flow is small than the distribution pump flow.

After the first load peak, the accumulator is again charged and the distribution pump flow is smaller than the internal heat flow. The accumulator only provides a minor discharge of energy during the second customer heat demand peak. Instead, the peak is met by the load of P2, which started to increase directly after P2 finished its starting sequence. This increase of P2 is also what helps to charge the accumulator.

During almost the whole optimization horizon, the supply temperature is lower than what the measurement data shows which saves the fuel burn of the units. However, this also requires the pump flows to be increased such that the heat load demand of the customers is met. The pump flows are, during the whole optimization horizon, larger than what the measurement data shows.

Beriden is not used at all as the customer heat demand is higher than the heat produced by P3 when it is producing maximum electricity.

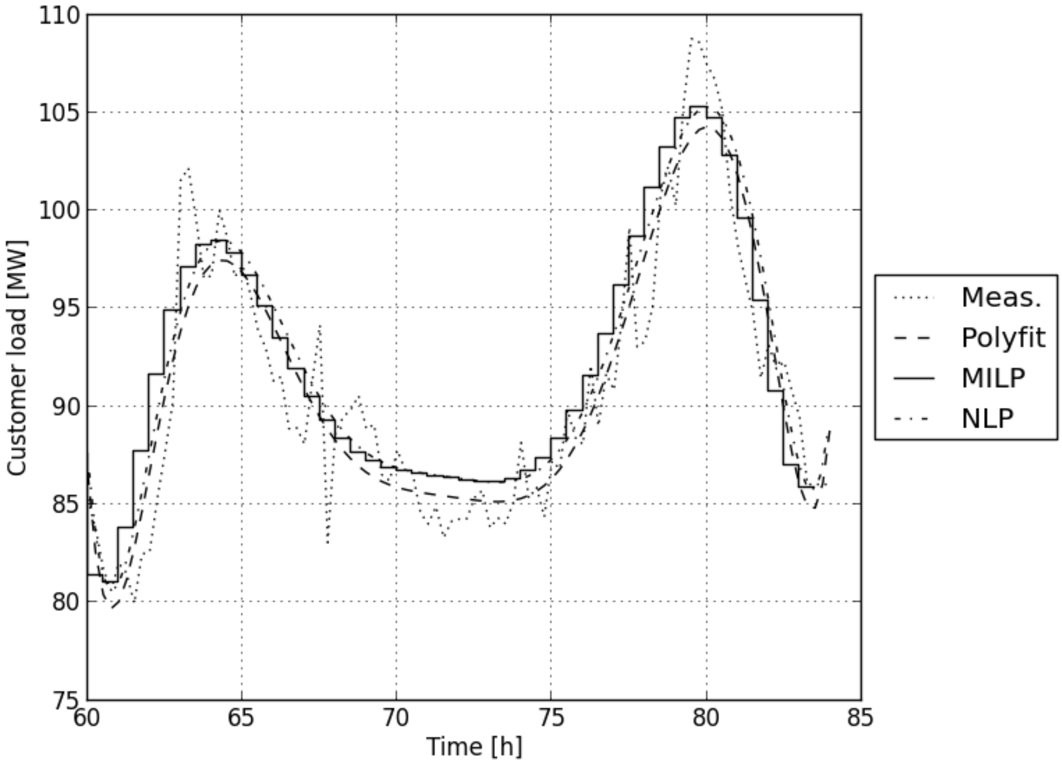


Figure 42 Case 4 measured and optimized customer heat load. The heat load demand is followed within specification of ±1MW.

Figur 42. Fall 4. Uppmätt och optimerad värmelast hos kunderna. Värmeproduktionen följer kundlasten enligt specifikationerna (±1MW).

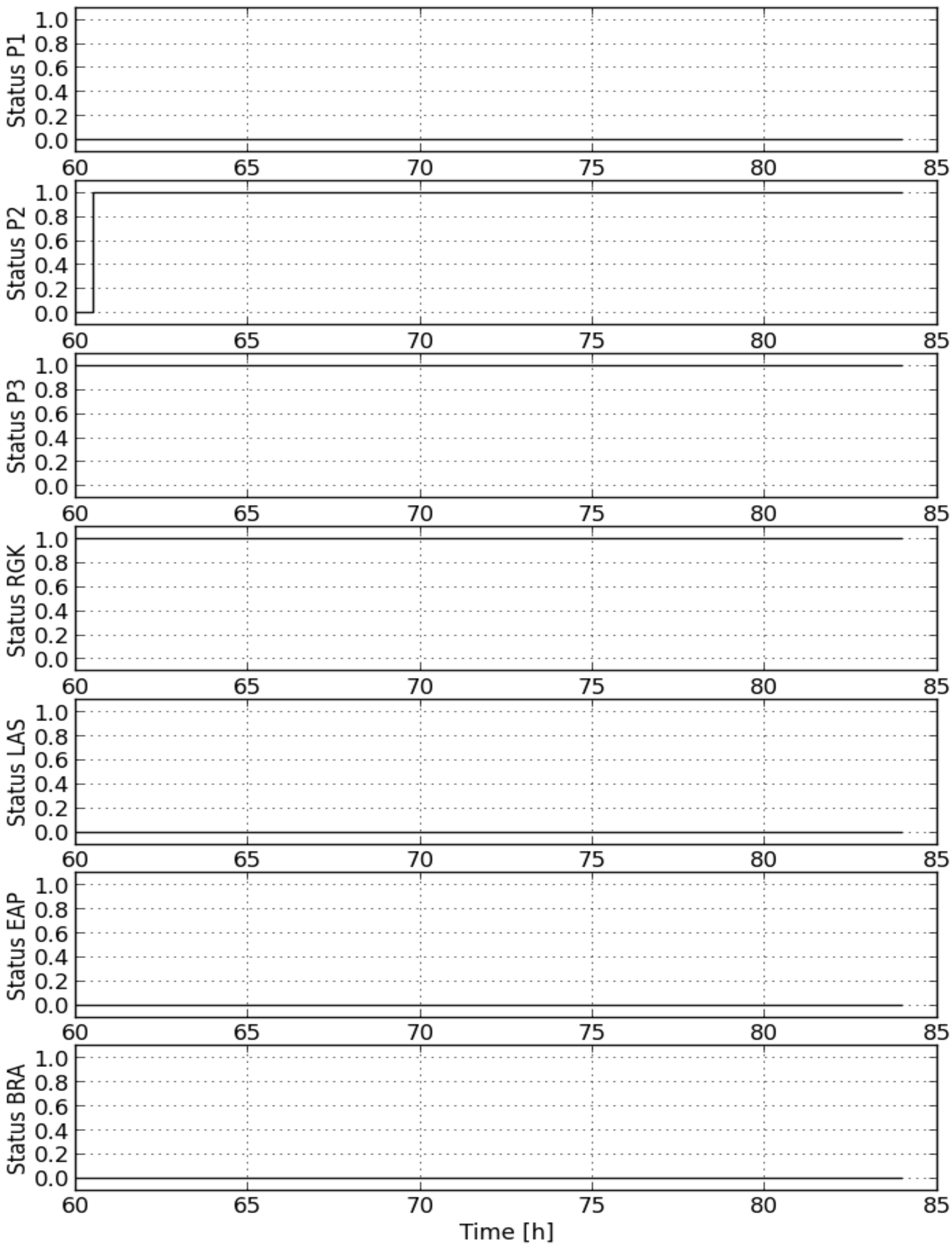


Figure 43 Case 4 optimized statuses of producers in the UCP. P2 is started in the beginning of the optimization interval.

Figur 43. Fall 4. Optimerade tillstånden för producenterna. P2 startar I början av optimeringen.

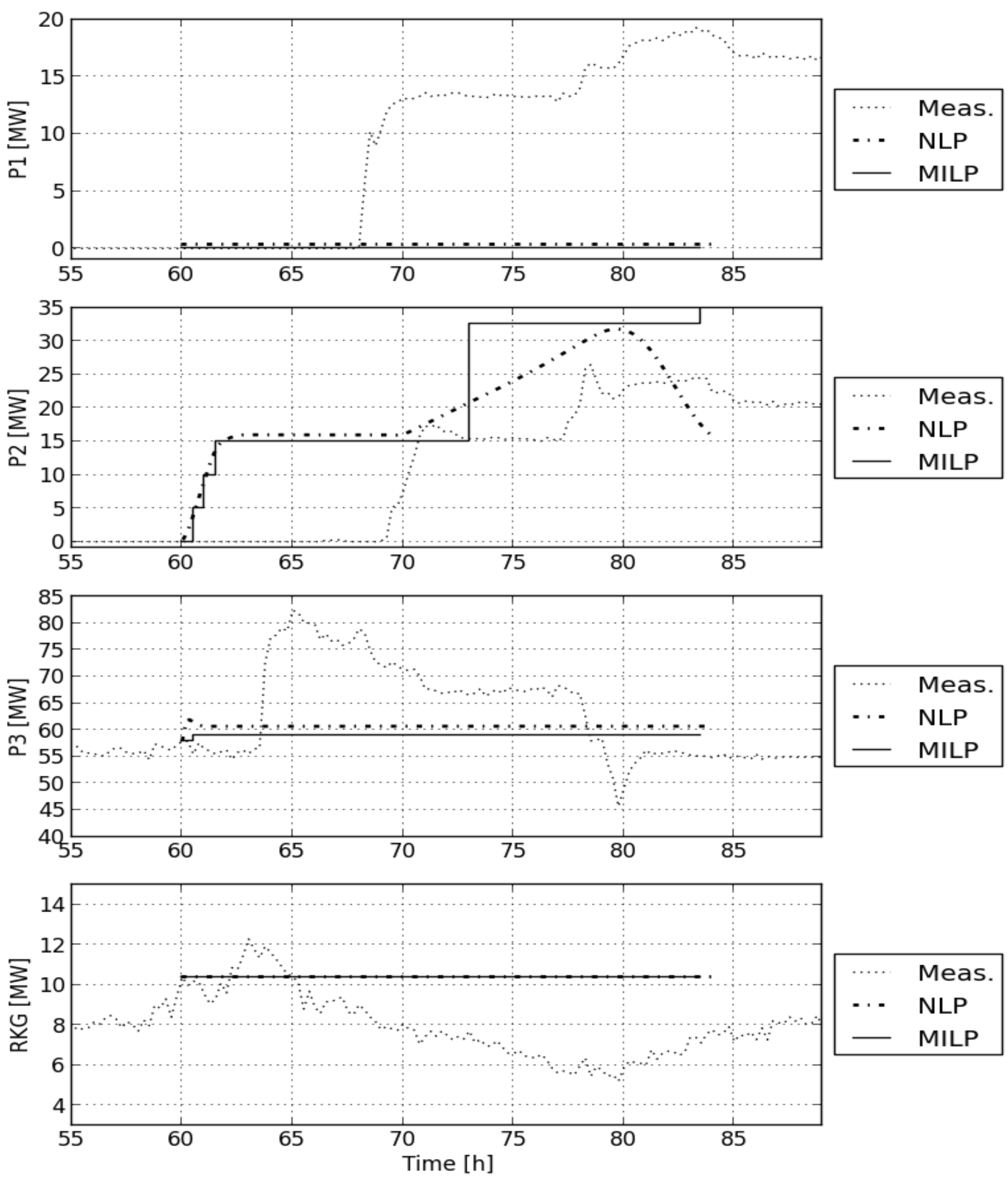


Figure 44 Case 4 measured and optimized heat production at P1, P2, P3 and RGK. Compared to measurement data, P1 is not started at all while P2 is started before. The peak in P2 heat production handles the second customer heat load peak.

Figur 44. Fall 4. Uppmätt och optimerad värmeproduktion för P1, P2, P3 och RGK. Jämfört med mätdata, P1 startar inte medan P2 startar tidigare. Toppen i P2's last hanterar den andra ökningen i kundernas last.

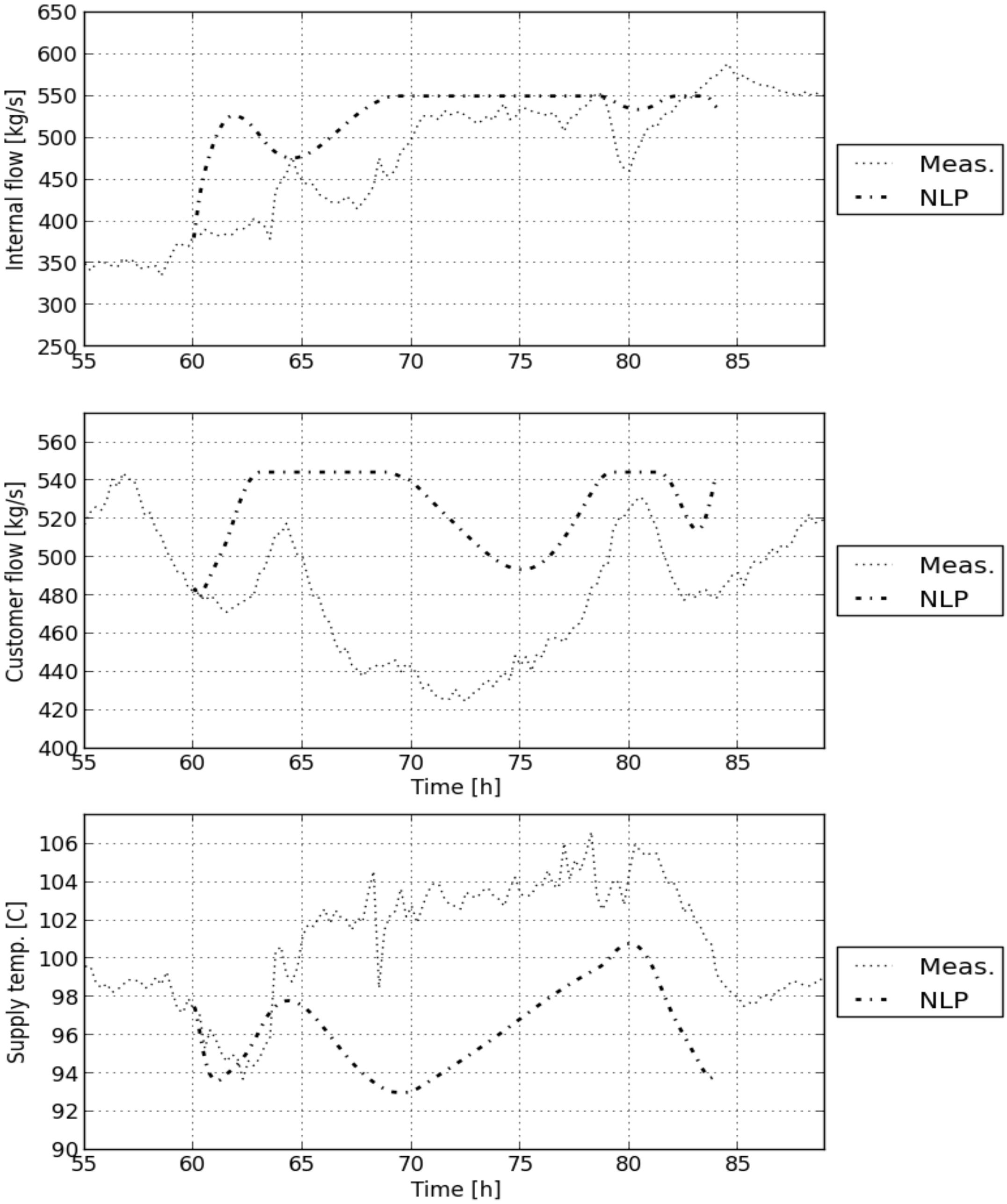


Figure 45 Case 4 measured and optimized internal and customer flow and supply temperature. Compared to measurement data, the supply temperature is decreased and the customer flow increased.

Figur 45. Fall 4. Uppmätt och optimerade flöden och temperatur i framledningen. Den optimerade temperaturen är lägre än i mätdata och det optimerade flödet ligger nära det maximala flödet.

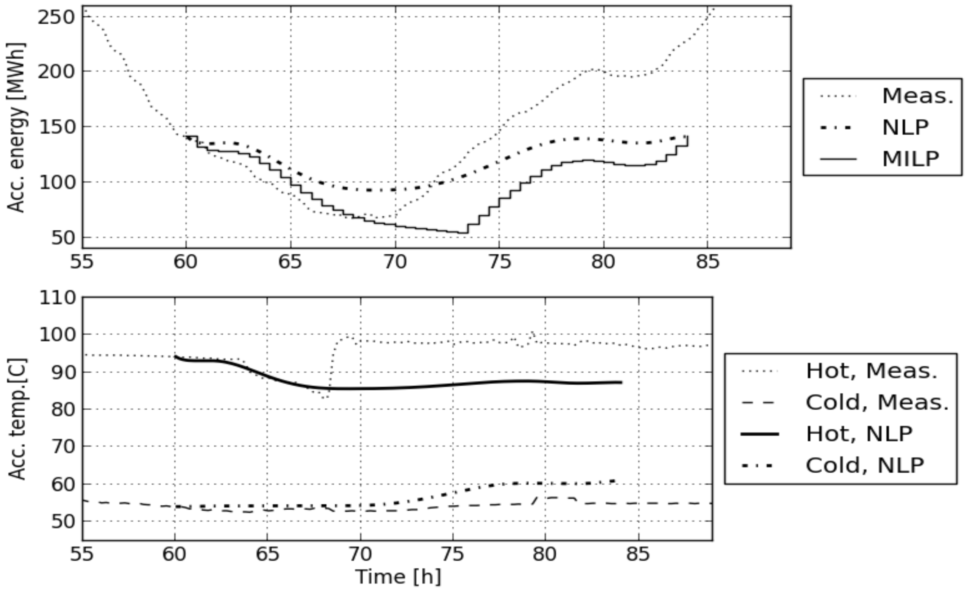


Figure 46 Case 4 measured and optimized accumulator energy and top (hot) and bottom (cold) accumulator temperatures. In both UCP and EDP, the accumulator is used for fulfilling the first customer heat load peak.

Figur 46. Fall 4. Uppmätt och optimerade energy samt temperatur i ackummulatorn. Både UCP och EDP-lösningarna laddar ackummulatorn för att hantera den 1:a ökningen i kundernas last.

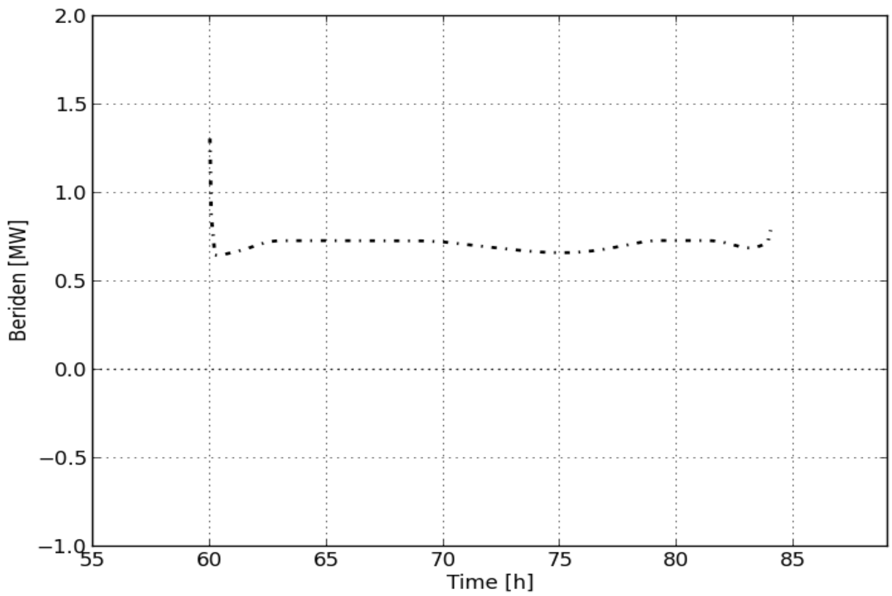


Figure 47 Case 4 measured and optimized Beriden cooling. No cooling by Beriden is performed.

Figur 47. Fall 4. Optimerad extern kylning. Ingen kylning sker.

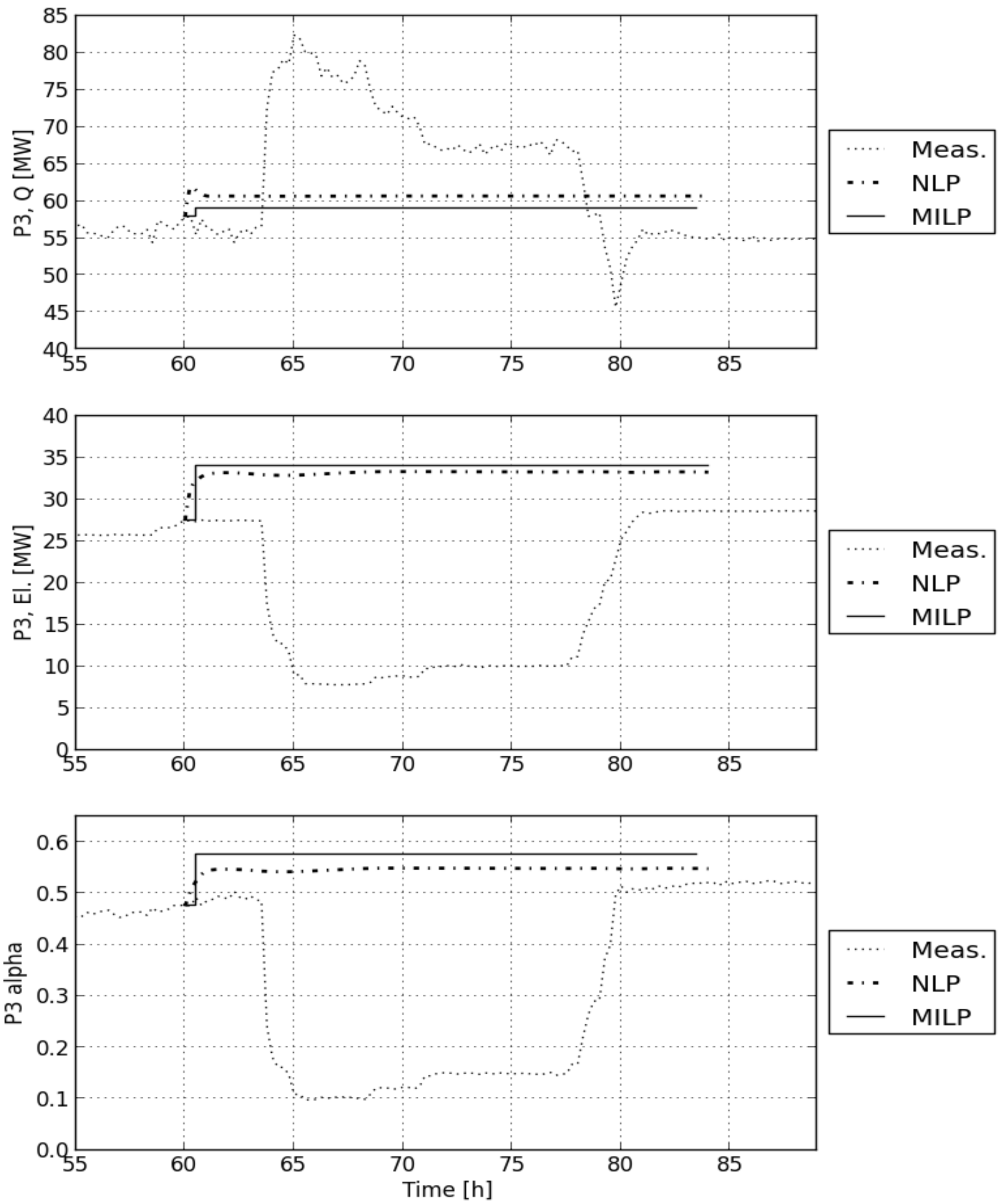


Figure 48 Case 4 measured and optimized P3 heat and electricity production and α value. Both the UCP and the EDP optimization shows a maximization of P3 electricity production.

Figur 48. Fall 4. Uppmätta och optimerade värden för el-, värmeproduktion samt alpha-värdet för P3. Både EDP och UCP-lösningarna maximerar el-produktionen.

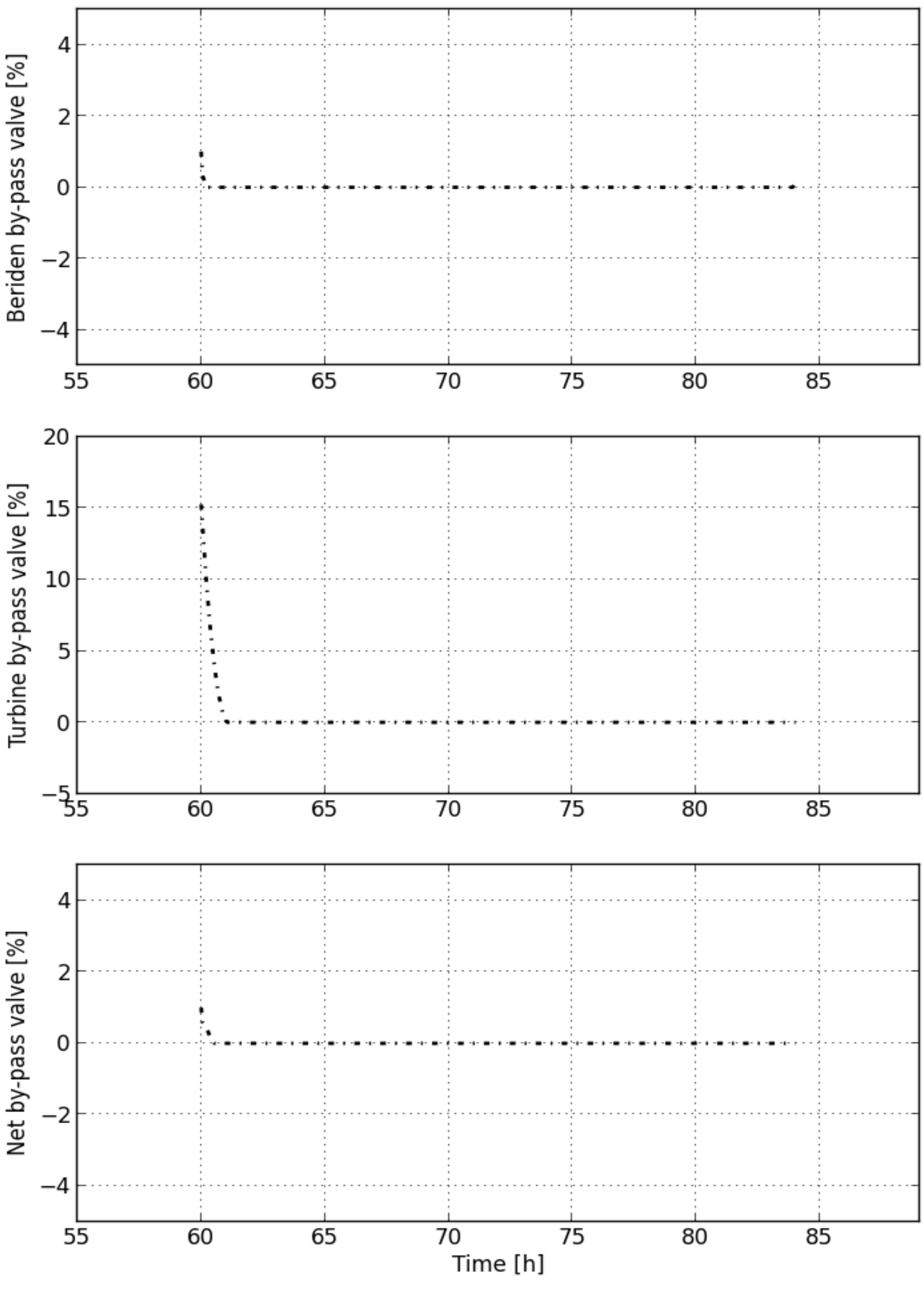


Figure 49 Case 4 optimized by-pass valve positions. All valves are closed.

Figur 49. Fall 4. Optimerade lägen för BP-ventiler: alla ventiler är stängda.

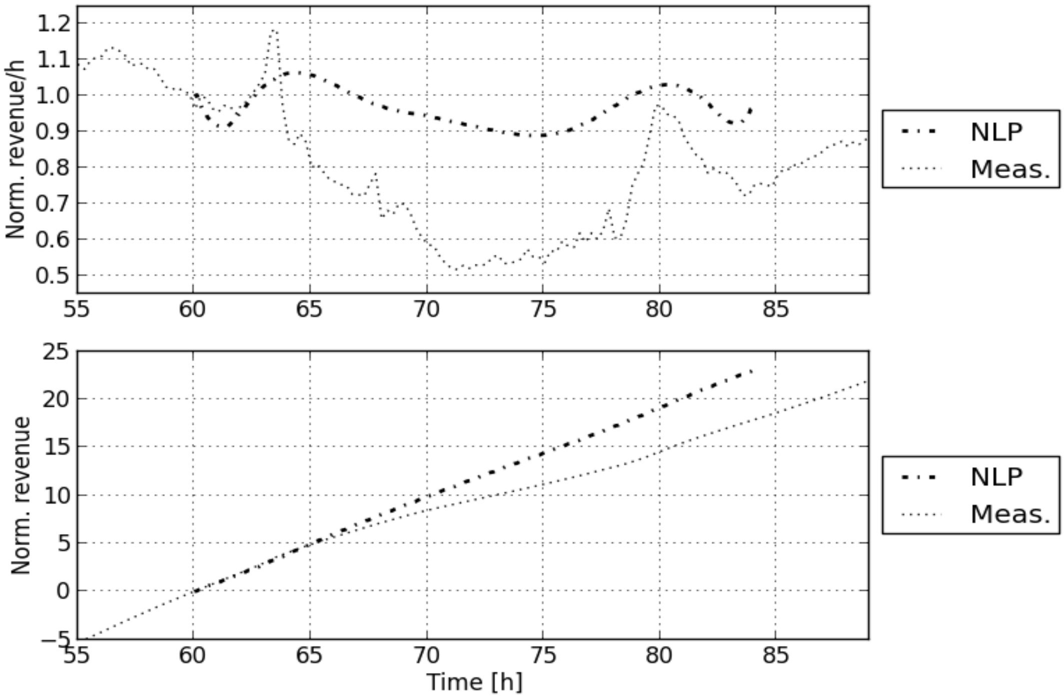


Figure 50 Case 4 measured and optimized normalized revenue/h and cumulative normalized revenue. The revenue/h is normalized such that the optimized solution starts at 1. The optimization shows a possibility of revenue increase.

Figur 50. Fall 4. Uppmätta och optimerade vinster (momentan och ackummulerad).

9.7 Case 5: Decreasing Customer Load and Shut-down of P1 and P2.

Data and figures

The measurement data used is 2013-01-25 15:00 - 2012-01-25 15:00 and the measurement data and optimization results can be found in Figure 51-Figure 59.

Decision variables

- UCP: See Section 6.2.1
- EDP: Derivatives of
  - P1 fuel load
  - P2 fuel load
  - P3 fuel load
  - Turbine by-pass valve position
  - District network by-pass valve position
  - Beriden by-pass valve position
  - Internal pump speed
  - Distribution pump speed

Main features
The electricity production is maximized in P3 and with a decreasing customer heat load demand both P1 and P2 are shut down. Compared to measurement data the customer supply temperature can be decreased and the customer supply flow can be increased. The accumulator is used during the shut-down of P1 and P2 and a revenue increase, compared to measurement data, may be done.

Economic benefit
A comparison between the optimized and measured cumulative revenues shows that the optimized revenue is approximately 13% higher than the measured.
The main reasons for this are:

1. Higher electricity production in P3.
2. Decreased supply temperature and increase supply flow rate.
3. Sooner shut-down of unneeded units.

Clarification of UCP plant operation
The load prediction is essentially decreasing during the whole optimization interval. Thus, the produced heat should decrease as well which is also the result of the UCP. In the initial point, P1, P2, P3 and the flue gas condenser are running. However, both P1 and P2 are running close their minimum load of 15 MW. As the optimization knows from the prediction that the heat demand is decreasing, it directly turns P1 off, making it follow the 5 h stop sequence of 4 h constant heat production at 15 MW and then a 1 h ramp down to 0 MW. Until the heat load prediction has become so low that it can be met by only P2, P3 and the flue gas condenser, the accumulator is discharged. When this point has been reached, the accumulator is again charged. At the end of the optimization interval, the heat demand can be met also without P2, which is turned off and follows a stop sequence identical to the one followed by P1.

Clarification of EDP plant operation
As the UCP results ordered P1 to shut down directly, the heat of P1 follows the stop constraints and then remains off. P2 decreases the heat production as well, but stays on the minimum load limit initially. The customer heat load demand in the optimization interval beginning is met as the pump flow is increased, i.e., a lower supply temperature may be used, and that the P3 load is increased. This load increase, together with closing the turbine by-pass valve, yields P3 to produce more, highly priced electricity which is at its maximum.

When P2 is completely turned off, the accumulator needs to be discharged to meet the customer heat load until the load has become even lower. This is seen in both accumulator energy and that the internal flow is smaller than the distribution flow. When the customer heat load has become so low that the accumulator is not needed any more, and that P2 together with P3 and the flue gas condenser are producing more heat than to only cover the customer heat load, it is charged again. The usage of the accumulator is thus similar to the usage in the UCP. At the end of the optimization interval, the customer load is so low that not even P2 is needed anymore. It is turned off, i.e., it follows the stop constraints, and remains turned off.

A comparison to the measurement data shows that the optimized supply temperature and flow rate are lower and larger, respectively. During the whole optimization interval, one of the two pump flows is always on its upper constraint, i.e., 550 kg/s.

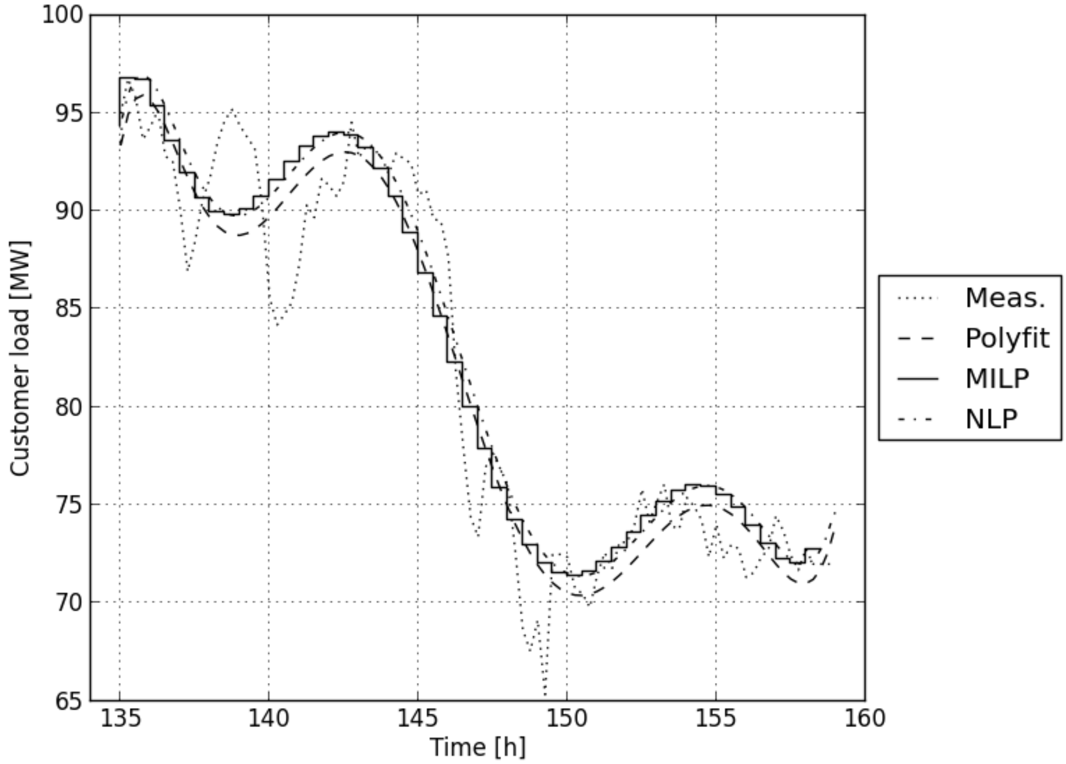


Figure 51 Case 5 measured and optimized customer heat load. Heat load demand is followed within specification of ±1MW.

Figur 51. Fall 5. Uppmätt och optimerad värmelast hos kunderna. Värmeproduktionen följer kundlasten enligt specifikationerna (±1MW).

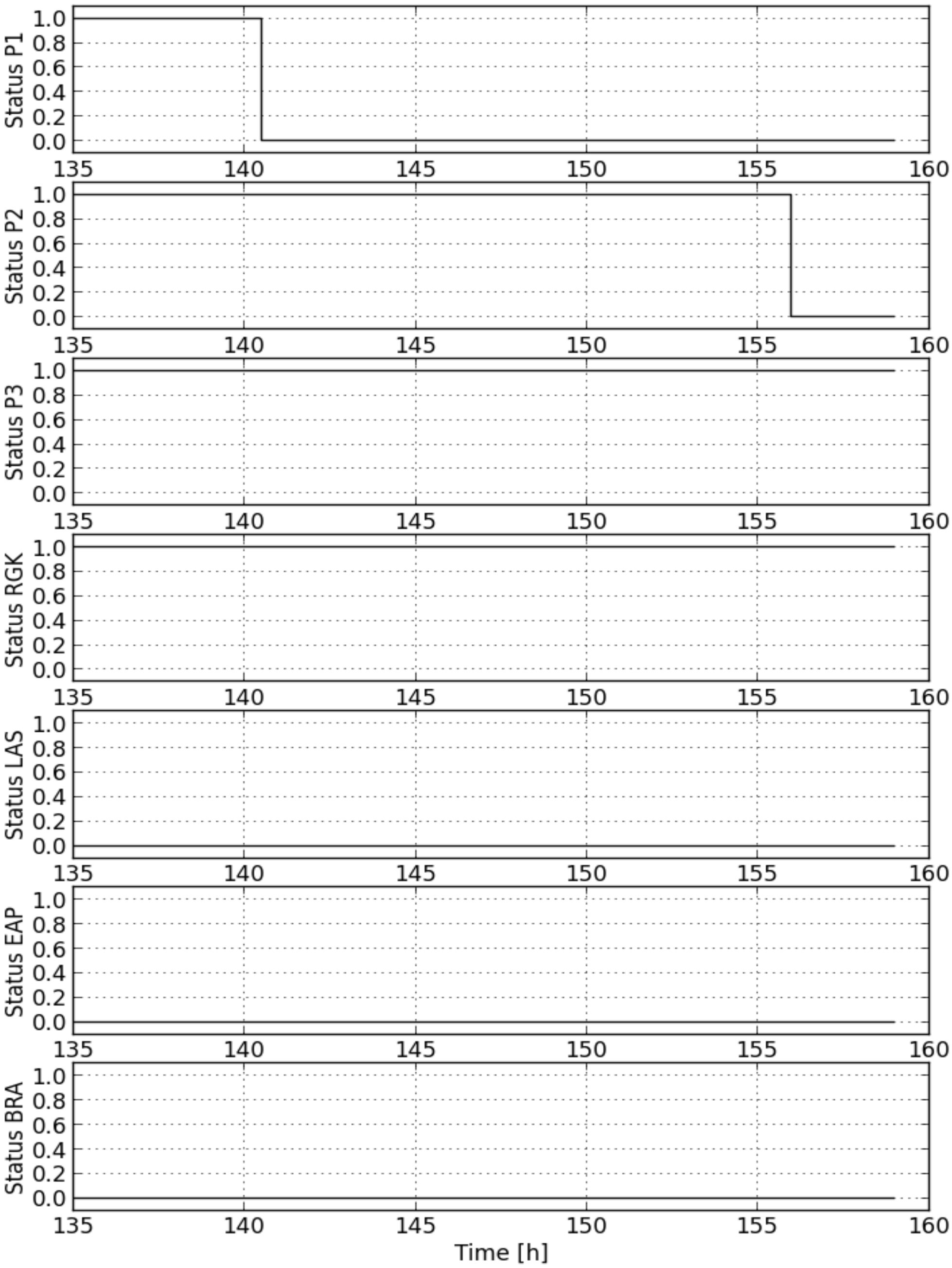


Figure 52 Case 5 optimized statuses of producers in the UCP. P1 and P2 are shut-down due to decreasing customer heat load.

Figur 52. Fall 5. Optimerade tillstånden I UCP-problemet. P1 och P2 stängs av på grund av den minskande kundlasten.

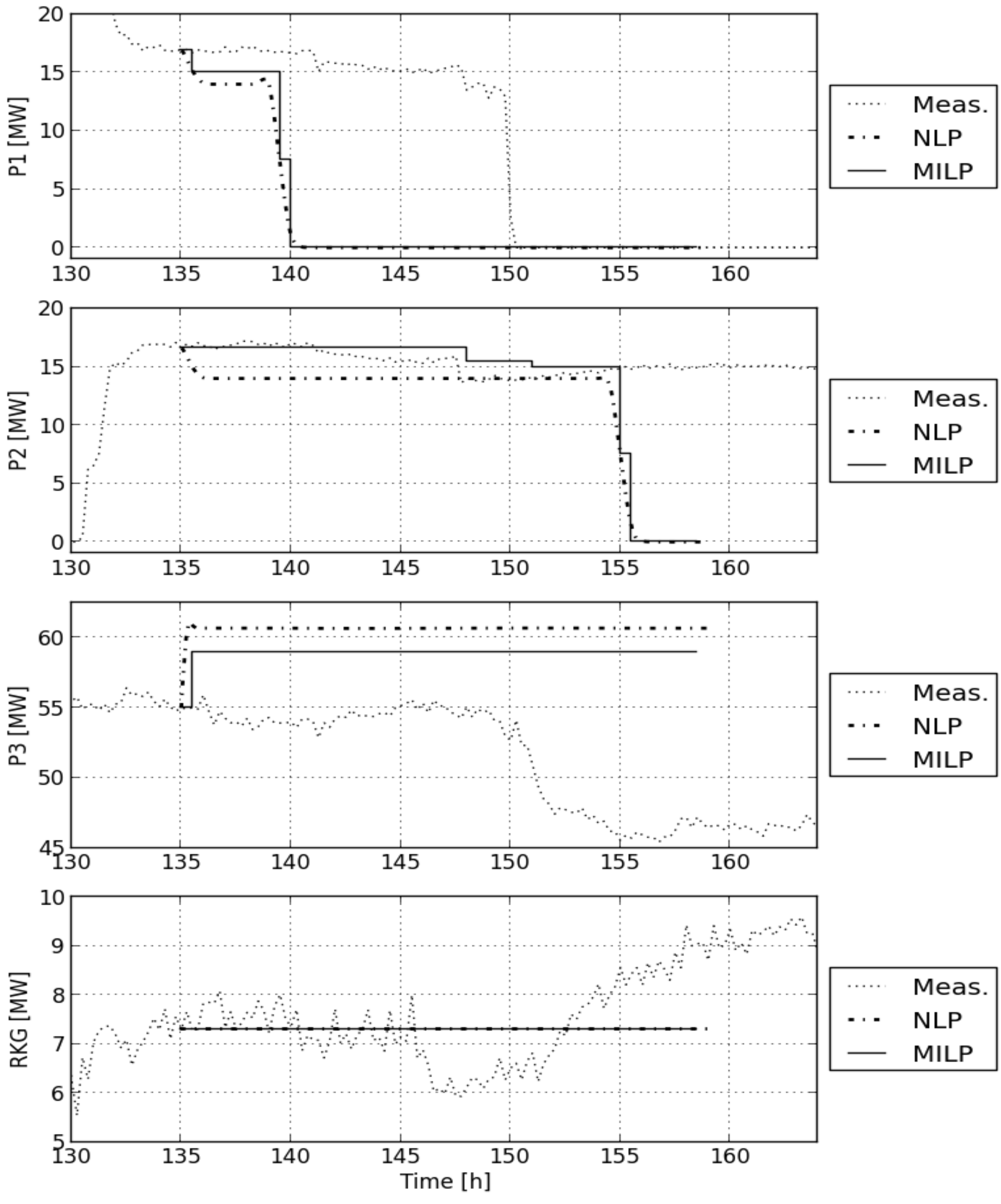


Figure 53 Case 5 measured and optimized heat production at P1, P2, P3 and RGK. P1 and P2 are shut down due to decreased customer heat demand and P3 heat production is increased due to increased electricity production.

Figur 53. Fall 5. Uppmätt och optimerad värmeproduktion för P1, P2, P3 och RGK. Panna 1 och 2 stängs av medan lasten till P3 ökar för en högre el-produktion.

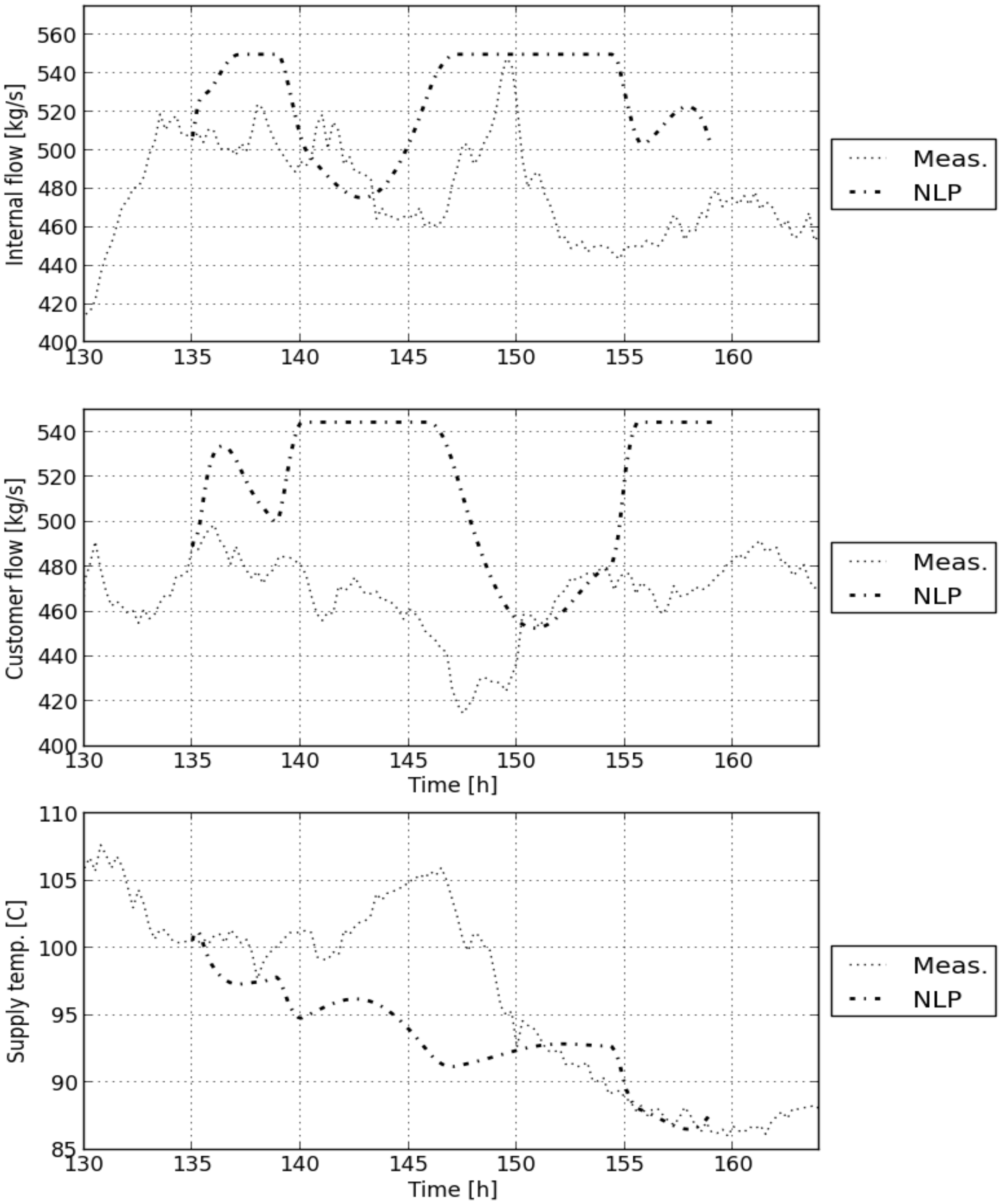


Figure 54 Case 5 measured and optimized internal and customer flow and supply temperature. Compared to measurement data, supply temperature is decreased and customer flow increased.

Figur 54. Fall 5. Uppmätt och optimerade flöden och temperatur i framledningen. Jämfört med mätdata, framledningstemperatur minskar och flödet ökar.

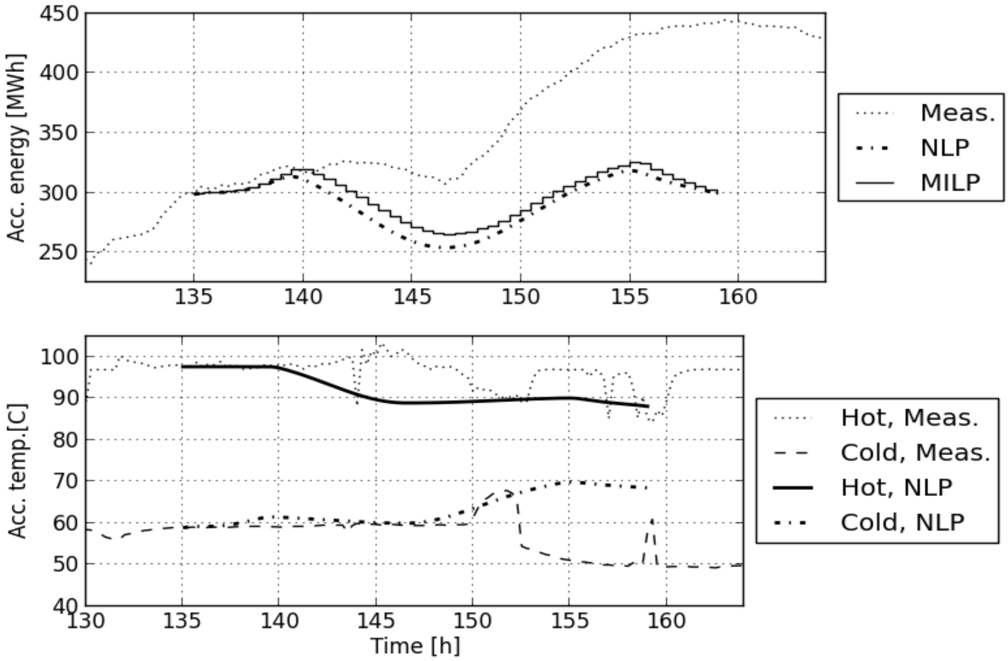


Figure 55 Case 5 measured and optimized accumulator energy and top (hot) and bottom (cold) accumulator temperatures. The accumulator is used in both UCP and EDP when P1 is turned off.

Figur 55. Fall 5. Uppmätt och optimerade energi och tmeperatur i ackummulatorn. Ackummulatorn används i både UCP och EDP-problemen när P1 stängs av.

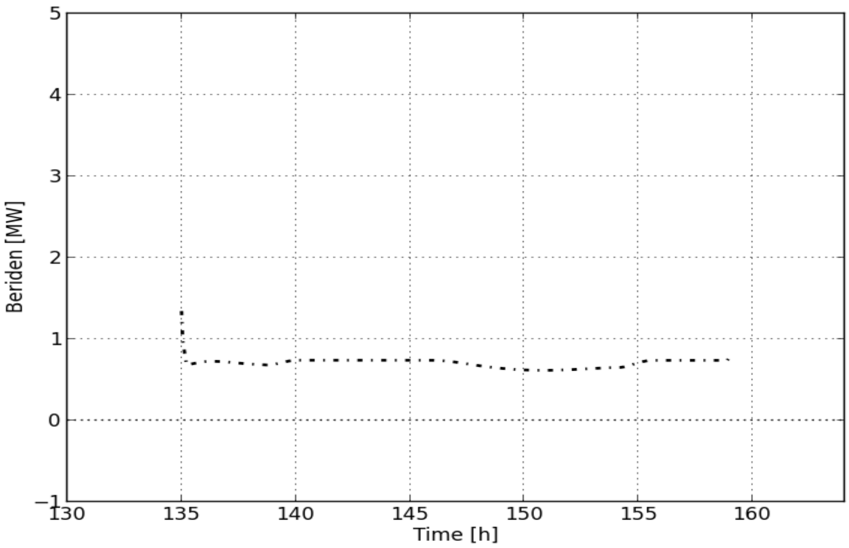


Figure 56 Case 5 measured and optimized Beriden cooling. No cooling by Beriden is performed.

Figur 56. Fall 5. Uppmätt och optimerad Beridens kylleffekt. Ingen kylning sker.

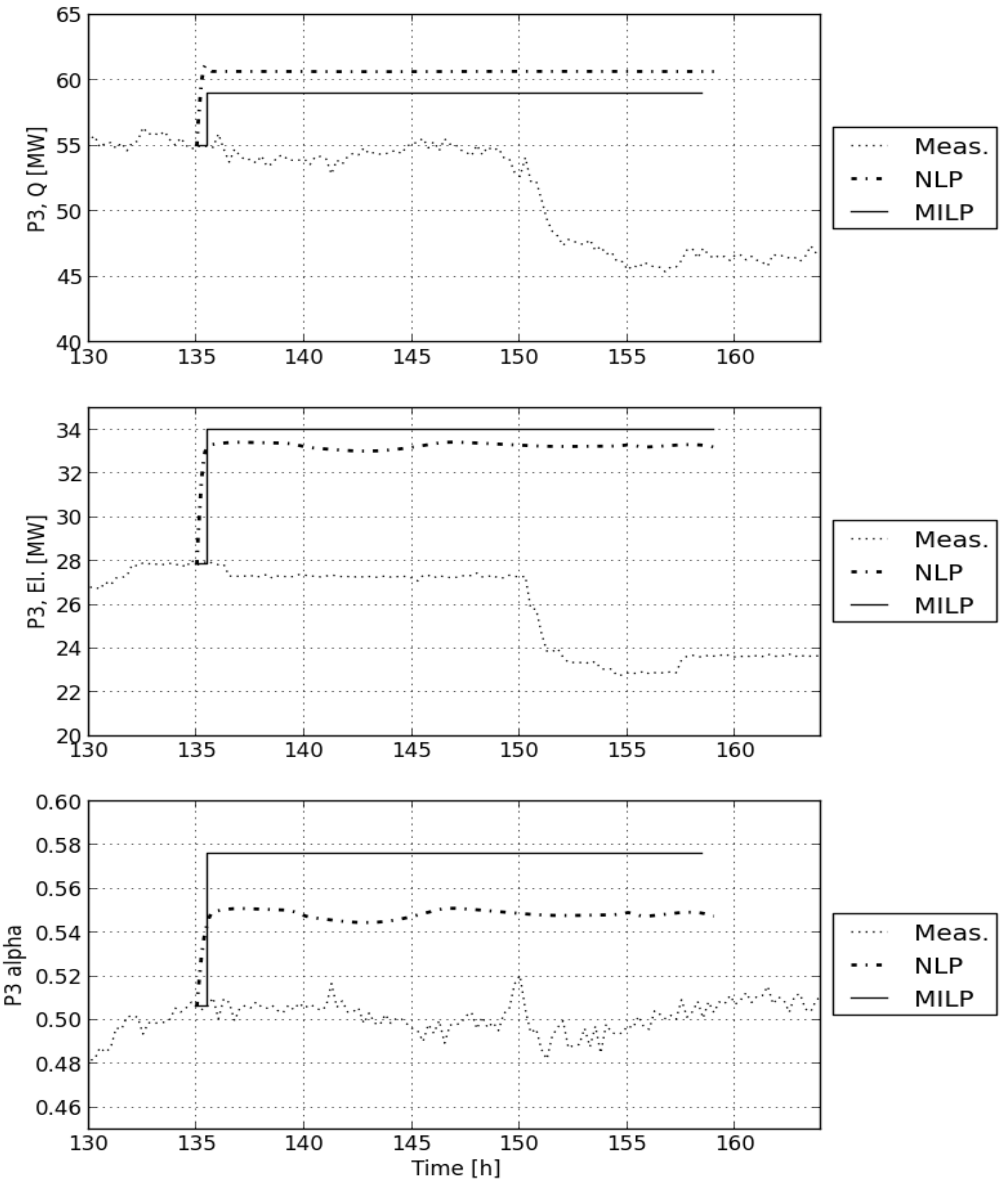


Figure 57 Case 5 measured and optimized P3 heat and electricity production and α value. Both UCP and ECP results in maximized electricity production.

Figur 57. Fall 5. Uppmätt och optimerade värden för el-, värmeproduktion samt alpha-värdet för P3. Både UCP och EDP maximerar elproduktionen.

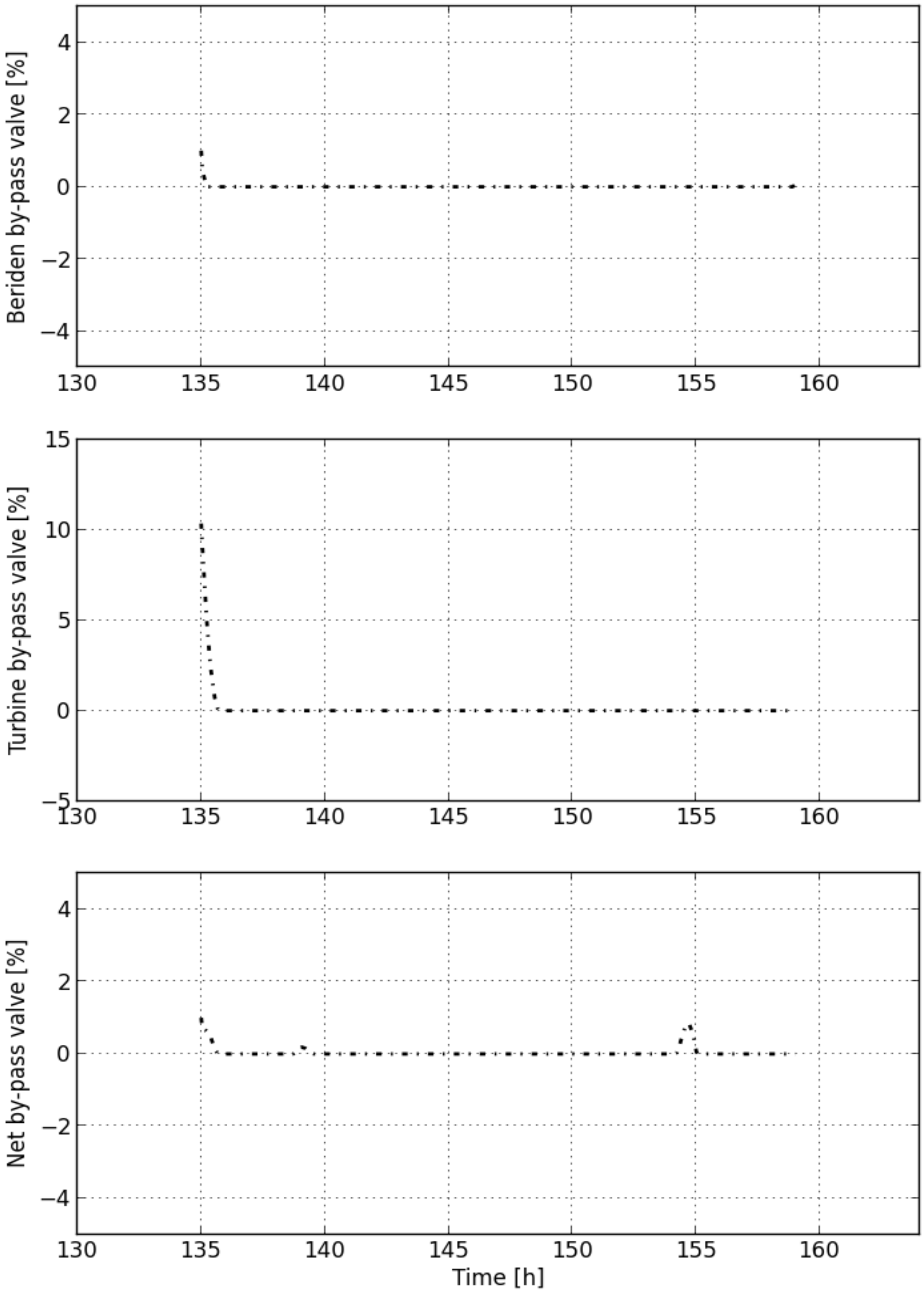


Figure 58 Case 5 optimized by-pass valve positions. All valves are closed.

Figur 58. Fall 5. Optimerade BP-ventiler: alla ventiler är stängda.

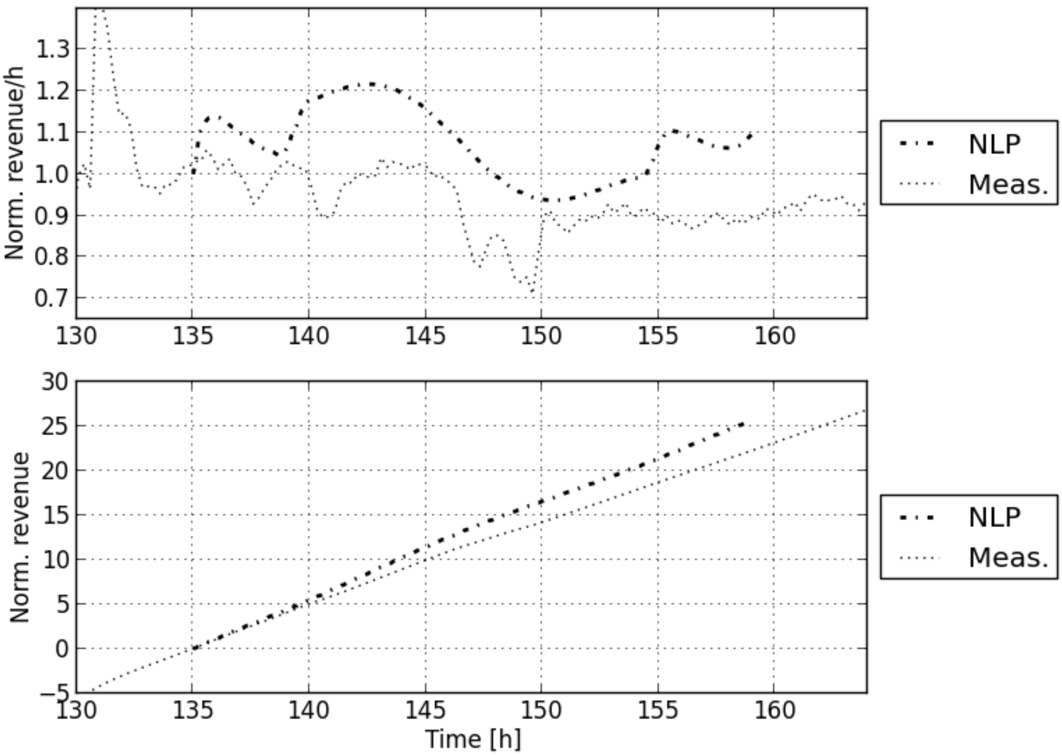


Figure 59 Case 5 measured and optimized normalized revenue/h and cumulative normalized revenue. The revenue/h is normalized such that the optimized solution starts at 1. The optimization shows a possibility of revenue increase.

Figur 59. Fall 5. Uppmätta och optimerade vinster (momentan och ackummulerad). Optimeringen visar en potentiell vinst.

9.8 Case 6: One-peaked Customer Load and Time-varying Electricity Price

In this case, the price of electricity is not constant during the optimization horizon as it is in the other optimization cases. During the first 10 h of the optimization interval, the price stated in Table 8 is used, while for the last 14 h the price is halved which makes the electricity worth less than heat to sell.

Data and figures

The measurement data used is 2012-01-13 12:00 - 2012-01-14 12:00 and the measurement data and optimization results can be found in Figure 60 Case 6 measured and optimized customer heat load. The heat load demand is followed within specification of ±1MW.Figure 60-Figure 68. This is the same measurement data as used in Case 3.

Decision variables

- UCP: See Section 6.2.1
- EDP: Derivatives of
  - P2 fuel load

- P3 fuel load
- Turbine by-pass valve position
- District network by-pass valve position
- Beriden by-pass valve position
- Internal pump speed
- Distribution pump speed

Main features

The electricity price is time varying, resulting in maximization electricity or increased heat production in P3 and usage of the accumulator. P2 remains at minimum load during whole optimization. Compared to measurement data, the customer supply temperature and flow may be decreased and increased, respectively, and a revenue increase is possible.

Economic benefit

A comparison between the optimized and measured cumulative revenues shows that the optimized revenue is approximately 8% higher than the measured. The main reasons for this are:

1. Higher electricity and heat production in P3.
2. Decreased supply temperature and increase supply flow rate.

The optimized result and the measurements are in this case not easily comparable since the drastic price change of electricity most certainly was not present when the measurements were taken.

Clarification of UCP plant operation

The optimization starts with P2, P3 and the flue gas condenser producing heat. The load on P3 is increased directly such that the high sell-priced electricity production is increased. The associated increased heat production of P3, together with P2 being on the minimum load constraint, the flue gas condenser and the accumulator can handle the first small peak in the customer heat load. However, it results in a clear decrease in accumulator energy.

As the electricity price is halved, a minor decrease is made in the electricity production and minor increase is made in the heat production of P3 such that the accumulator is charged slightly. However, before the high customer heat load peak, the electricity production is heavily decreased and heat production heavily increased such that the accumulator is appropriately loaded to be able to help P3 and P2 handling the peak. The charging is performed such that when the accumulator is discharged its energy ends at the end point constraint.

Compared to Case 3, the change in electricity price gives a different optimization result. As the optimization procedure knows that it is beneficial to have high heat production after 10 h, it can focus on electricity production and use the accumulator the first 10 h and charge it afterwards.

Even though P2 is at minimum load during the whole optimization, its contribution is needed for loading of the accumulator before the high customer heat load peak and meeting the high customer heat load peak. Optimizations have been performed with a much lower start-up cost of P2, which results in a stop and start of P2 during the first minor peak. However, when using the more reasonable start-up cost of P2 it is too expensive to perform the stop and start of it and instead it is kept at minimum load during the whole optimization.

Clarification of EDP plant operation

As for the UCP optimization, the electricity production in P3 is increased directly. The first heat load peak is handled by the associated increased heat production in P3, the flue gas condenser, P2 on its minimum heat load and the accumulator. The supply temperature is decreased directly, which requires an increase in supply flow to meet the customer heat load demand.

When the electricity price is halved, i.e., the heat has a higher sell-price than electricity, at 10 h into the optimization, P3 starts to emphasize its heat production instead of electricity production. This is made by holding the fuel load high and opening the turbine by-pass valve, i.e., more steam is routed to the condensers instead of the turbines. The heat increase is not made to maximum heat production; instead it is only increased such that the customer heat load can be met together with P2 on minimum load and at the same time charge the accumulator. The charging of the accumulator is performed such that it can be discharged appropriately during the high customer load peak at the end, yielding an accumulator energy ending at the end point constraint and P2 being at minimum load during the whole optimization.

Just as in the previous five optimization cases, the supply temperature is decreased and supply flow increased compared to the measurement data. The supply flow is close to its maximum limit during the whole optimization interval.

The external cooler Beriden is not used at all as all the heat produced in P3 when producing maximum electricity may be used to supply customers or load the accumulator.

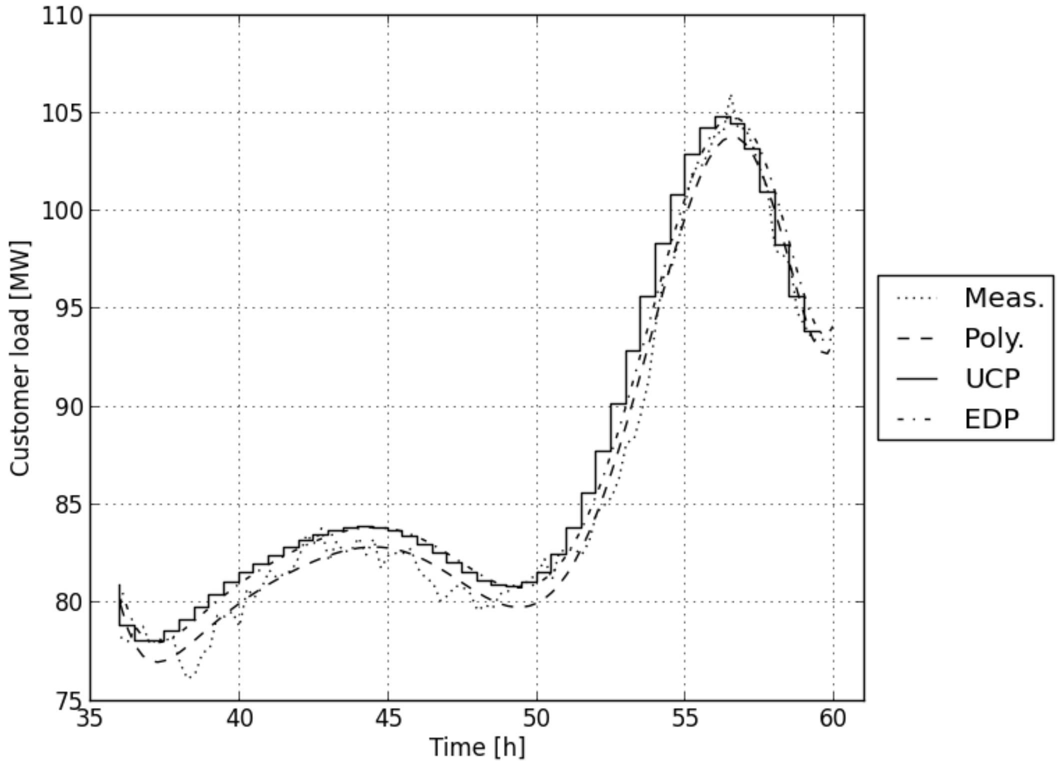


Figure 60 Case 6 measured and optimized customer heat load. The heat load demand is followed within specification of ±1MW.

Figur 60. Fall 6. Uppmätt och optimerad värmelast hos kunderna. Värmeproduktionen följer kundlasten enligt specifikationerna (±1MW).

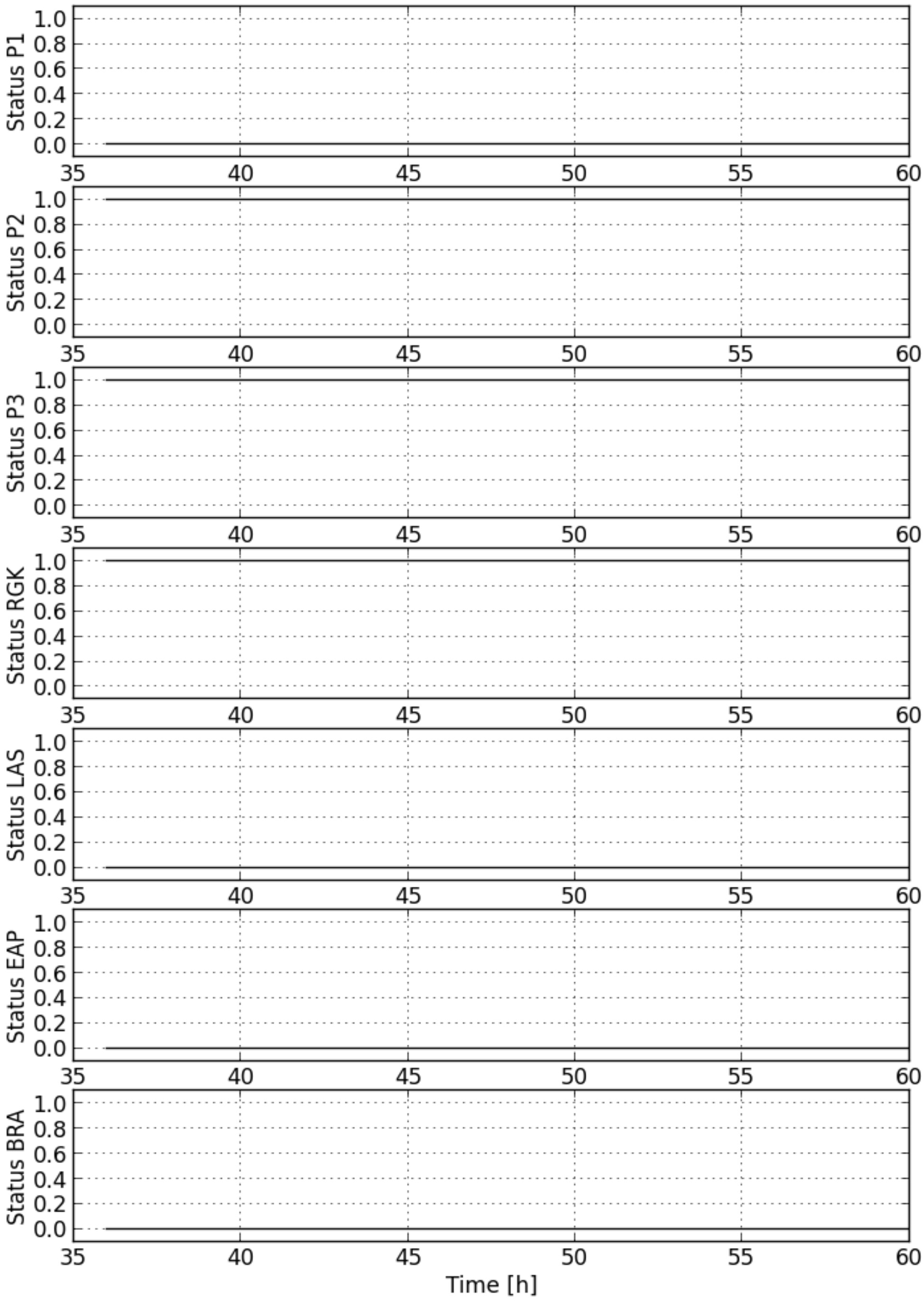


Figure 61 Case 6 optimized statuses of producers in the UCP. No unit changes its status.

Figur 61. Fall 6. Optimerade tillstånden för producenterna. Ingen tillståndsförändring.

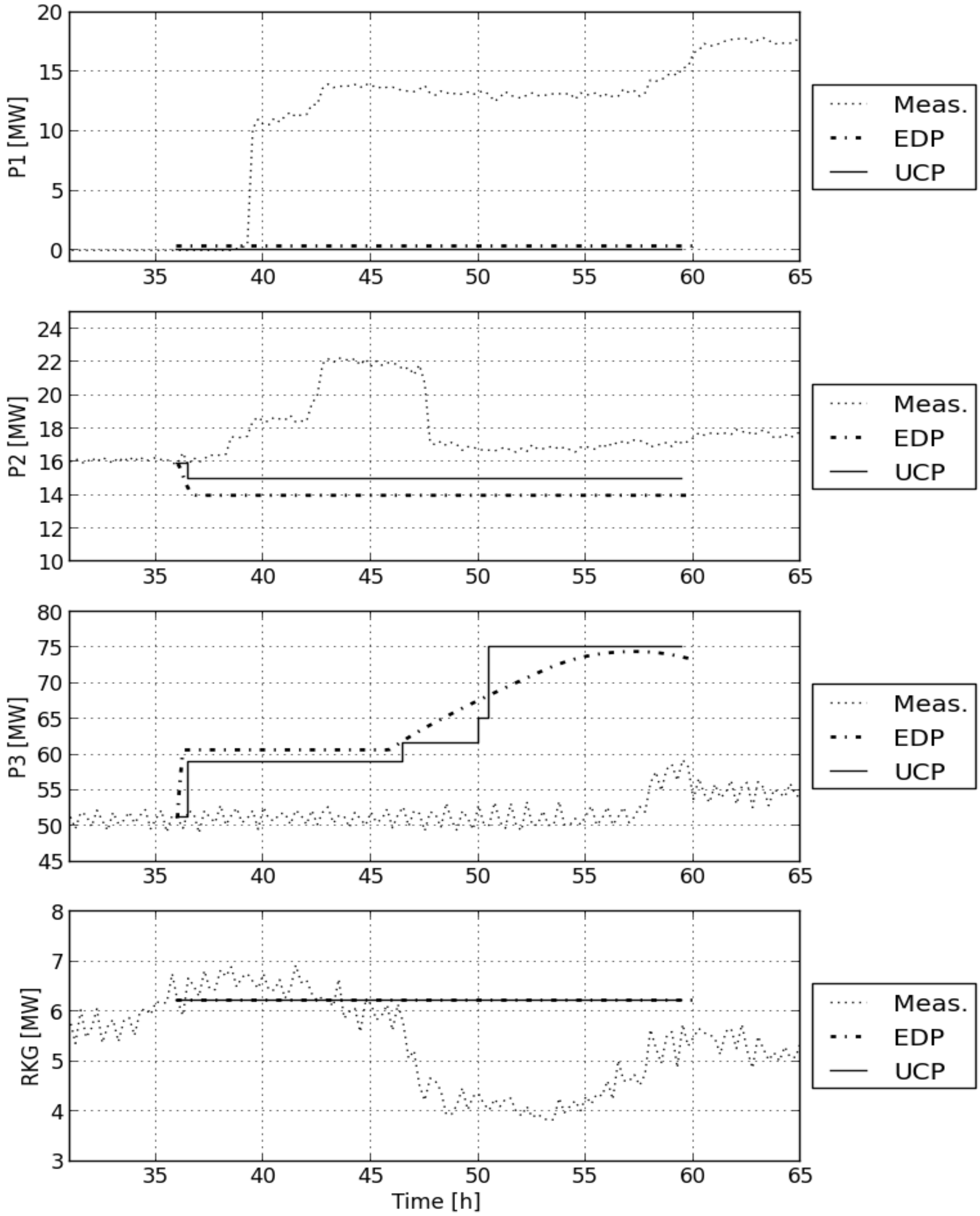


Figure 62 Case 6 measured and optimized heat production at P1, P2, P3 and RGK. P2 is at lower limit and P3 increases its heat production when the electricity price is halved.

Figur 62. Fall 6. Uppmätt och optimerad värmeproduktion för P1, P2, P3 samt RGK. P2 ligger vid min-ast medan P3 ökar sin värmeproduktion när elpriset halveras.

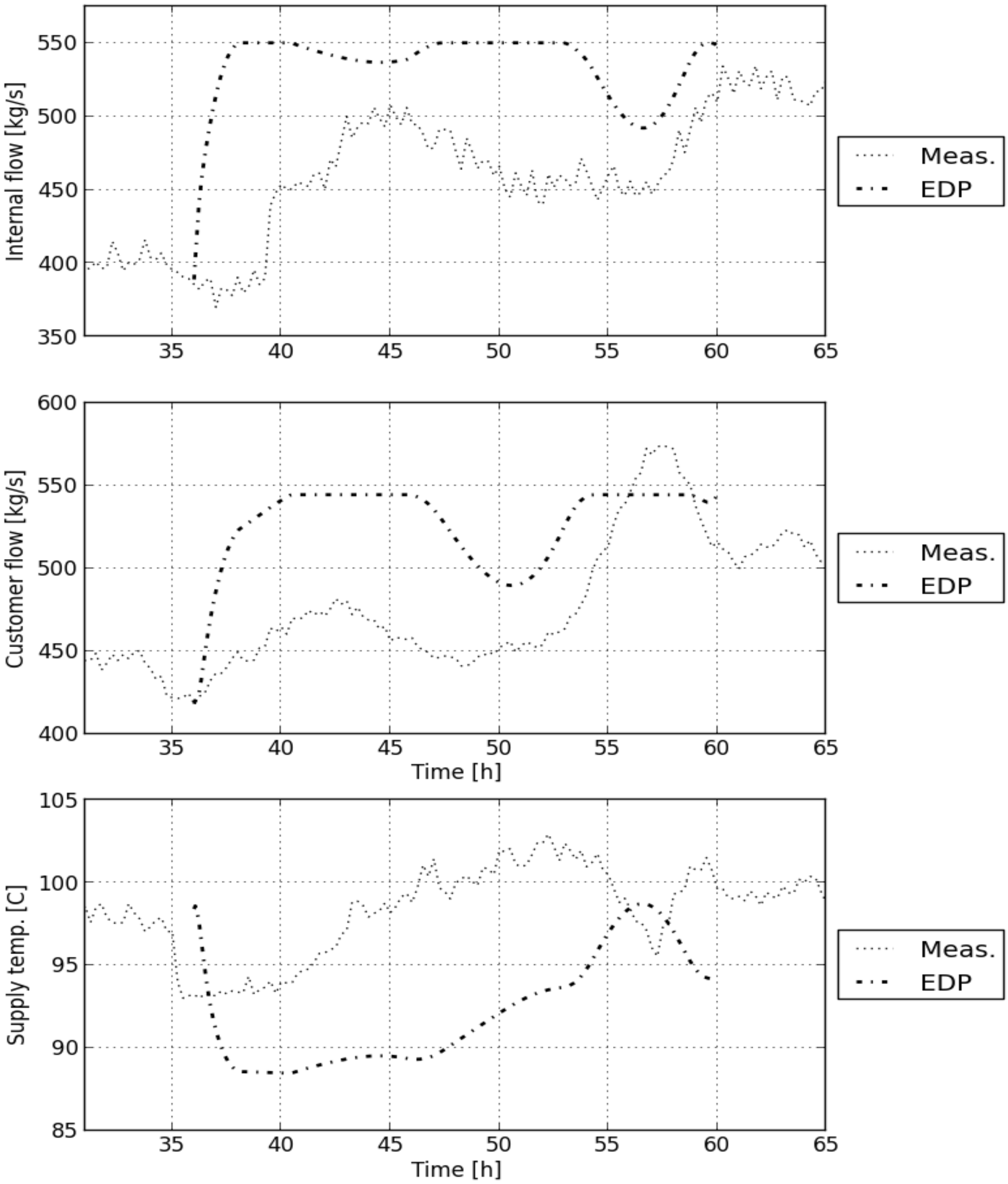


Figure 63 Case 6 measured and optimized internal and customer flow and supply temperature. Compared to measurement data, the supply temperature is decreased and the customer flow is increased.

Figur 63. Fall 6. Uppmätt och optimerade värden för framledningstemperatur och flöde. Jämfört med mätdata är flödet högre och temperaturen mindre.

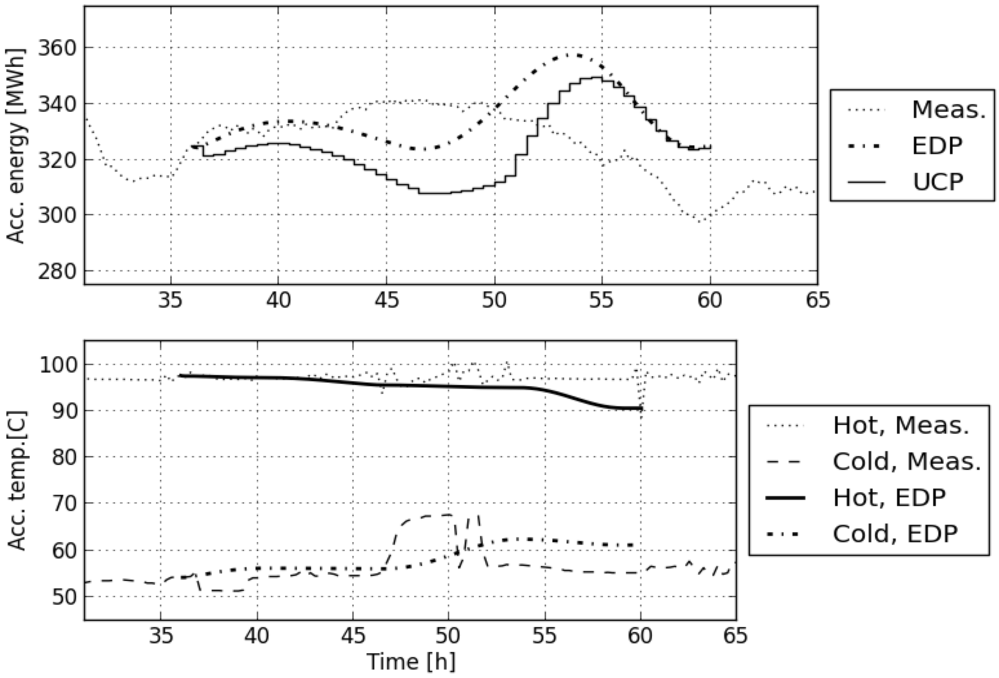


Figure 64 Case 6 measured and optimized accumulator energy and top (hot) and bottom (cold) accumulator temperatures. The accumulator is charged before the high customer heat load and discharged during the high customer heat load.

Figur 64. Fall 6. Uppmätta och optimerade värden för ackummulatorenergi och temperatur. Ackummulatorn laddas före den ökningen i kundlasten och laddas ur därefter.

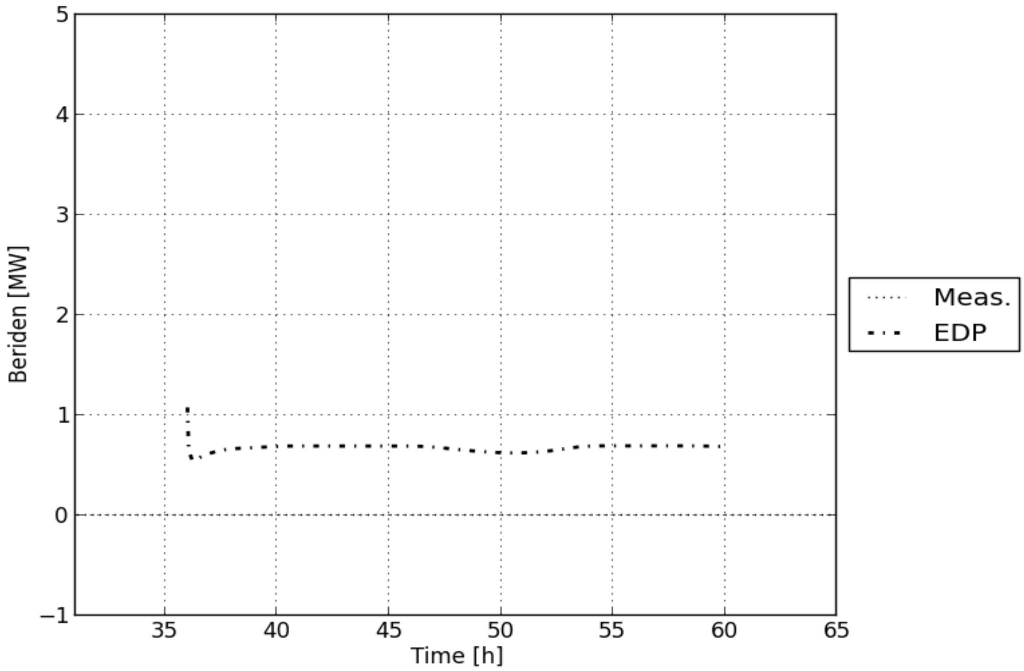


Figure 65 Case 6 measured and optimized Beriden cooling. No cooling by Beriden is performed.

Figur 65. Fall 6. Uppmätt och optimerad Beriden kylning. Ingen kylning sker.

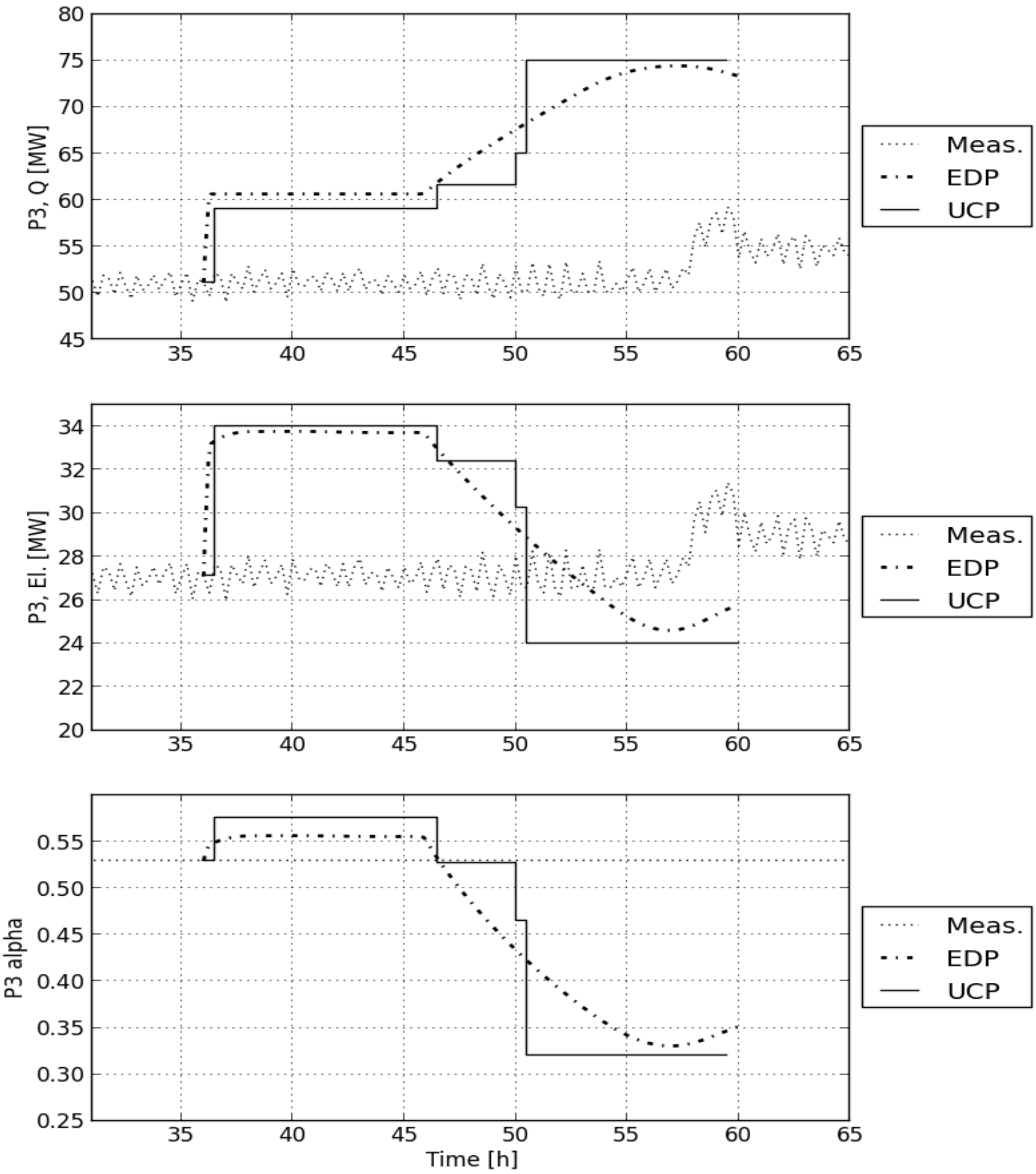


Figure 66 Case 6 measured and optimized P3 heat and electricity production and α value. P3 electricity production is maximized in the beginning when electricity price is high, while heat production is emphasized when electricity price is halved.

Figur 66. Fall 6. Uppmätta och optimerade värden för el- och värmeproduktion samt alpha-värdet för P3. Elproduktionen är maximal då elpriset är högst i början av optimeringen medan värmeproduktionen ökar vid lägre elpriser.

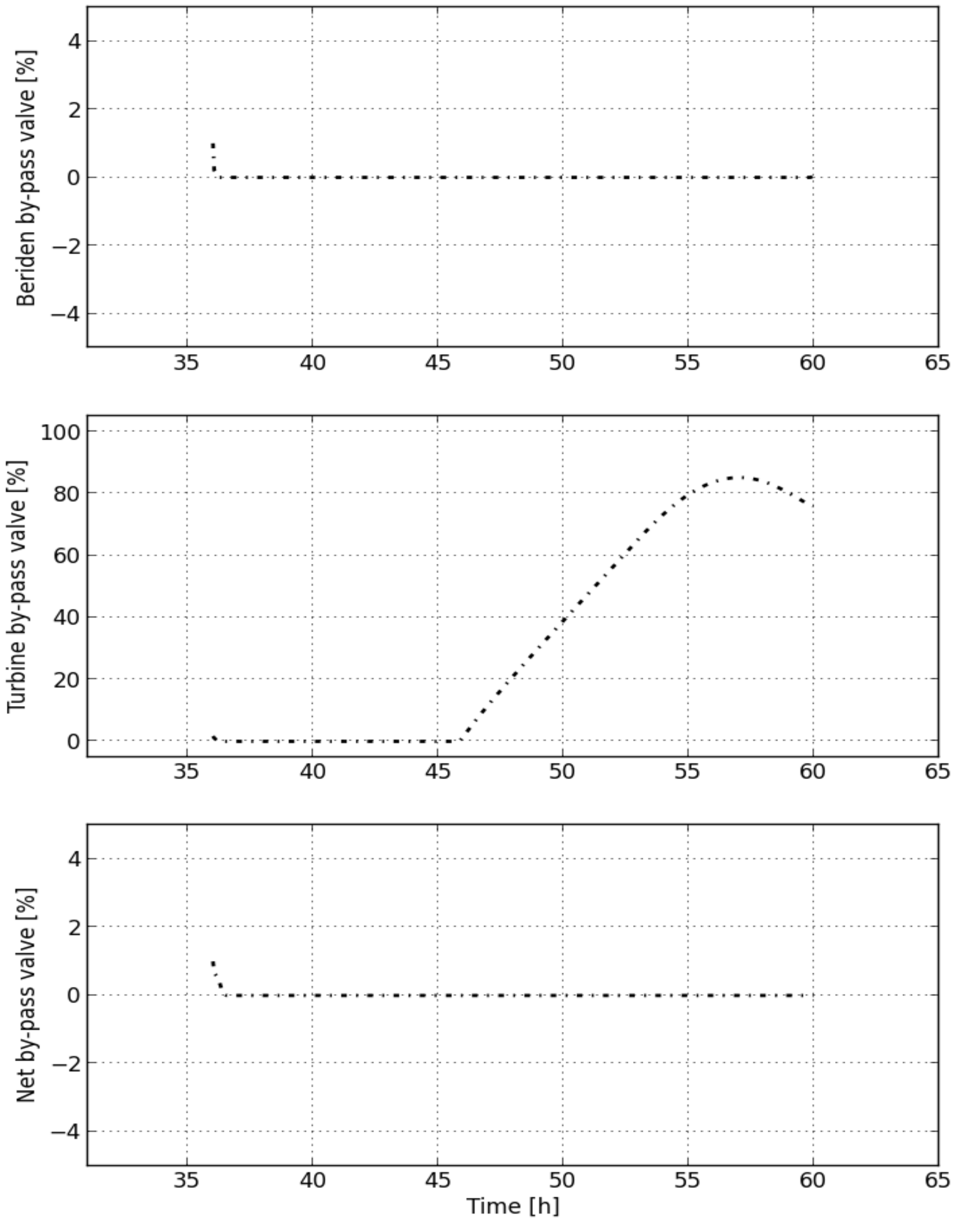


Figure 67 Case 6 optimized by-pass valve positions. The turbine by-pass valve is closed when P3 is maximizing electricity production and open when emphasizing heat production.

Figur 67. Fall 6. Optimerade lägen för bypass-ventilerna. Turbinens BP-ventil stängs för att maximera elproduktionen och öppnar därefter för att öka värmeproduktionen.

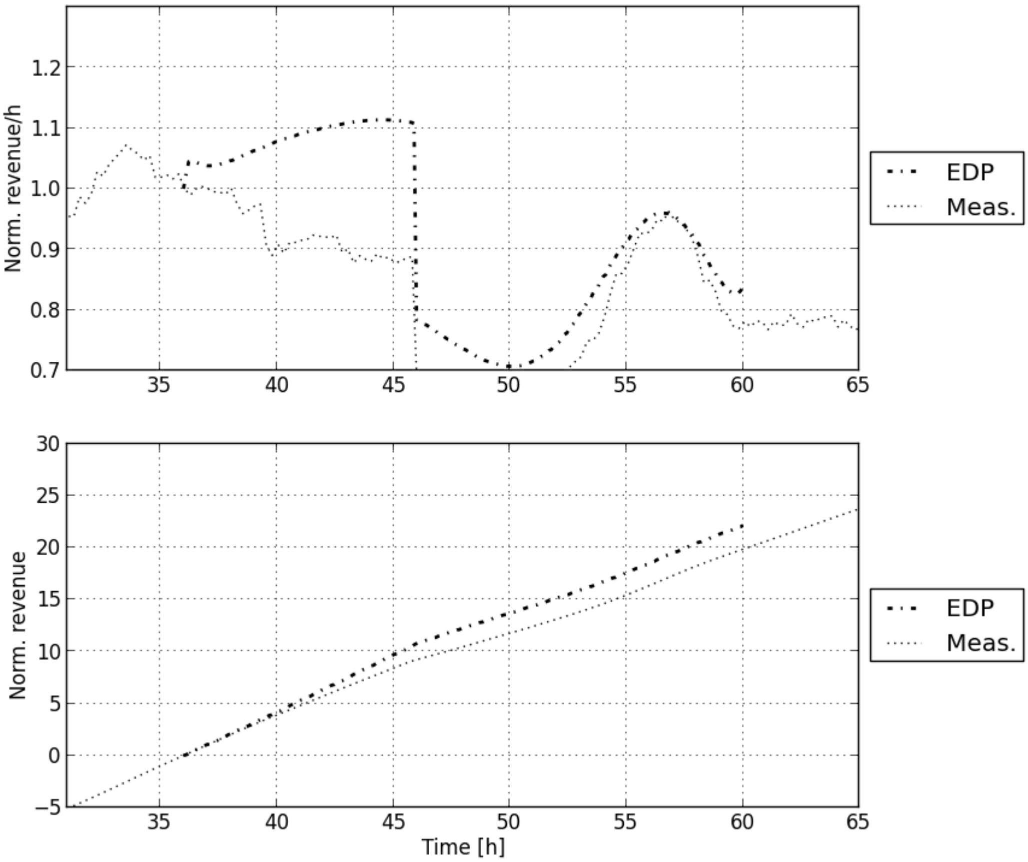


Figure 68 Case 6 measured and optimized normalized revenue/h and cumulative normalized revenue. The revenue/h is normalized such that the optimized solution starts at 1. The optimization shows a possibility of revenue increase.

Figur 68. Fall 6. Uppmätt och optimerade vinster (momentan och ackummulerad). Optimeringen visar en potentiell vinst.

10 Summary

This section summarizes the main contributions of the work presented in the report.

10.1 General

In the overall work, it has been shown that

- there is a high economical potential in the methods presented.
- it is fully possible to utilize status optimization results from the unit commitment problem (UCP) in the economic dispatch problem (EDP) optimization formulations.
- it is fully possible to use physics-based Modelica models of heat and electricity producing units together with non-linear optimization routines.

However, there is still need for further improvement before the framework is ready for implementation as a decision support tool. Such improvements are listed in Section 11.

10.2 Optimization

Results from the EDP of specific optimization cases have shown that maximum electricity production of the co-generation plant P3 can be reached by

- closing the turbine by-pass valve for a higher α value, see e.g. Case 4 and sub-figure 3 in Figure 48 and sub-figure 2 in Figure 49.
- maximizing the fuel load while using with the external cooler Beriden or the accumulator to remove the undesired heat production, see Figure 20 and Figure 29 for Case 1 and 2 and Figure 46 for case 4. Note that such units may not be present in all heat district networks.

On the contrary, the EDP result of Case 6 shows that when the electricity price is lower than the heat price, emphasized heat production in the co-generation plant P3 is performed by fully opening the turbine by-pass valve together with full fuel load, see sub-figure 2 in Figure 67.

Usage of the physics-based Modelica EDP models together with JModelica.org gives the user availability of flexible and high-level formulation of non-linear optimization formulations, see Figure 13 and Figure 14. In all optimization cases, the EDP results from these formulations have shown that

- the customer supply temperature may be decreased compared to measurement data. The decrease varies up to 10^{o}C, but the mean decrease is approximately 4 ^{o}C which is comparable to the results in [15].
- the customer supply flow may be increased compared to measurement data
- constraints of pump capacity and supply temperature are complied

It should be kept in mind that these results are based on a perfect heat load prediction as the prediction model was decided infeasible to use. Some uncertainty in the load prediction should be handled by the accumulator, but this needs to be further investigated.

The usage of optimization has in specific use cases shown that

- delays or profiles in produced heat can be followed when starting or stopping a unit, see e.g. Case 4 and sub-figure 2 in Figure 44.
- fewer starts of boilers are possible, see Case 4 and sub-figure 1 in Figure 44.
- time-varying electricity price can be handled, see Case 6.

The usage of optimization, both in the UCP and EDP, has resulted in planning schedules that for all optimization cases has

- a balance between heat production and heat consumption of customers
- given priority to boilers with low fuel cost
- handling of minimum and maximum capacity as well as minimum and maximum rate of load change of each unit.
- maximized the revenue of the plant considering fuel prices, heat and electricity sell prices and start costs.

10.3 Physical Modeling

The physics-based modeling performed in the Modelica language has the advantage that a high level formulation and graphical representation of the system to be optimized are used, see Figure 4 and Figure 5. The main results of the physics-based modeling are

- an optimization-friendly water media package with polynomials approximating IF97 property functions.
- proper description of co-generation plant P3, which generation of heat and electricity depend on fuel load and turbine by-pass valve, but also temperature and flow rate of return water.
- proper descriptions of the accumulator and the Beriden cooler, showing that
    - the energy in the proper physics-based accumulator model may differ significantly compared to the simplistic UCP model which is included in standard work today. This has been noted in all optimization cases.
    - the Beriden cooler may be used more, compared to measurement data, to increase electricity production when customer heat load is low, see optimization case 1 and 2.
    - correct temperature in the accumulator may avoid start of boilers
- handling of transport delays that are mass flow dependent, which is not possible in the standard tools for production planning, see pipe model.

11 Future Work

The project has shown the feasibility and the potential of the proposed approach. There are however needs for further improvements before implementing the approach as a decision support tool in arbitrary district heating networks. Apart from an improved load prediction, future work would aim at increasing both the generality and the robustness of the optimization method.

In this section, a list of possible improvements as well as some ideas for their implementation is given.

11.1 Generality

The present report considers general plant models that can be used to describe any heat and electricity generation unit. The optimization method is currently applicable to small networks with centralized heat production. To increase the scope of application of the method, the following is required:

- A distribution network model. This is needed to
  - describe the energy stored in the network, which is useful when the network is used as an accumulator
  - express the optimization constraint related to the minimum supply temperature at the customer (instead of at the plant)
  - describe a heat production that is geographically distributed in the network
- A more detailed model for the formulation of the unit commitment problem, with for instance start-costs that depend on the time a boiler has been shut-down (warm start), with eventually a description of the temperature influence
- Handling of time varying prices on fuels, heat and electricity. Case 6 shows that time varying price on electricity can be handled but it might be of interest to model different kind of contracts with suppliers and customers.
- Longer optimization horizons such that:
  - boilers with very long starting and stopping sequences can be considered
  - the accumulator can be used without setting an end point constraint on available energy.

11.2 Robustness

When the method is to be applied on larger and more general configurations, it is critical to dispose of a robust optimization process. This can be achieved by

- A tighter integration between the Economic Dispatch and the Unit Commitment problems, which requires
  - An improved consistency between the physical models and the simplified ones. It has been shown that the simplified models in the unit commitment formulation can result in significantly different results compared to the economic dispatch. It would be beneficial to improve

    the consistency between the different model sets by automatically deriving the simplified models from the physical ones.
  - An iterative process to cope with sub-optimality or infeasibility problems due to the difference in the models used in the sub-problems.
- A robust optimization that
  - accounts for the uncertainty in the load prediction. This could be easily implemented using stochastic MILP formulations which is available in PySP, which is an open source modeling and solver library for stochastic programming, see [24].
  - removes the effect of finite horizon optimization. This can easily be removed by increasing the horizon, but not using the full optimization result. However, more direct methods exist, see e.g., [23].
- Estimation techniques for an increased consistency between data and plant models. With increased fidelity level, more model parameters are often introduced. These should be estimated using measurement data, either online or offline.
- Improved load prediction model to minimize the uncertainty in the load prediction. The prediction model designed for supply temperature control with 2 hour horizon could be improved when the purpose is 24 hour predictions and longer.

11.3 Requirements on Methods and Tools

The aforementioned improvements increase the optimization complexity and size, both for UCP and EDP, which requires increased robustness of the optimization algorithms. Such requirements can be fulfilled by using

- a more advanced MILP solver such as the commercial solvers CPLEX and Gurobi, see [25] [26].
- CasADi, a symbolic framework for automatic differentiation that can be well integrated with JModelica.org, see [27]. Usage of CasADi would give symbolic Jacobian and Hessian of the resulting NLP after collocation discretization, which would increase robustness and convergence considerably.

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A. Collocation

The collocation scheme very briefly presented below is the one implemented in the JModelica.org platform, see [28].

The optimization interval $[t_0, t_f]$ is divided into N_e elements, each with a normalized length of $h_0, \dots, h_{N_e-1}$. The element junction points t_i may then be written as

$$t_i = t_0 + (t_f - t_0)\sum_{k=0}^{i-1} h_k, \; i = 1, \dots, N_e - 1.$$

In each element, N_c Radau based collocation points $\tau_j \in (0, 1], j \in \{1, \dots, N_c\}$, are introduced which gives the collocation time points

$$t_{i,j} = t_0 + (t_f - t_0)(\sum_{k=0}^{i-1} h_k + \tau_j h_i), \; i = 1, \dots, N_e - 1, j = 1, \dots, N_c,$$

where the indices *i* and *j* correspond to element *i* and collocation point *j* in the element, respectively. The state, algebraic variable and control profiles are approximated by Lagrange polynomials based on the Radau collocation points. A Lagrange polynomial of order $N_c - 1$ is defined by

$$L_j^{N_c}(t_k) = \begin{cases} 1, & N_c = 1 \\ \prod_{k=1,k\neq j}^{N_c} \frac{\tau - \tau_k}{\tau_j - \tau_k}, & N_c \geq 2 \end{cases}$$

and thus at the collocations points it has the value

$$L_j^{N_c}(\tau_k) = \begin{cases} 1, & N_c = 1 \\ 0, & N_c \geq 2 \end{cases}$$

The state variables are approximated by Lagrange polynomials of order N_c, where the extra point τ_0 is used, while algebraic variables and control profiles are approximated by Lagrange polynomials of order $N_c - 1$, in each element, see Figure 69.

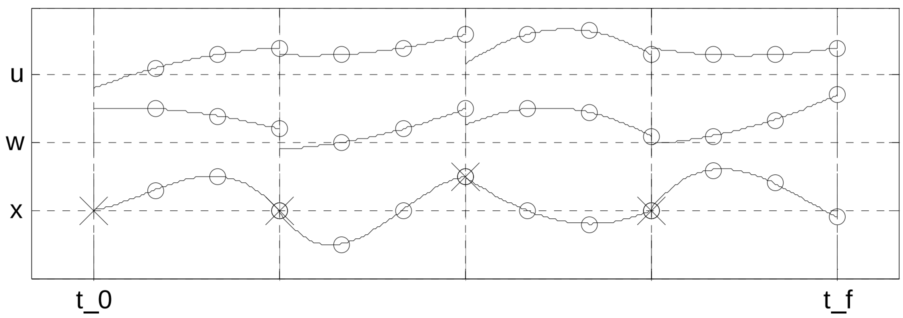

Figure 69 Example of approximations of control, algebraic and state variable profiles

Figur 69. Exampell på approximation för regler-, algebraiska, och tillståndsvariabler.

The approximated profiles on the time interval $[t_i, t_{i+1}]$ are written as

$$x(t) = \sum_{k=1}^{N_c} x_{i,k} L_k^{N_c+1} \frac{t - t_i}{h_i(t_f - t_0)}, t \in [t_i, t_{i+1}]$$

$$w(t) = \sum_{k=1}^{N_c} w_{i,k} L_k^{N_c} \frac{t - t_i}{h_i(t_f - t_0)} \; t \in [t_i, t_{i+1}]$$

$$u(t) = \sum_{k=1}^{N_c} u_{i,k} L_k^{N_c} \frac{t - t_i}{h_i(t_f - t_0)} \; t \in [t_i, t_{i+1}]$$

and with the properties of the Lagrange polynomial it follows that

$$x(t_{i,j}) = x_{i,j}, i = 0, \dots, N_e - 1, j = 0, \dots, N_c$$

$$w(t_{i,j}) = w_{i,j}, i = 0, \dots, N_e - 1, j = 1, \dots, N_c$$

$$u(t_{i,j}) = u_{i,j}, i = 0, \dots, N_e - 1, j = 1, \dots, N_c$$

where the variables $x_{i,j}$, $w_{i,j}$ and $u_{i,j}$ now represent the discretized profiles.

Continuity of the state profiles over element borders can be enforced by the equality constraint

$$x_{i,N_c} - x_{i+1,0} = 0, i = 0, \dots, N_e - 1$$

while state derivatives are approximated by differentiating the approximated profiles, giving the variables $\dot{x}_{i,j}$. The variables $\dot{x}_{0,0}, w_{0,0}$ and $u_{0,0}$ are introduced for the initial value problem. An equation for the control profile initial value $u_{0,0}$ is given using interpolation of the first Lagrange interpolation polynomial.

Using the discretized description of the trajectories, the constraints of the original dynamic optimization problem may then be written as

$$F(\dot{x}_{i,j}, x_{i,j}, w_{i,j}, u_{i,j}) = 0$$

$$F_0(\dot{x}_{0,0}, x_{0,0}, w_{0,0}, u_{0,0}) = 0$$

$$C_{ineq}(\dot{x}_{i,j}, x_{i,j}, w_{i,j}, u_{i,j}) \leq 0$$

$$C_{end}(\dot{x}_{N_e-1,N_c}, x_{N_e-1,N_c}, w_{N_e-1,N_c}, u_{N_e-1,N_c}) \leq 0$$

If all variables defining the trajectories are collected in a vector $\bar{x}$ and all equality and inequality conditions are collected in $h(\bar{x}) = 0$ and $g(\bar{x}) \leq 0$, respectively, and only considered being valid at the collocation points, then the dynamic optimization problem is transformed into an NLP defined as

$$\begin{aligned} \min_{\bar{x}} \;\; & f(\bar{x}) \\ s.t. \;\; & h(\bar{x}) = 0 \\ & g(\bar{x}) \leq 0 \end{aligned}$$

where $f(\bar{x}) = J(x_{N_e-1,N_c})$.

This NLP may be solved directly and simultaneously for all state, algebraic and control variables representing the approximated trajectories. In JModelica.org, and used here, is the large-scale NLP solver IPOPT, see [19].

B. Abbreviations

Abbreviation	Meaning
BRA	Brandkärr
EDP	Economic Dispatch Problem
EÅP	Elångpanna
IPOPT	Interior POint OPtimizer
LAS	Lasarettet
LR	Lagrangian Relaxation
MILP	Mixed-Integer Linear Problem
MINLP	Mixed-Integer Non-linear Problem
NLP	Non-linear program
UCP	Unit Commitment problem