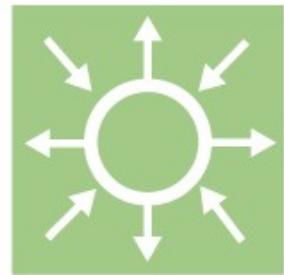




Crack propagation in buttress dams

Application of non-linear models – Part II

Elforsk report 10:69



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ELFORSK

Förord

Vattenkraftföretagen*) har via Elforsk stöttat forskning och utveckling inom det betongtekniska området sedan början av 90-talet.

Programmet är inriktat på ett kostnadseffektivt förvaltande av vattenkraftindustrins betongkonstruktioner. Syftet är att ge ett kvalificerat stöd till vattenkraftföretagen.

Målet är att ta fram verktyg, riktlinjer, utförandebeskrivningar och teknik som fyller industrins behov. Målet är också att bygga kompetens. En uttalad ambition är att samarbeta med övrig industri och landets tekniska högskolor.

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Sammanfattning

De största och viktigaste vattenkraftsdammarna i Sverige är lamelldammar av betong. De sätts samman av ett stort antal betongmonoliter som i sin tur består av en frontplatta och en bärande dammpelare. I några av dessa monoliter har sprickor observerats, vilka med tiden kan komma att påverka dammarnas säkerhet.

Dammar i områden där vinterklimat säsongvis förekommer utsätts ofta för extrem miljöpåverkan. Vid en nyligen utförd besiktning av en lamelldamm byggd 1954 i norra Sverige identifierades flera olika typer av sprickor. Dammens frontplatta var ursprungligen inte värmeisolerad på nedströmssidan vilket har lett till frysskador i plattan. År 1994 uppfördes en isolerande vägg för att förhindra isbildning och skydda frontplattan mot ytterligare frysskador. Väggen monteras mellan närliggande dammpelare och sträcker sig från berggrunden upp till dammkrönet. Det är troligt att väggen har ökat de spänningar som uppstår i dammpelarna på grund av ojämn volymförändring vid säsongvariationer i den omgivande lufttemperaturen.

En finit elementmodell baserad på brottmekanik, plasticitetsteori och skademekanik har använts för att beskriva uppsprickningen i en lamelldamm. Den kombinerade effekten av förhindrade temperaturrörelser och vattenlast studeras. Uppkomsten av sprickor orsakade av säsongsvierande temperaturer simulerades och särskilt undersöktes effekten av en isolerande vägg som antogs ha installerats ca 40 år efter att dammen byggdes. Resultaten visar att temperaturvariationerna över året ger upphov till stora dragspänningar på flera ställen i dammkonstruktionen och att sprickor kan uppkomma vid minst fyra olika platser. Temperatur-spänningar i kombination med vattenlast visades vara orsaken till sprickbildningen. Resultaten visar att installationen av den isolerande väggen i hög grad har bidragit till sprickbildning i dammpelarna. En lämpligare placering av väggen hade kunna förhindrat att de sprickorna uppstod.

Summary

The largest and most important concrete dams in Sweden are buttress dams. These consist of a large number of concrete monoliths formed by a front-plate with a supporting buttress. Cracks have been observed in some of these dams which in a long-term perspective may affect their safety.

Concrete dams located in cool areas are often subjected to severe environmental impacts. Recent assessments and investigations of a buttress dam built 1954 in northern Sweden points out several types of cracks. The front-plate of the dam was not heat insulated on the downstream side when constructed, which has led to freeze-thawing damages in the plate. However, in 1994 a heat insulation wall that prevents ice-formation and protects the front-plate against frost damage was installed. It is located between two buttresses in the dam, from the rock up to the dam crest. The wall has most probably led to increased mechanical stresses in the pillars as a result of contraction and expansion due to seasonal temperature changes.

A finite element model based on non-linear fracture mechanic, plasticity theory and damage mechanics has been utilized to study crack development in a buttress dam. The combined effects of restrained thermal displacements and loads caused by water were studied. The development of cracks due to seasonal temperature variations was simulated, especially with respect to the effect of the insulating wall installed some 40 years after the completion of the dam. The results show that the seasonal temperature variation causes high tensile stresses at different locations on the dam, and that the cracks can be initiated from at least four locations. Thermal stresses in combination with the load caused by water were shown to be the reason for cracking. The results point out that the addition of the insulating wall greatly contributed to the development of cracks in the buttress. A more suitable placement of the insulation wall could have prevented the cracking of the pillars.

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1 Introduction

1.1 Background

Many of the major Swedish dams are concrete dams and in some of them, cracks have been observed. The type of cracks and the causes of cracking vary for different types of dams and the influence that concrete cracks have on the dam safety also varies for different types of dams. Cracks appear due to restrained thermal strain during cooling after casting, but also due to large stresses from loads in the serviceability state, for example from water pressure, ice-load and seasonal temperature changes. The cracks are often unstable as the creep effects, caused by long-term loads, lead to cracking at stresses lower than the concrete short-term strength. A cracked concrete structure is more likely to be damaged by environmental agents, which in turn can lead to further cracking or widening of the existing ones. The present conditions of many concrete dams therefore need to be evaluated.

Concrete buttress dams located in cold areas are often subjected to severe environmental impacts. Recent assessments and investigations of the Storfinnforsen buttress dam in northern Sweden point out several types of cracks in the concrete structure. The front-plate of this dam was not heat insulated on the downstream side when it was constructed in 1954 and this has led to freeze-thawing damage in the plate. However, in 1994 a heat insulation wall protecting the front-plate was installed. It is placed between each pillar in the dam, from the bedrock up to the crest and is located approximately in the middle of the pillars. The insulation wall has most probably led to increased mechanical stresses in the pillars as a result of the contraction and expansion of the outdoor part of the pillars due to the seasonal temperature changes.

Numerical models should be used in order to find the cause of cracking and follow the propagation of existing cracks in concrete dams. This is possible with the finite element method (FEM) which also makes the use of non-linear material models possible. Such a non-linear analysis does, however, rely on advanced numerical routines and may suffer from problems with convergence and numerical stability. Combinations of loads from external water pressure and temperatures in the surrounding air, water and ground can be used as input for an analysis. The results show concrete cracks that may occur, which in turn can be used for estimation of the stability of the analyzed dam. This has great importance in risk analysis of large hydropower dams. The use of non-linear analysis tools can be demanding for the practicing engineer as the validity of the results relies on the accuracy of the advanced models used. There is thus a need to evaluate the use of FEM for analysis of large concrete dams and to show possibilities and restrictions associated with the method. The Swedish guidelines in the field of hydropower are adapted to old design and calculation models based on simplified assumptions and there is no guidance for using numerical models. There is thus a need to evaluate the use of FEM for analysis of large concrete dams and to show possibilities and restrictions associated with the method. In Sweden there are today

approximately 25 large concrete dams and most of these are old structures and their conditions need to be maintained, in some cases through costly upgrading and repairs. Advanced finite element analysis will be an important tool in the condition assessment of these dams, and similar dams around the world.

A first step towards a finite element based analysis tool for concrete buttress dams was taken By Björnström et al (2006) through linear elastic analyses aiming at showing whether or not deformations and stresses caused by seasonal variation of temperatures could initiate cracks in a concrete dam. The main purpose was to demonstrate the use of FEM as a tool for analysis. The results revealed areas of stress concentrations and high stress levels in parts of a dam from where cracks could initiate. The analysis was performed with a large three-dimensional model of solid elements that could not be practically used in a non-linear analysis. This was followed by analyses using non-linear fracture mechanic and plasticity theory in combination with a two-dimensional finite element model, presented by Ansell et al (2008). These results showed that it is possible to model and follow crack development in a buttress dam and that the seasonal temperature variation was the cause of cracking. The present report summarizes a following project that included non-linear analysis using a three-dimensional finite element model based on shell elements.

1.2 Project organisation

The project group was formed in 2006 and at the present consists of representatives from Vattenfall Research and Development (Älvkarleby), KTH, The Royal Institute of Technology (Stockholm) and Energoretea (Malmö). The project is financially supported by Elforsk AB, the Swedish Power Companies R&D association.

The project is coordinated by Dr. Manouchehr Hassanzadeh from Vattenfall R&D, who also contributes with knowledge on materials formulation and fracture mechanics for concrete. He also maintains international contacts taken by the research group. From Vattenfall did also Mr. Jonas Björnström at Vattenfall Power Consultants contribute with finite element during the first parts of the project. Dr. Tomas Ekström from Energoretea works with monitoring and evaluation of the in-situ measurements on the Storfinnforsen hydropower dam. These results are used in for calibrations during the modelling work. The contributions from the Department of Civil and Architectural Engineering at KTH are coordinated by Dr. Anders Ansell who also is responsible for formulation and evaluation of the numerical results presented in this report. At KTH does Dr. Richard Malm contribute with knowledge on advanced finite element modelling and have also done a majority of the numerical simulations presented.

The project continues during 2008–09 with a fourth part, still as a collaboration support from Elforsk AB.

1.3 Aims, goals and contents of report

This reports aims at investigating the possibilities of developing an analysis method for concrete buttress dams, based on a finite element model using non-linear material properties for reinforced concrete. The model describes one concrete monolith of a large dam with long extension, is three-dimensional and built up by shell elements.

The goal is to show that choices of realistic material properties, outside temperature variations and loads will lead to displacements and concrete cracks as observed in-situ. The goal is also to show that it possible to accurately calculate the direction and the extension of thermally induced cracks.

The report summarizes the work done in this third step of the ongoing project, carried out during 2007–08. The background, descriptions of the finite element model, the assumed loads, material formulations and results have also been presented in a conference paper by Ansell et al (2009) and in two papers by Ansell and Malm (2008) and Malm and Ansell (2010). The full formulation of the non-linear material model is given in these references, but only briefly described in the present report.

2 Concrete buttress dams

2.1 Principles of design

The studied dam is a buttress dam, consisting of tall concrete monoliths, each with a front-plate facing the water and supported by a buttress. Such a buttress is sometimes also called dam pillar. A section of a buttress dam is shown in Figure 2.1 where three adjacent monoliths are seen, each with a front-plate supported by a buttress. There is a passage with an inspection gallery through openings in each buttress, from where inspections of the concrete structure can be conducted. There are also vertical insulating walls, which together with the front-plate forms spaces with relatively small variations in temperature and humidity. This type of walls has been installed in some dams after formation of ice at the upstream side of the front-plates was observed. These dams were 30–40 years old when these modifications were done.

A typical, large buttress dam can consist of up to 70–100 concrete monoliths that are up to 40 m high. Each supporting buttress can be 2 m thick and about 30–35 m wide near the bedrock. The section of the front-plate supported by one buttress is 8–10 m wide and has a varying thickness of about 1–2 m, where the thickest part of the front-plate is found near the bedrock.

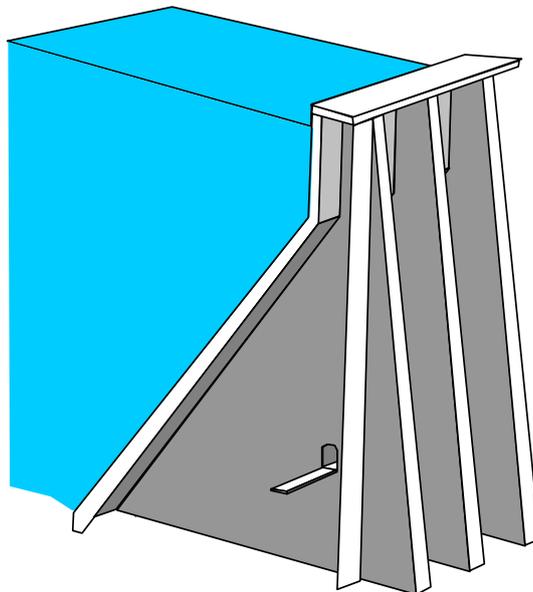


Figure 2.1 A section of a concrete buttress dam. From Malm and Ansell (2010).

2.2 Major Swedish dams

There are approximately 25 major concrete dams in Sweden (FOI, 2000) of which about 15 are buttress dams and the remaining are gravity dams and arch dams. The largest Swedish buttress dam is a part of Storfinnforsen hydropower plant that was built between 1949 and 1954, located in the Faxälven River in northern Sweden. The entire dam is about 1200 metres long with a concrete part that is about 800 metres, here shown in Figures 2.2–4. The dam consists of 66 concrete monoliths, excluding intakes, spillways and a concrete support wall dam. The tallest monolith is about 40 m high and consists of an 8 m wide front-plate with a downstream supporting block, the buttress. Each buttress is 2 m thick and the width near the bedrock is about 34 m in the largest monolith. The front-plates are about 2.5 m thick near the bedrock and 1.2 m at the top of the dam. The gap between the front-plates of two adjacent monoliths has a water tight seal.



**Figure 2.2 Storfinnforsen hydropower plant in the Faxälven River.
Photo from P. Stenström, WSP.**



Figure 2.3 The concrete dam of Storfinnforsen hydropower plant. From Ansell et al (2008).



Figure 2.4 The crest of Storfinnforsen hydropower dam. Photo from T. Ekström, Energoretea.

2.3 Observed cracks and damage

The Storfinnforsen hydropower dam is the largest concrete buttress dam in Sweden and has therefore been subjected to recurring inspections. Soon after the completion of some of the large dams, horizontal cracks were detected in the lower part of the front-plates. Through these cracks have water started to leak from the reservoirs. Cracks also appeared early in contraction joints between front-plates and the buttresses. Such a joint is basically a thin, nearly vertical band of concrete that was cast subsequent to the front-plate and the dam pillar. The concrete was hand-rammed only and not vibrated. It is thus possible that some of the cracks have partly arisen, or at least been initiated, already during the cooling phase that follows casting of the concrete, see Fahlén and Näslund (1991), Melander (1997) and also Björnström et al (2006).

An inspection of the dam in 2004 revealed that the cracks in the front-plates had increased even further since the previous mapping in 1986. The inspection also revealed that inclined cracks had appeared in the buttress. These cracks, of types (2)–(4) in Figure 2.5, were not reported in 1986 which implies that these cracks developed after an insulation wall was installed in 1994. The mapping in 1986 showed that the cracks had increased by a factor of three since 1961 and the length of the cracks had doubled.

A summary of typical observed cracks are here given in Figure 2.5. The cracks are classified as four different types where type 1 is horizontal cracks in the front-plate. The type 2 is inclined cracks from the front-plate that has propagated in the buttress towards the foundation. The type 3 is an inclined crack that has propagated from the inspection passage towards the front-plate and the type 4 is vertical cracks originating from the foundation. A typical crack in the monolith starts at the front-plate, from a horizontal crack, and continues diagonally downwards in the downstream direction. Such a crack often goes through the entire concrete cross section.

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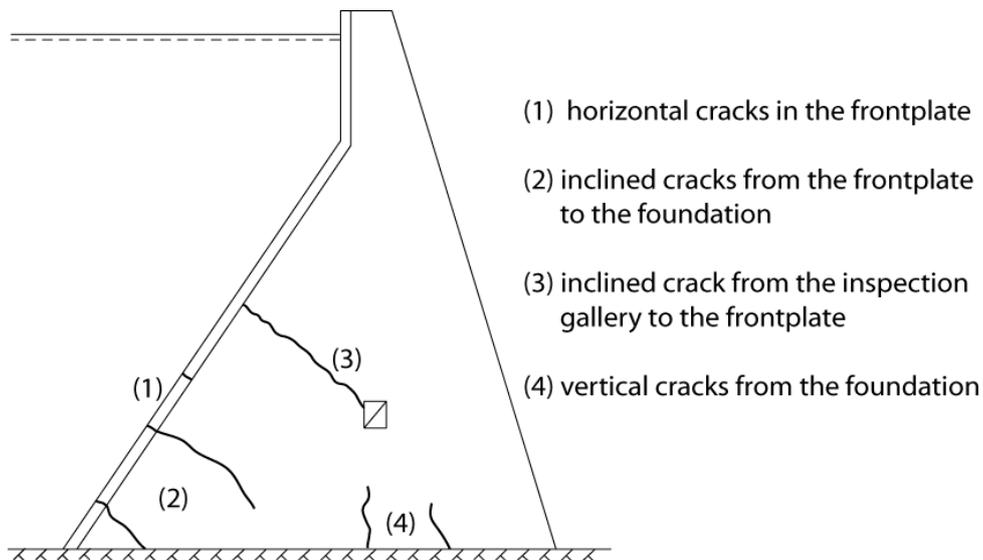


Figure 2.5 Location of cracks observed in situ. From Malm and Ansell (2010).

2.4 In-situ measurements

To follow up the mapping of observed cracks and further investigate if there are further changes in crack widths have two of the tallest monoliths of the Storfinnforsen dam been monitored since March 2006 (Björnström et al, 2006) Displacement and temperature variations have been monitored using the set-up of transducers shown in Figures 2.6–7, with details in Figure 2.8. The temperature was measured both inside and outside the heat insulation wall. Some transducers were placed inside a 50 mm deep drilled hole, and others outside the concrete body. Displacement transducers were placed across the cracks and mainly perpendicular to the cracks. It should be noted that no monitoring of temperature or displacement occurred before installation of the insulation wall.

Temperatures monitored during 2006 are shown in Figure 2.9. The results show that the temperature inside the insulation wall varies between +3 °C and +11 °C. The temperature outside the wall follows the ambient temperature, which vary between -20 °C and +25 °C. The water temperature has not been monitored but it can be assumed to be about +4 °C all the time from bottom up to approximately 8–10 m from the water surface. In the 8–10 m zone nearest the surface the temperature changes during the year. Furthermore, the results show that the temperature inside the drilled holes follows the ambient air temperature.

Monitored displacements are shown in Figure 2.10. The displacements vary during the period, probably as a result of the seasonal variation in temperature. Note that the transducer 7 is placed on the downstream side of the front-plate and that the transducer 5 is placed between the rock and the

buttress. The crack movement recorded by this transducer is large and varies during the period in spite of a rather small variation of the water temperature at that level.

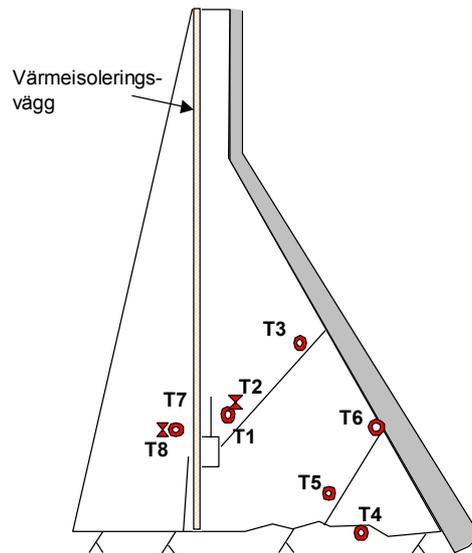


Figure 2.6 Set-up of transducers for temperatures. The symbols \diamond denote surface mounted transducers and \circ transducers 50 mm into the concrete. From Ansell et al (2008).

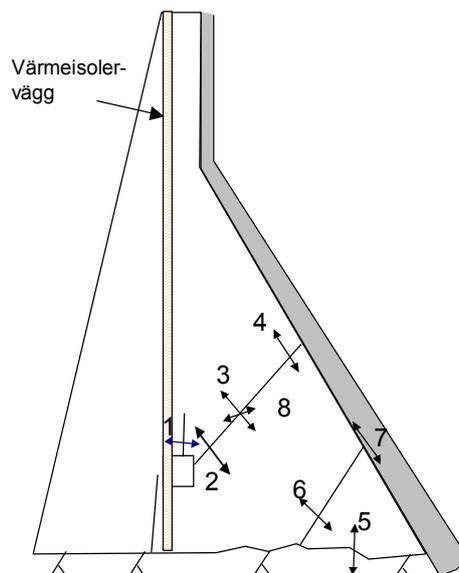


Figure 2.7 Set-up of transducers displacements. LVDT transducers marked with \leftrightarrow . From Ansell et al (2008).



Figure 2.8 Displacement LVDT transducers on the buttress of the Storfinnforsen dam. From Ansell et al (2008).

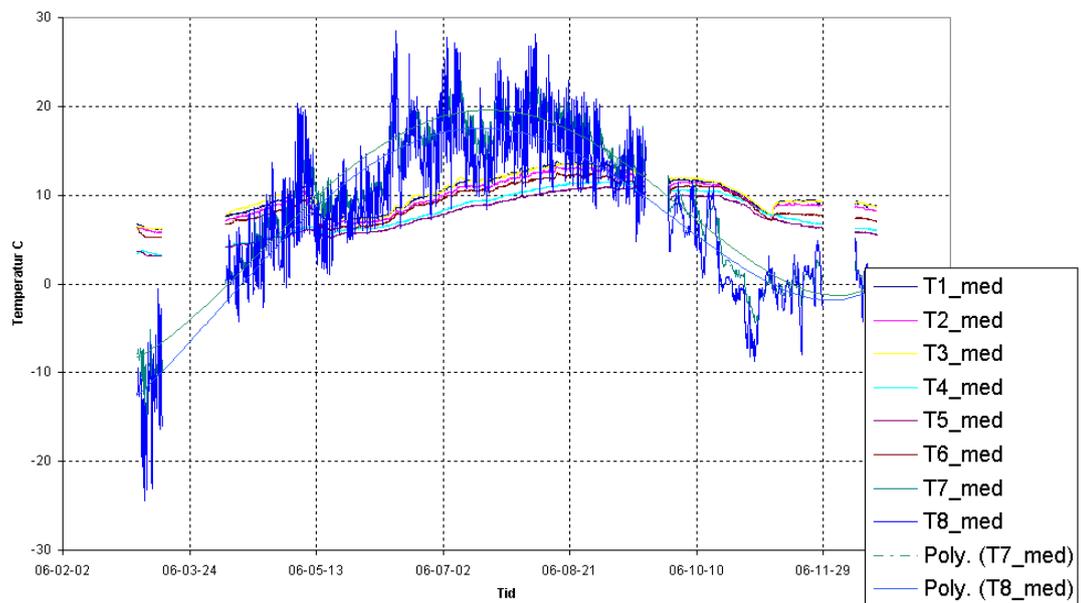


Figure 2.9 Temperatures monitored during 2006. Transducers mounted on monolith 43 of the Storfinnforsen dam. From Malm and Ansell (2010).

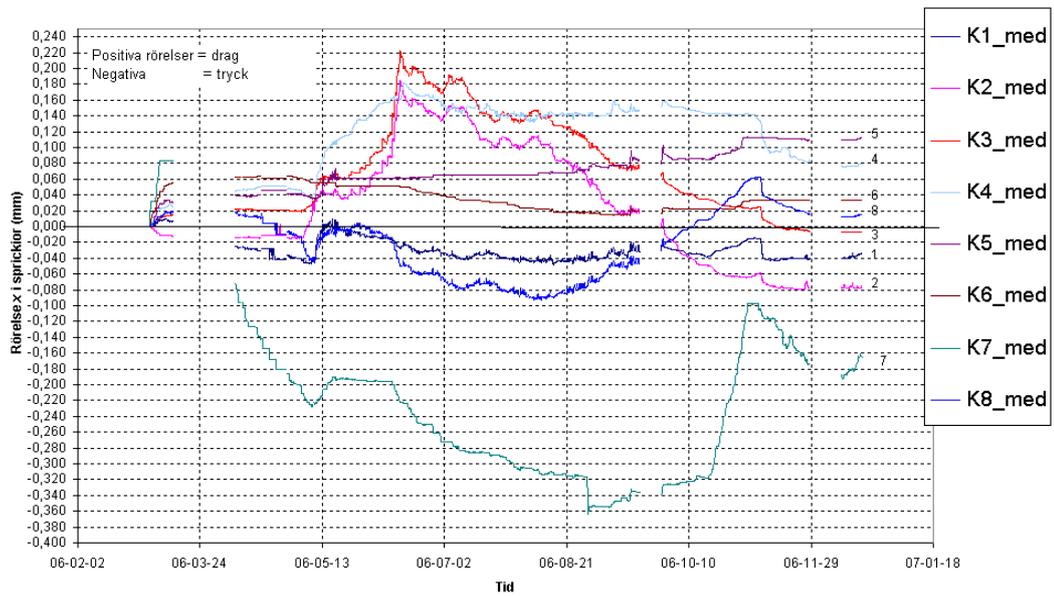


Figure 2.10 Displacements monitored during 2006. Transducers (LVDT) mounted on monolith 43 of the Storfinnforsen dam. From Malm and Ansell (2010).

3 Thermally loaded concrete dams

3.1 Mechanisms causing the cracks

Older studies mention only cooling stresses during the moulding stage as the cause of the cracks in large concrete dams of buttress type. It is possible that cracks have partly arisen, or at least been initiated, during the cooling phase. Calculations of concrete stresses during the cooling period of concrete plates in dams have been made by Fahlén and Näslund (1991) and Melander (1997). The conclusion is that there is a risk of concrete cracks if the front-plate is fastened in two opposite edges, but not if fastened along one single edge. The results may explain the initiation of some cracks observed in situ but they are not able to elucidate the entire crack pattern. No other factors such as loads caused by water, seasonal temperature variation, etc. have been considered as possible mechanisms causing the cracks. There are no early analyses regarding the synergetic effects of several involving mechanisms.

In addition to the initial stresses caused by the cooling process of the dam, the most important loads on the dam are drying shrinkage, water pressure, self-weight, ice-load and stresses due to seasonal temperature variations. All or perhaps some of the mentioned mechanisms must be considered when investigating the formation of crack pattern of a large concrete dam.

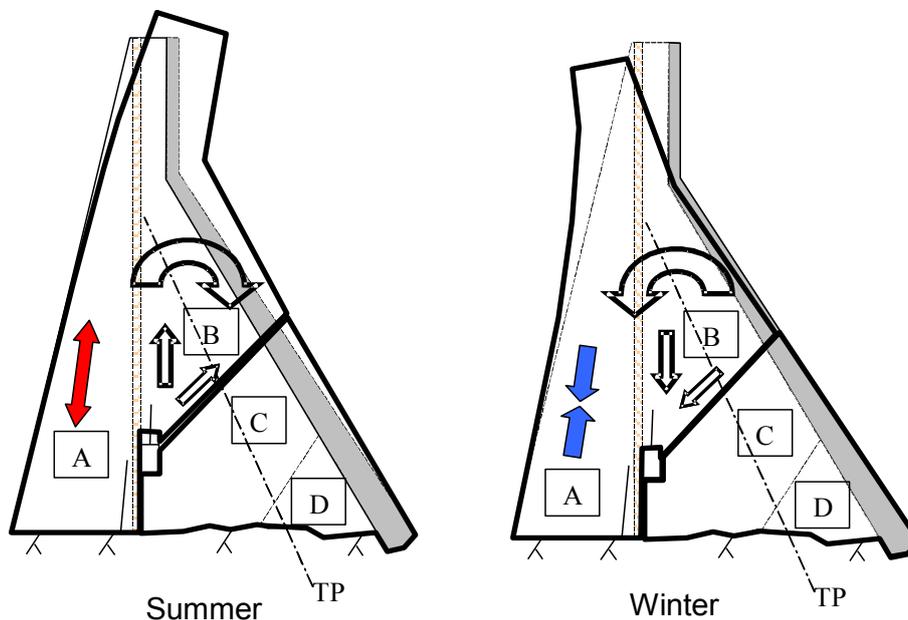


Figure 3.1 Geometry of the analysed concrete monolith. From Ansell et al (2009).

The principles of the structural behaviour are described by Björnström et al (2006) and Ansell et al (2008) and here shown in Figure 3.1. It can be seen that when the ambient air temperature increases, the part A of the buttress expands and forces part B to follow. The front-plate thus becomes subjected to compression when the air temperature increases. The opposite situation is to be expected during the winter, when the ambient air temperature decreases.

3.2 Previous modelling

Previous numerical modelling of the dam behaviour verified that the variation in seasonal temperature together with the water pressure is important mechanisms behind the observed crack pattern. A first step was taken by Björnström et al (2006), through linear elastic analysis aiming at showing whether or not deformations and stresses caused by seasonal variation of temperatures could initiate cracks in a concrete dam. The main purpose was to demonstrate the finite element method (FEM) as a tool for analysis. The results revealed areas of stress concentrations and high stress levels in parts of a dam from where cracks could initiate. The analysis was performed with a large three-dimensional model of solid elements that however could not be efficiently used in a non-linear analysis. The model consisting of approximately 9,000 brick-elements are here shown in Figure 3.2.

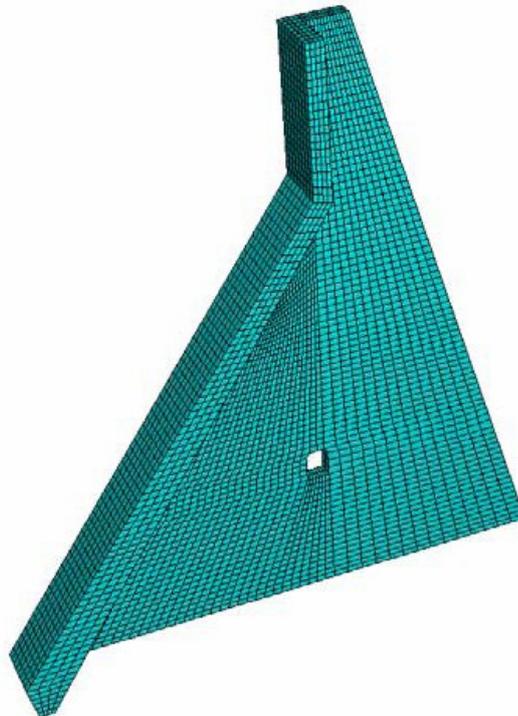


Figure 3.2 Three-dimensional finite element model with 9,000 brick-elements. From Björnström et al (2006).

In a following project, Ansell et al (2008) tested two-dimensional models of triangular continuum elements and non-linear material properties for the concrete. A fictitious buttress dam structure is studied, with measurements typical of large buttress dams in Sweden. The front-plate and the buttress were relatively thin in comparison with the height and width of the analysed dam and therefore plane stress conditions were assumed, in contrast to plane strain conditions that are often used for two-dimensional models of massive structures, such as gravity dams. For the combined analysis of displacements, stresses and temperatures thermally coupled elements were chosen. A typical model of the dam consisted of nearly 6,000 triangular elements with almost 53,000 degrees of freedom, as shown in Figure 3.3. The depth (thickness) of buttress was 2 m with a 8 m deep, 2m wide band representing the front-plate. The monolith was assumed to be rigidly connected to the underground. The effect of reinforcement was not included in this case. The analyses focused on the determination of the direction and the extension of thermally induced cracks. The results showed that it is possible, by means of numerical non-linear analysis, to describe and follow the initiation and propagation of cracks found in situ on the studied type of concrete buttress dams.

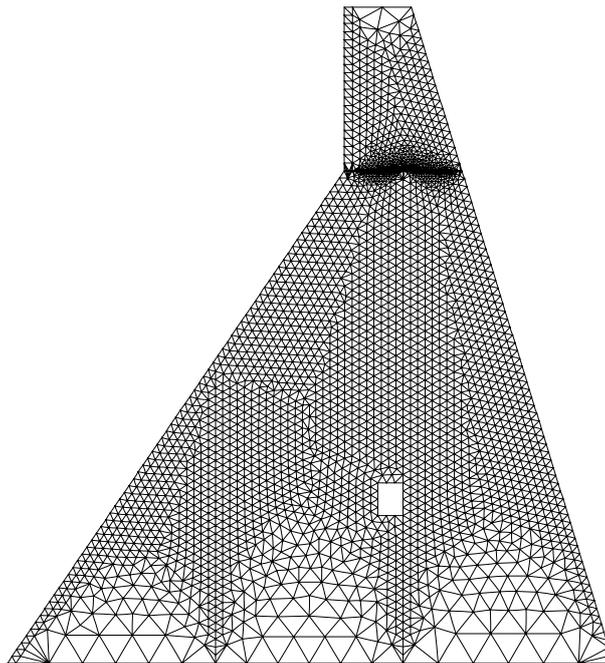


Figure 3.3 Two-dimensional finite element model with 5,800 triangular elements. From Ansell and Malm (2008).

3.3 Structural behaviour

For the previous numerical modelling of the stresses and deformation of the dam behaviour was extreme summer and winter temperatures assumed. It was shown that summer temperatures give rise to large stresses inside insulated area of the buttress. Winter conditions result in high stresses along the downstream side of the front-plate. An example of applied temperatures is shown in Figure 3.4. It is the assumed temperatures used in the analysis with a two-dimensional model, presented by Ansell et al (2008) and Ansell and Malm (2008). The air temperatures were not given in this case, but instead the temperatures within the concrete structure which is divided into four zones, approximately corresponding to the dam with a heat insulation wall vertically from rock to the dam crest, just downstream from the inspection gallery. The two cases summer and winter should be seen as possible extreme temperature distributions. The surface water was in this case given the same temperature as water at greater depth and there was no need to insert rock temperatures for the cases studied with these temperature distributions. The temperature of the centre of the buttress was kept close to $+10^{\circ}\text{C}$, the assumed initial temperature of the concrete that corresponds to zero temperature strain. The part facing downstream, i.e. away from the front-plate, was given high positive and negative temperatures, depending on season. One drawback with a two-dimensional model of this type is that it is not possible to assign temperatures to surfaces that are parallel with the plane considered in the analysis. It is thus impossible to describe temperature variations across the thickness of the front-plate and the buttress. This problem is avoided when using a three-dimensional model, as will be demonstrated in the examples presented in the following.

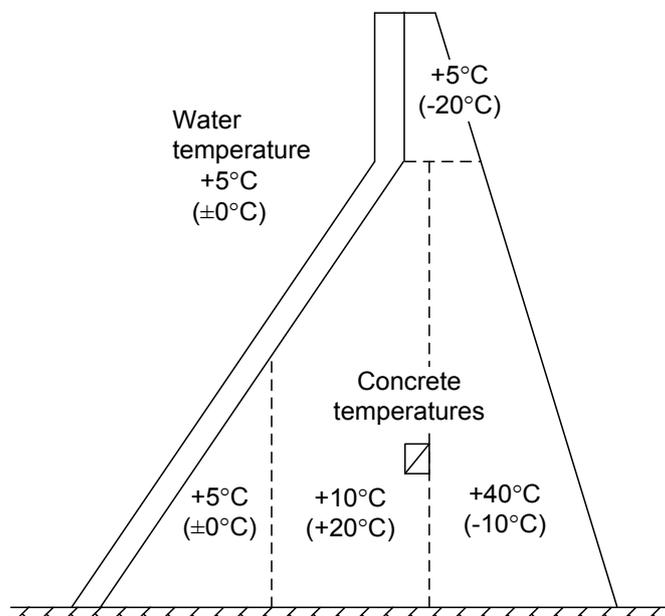


Figure 3.4 Example of previously assumed extreme temperatures in water and concrete during summer (and winter). From Ansell and Malm (2008).

The tests with the two-dimensional model showed that the extreme temperatures, positive and negative, must be applied to parts of the structure before cracks were initiated. The extreme summer temperatures gave rise to large stresses in the buttress and tensile cracks propagated from the opening towards the front-plate, and towards the lower right corner of the structure. For the winter conditions there were low or zero stresses within the interior (insulated) zone with temperatures close to the initial temperature of the concrete, i.e. $+10^{\circ}\text{C}$ which corresponds to zero temperature strain. A vertical band of stresses appeared along the interface between positive and negative temperature zones. There were also high stresses along the front-plate. The assumed full coupling to the underground in both cases gave rise to large stresses and some damage at the lower corners of the structure. The deformed shapes are shown in Figure 3.5 where it is evident that the buttress deforms towards the side with extreme temperatures, towards the relatively cold water during the summer and vice versa during the winter. The horizontal displacement of the crest was in this case approximately 5 mm during summer and 9 mm during winter.

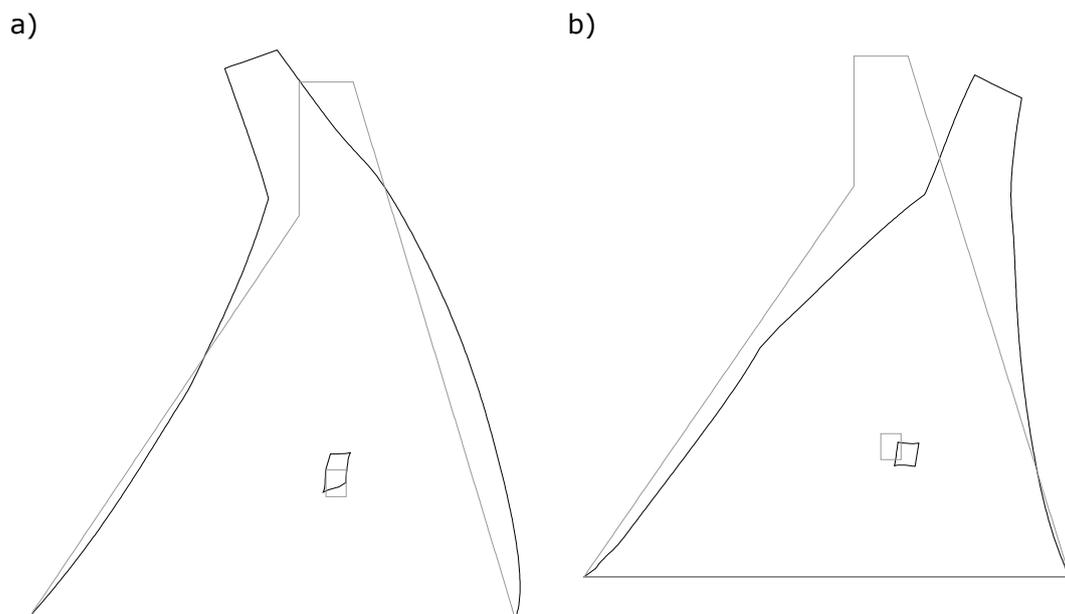


Figure 3.5 Deformation due to extreme (a) summer and (b) winter temperatures. Deformation scale factor 1000. From Ansell and Malm (2008).

4 Non-linear finite element model

4.1 Geometry

One concrete monolith with dimensions similar to that of the Storfinnforsen buttress dam was analyzed. The analysed structure is a 40 m high section of a buttress dam, as shown in Figure 4.1. It consists of a 2 m thick and 8 m wide front-plate facing the water and a 40 m high triangular supporting buttress. The upper 10 m of the front-plate is vertical while the lower part is inclined, with an angle of 56.3° versus the horizontal axis. The downstream edge of the buttress slopes 68.2° versus the horizontal. The dam crest, the horizontal upper part of the buttress and front-plate, has a width of 4 m and the water surface is assumed to be situated 1.5 m below this level. The front-plate is rigidly connected to the buttress, but free at its outer ends that are in contact with the front-plates of the adjacent monoliths. The buttress is 2 m thick and rigidly connected to the rock underground, which is assumed to be of good quality granite. A slab of granite is included in the model which represents the elasticity of the bedrock. This slab is 2 m thick with an $8 \times 34 \text{ m}^2$ base. There is an inspection passage through a $2 \times 1.5 \text{ m}^2$ rectangular opening in the buttress, situated 9 m above the ground.

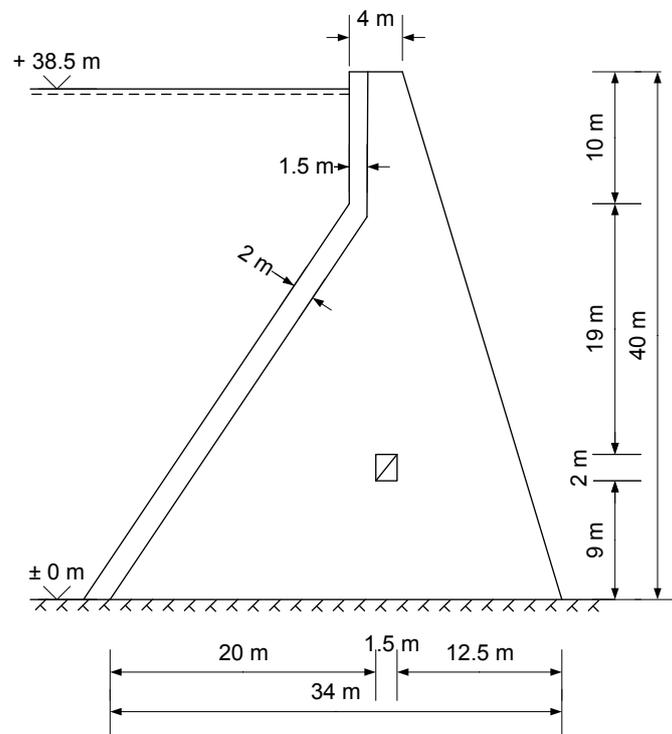


Figure 4.1 Geometry of the analysed concrete monolith.

4.2 Material properties

It is assumed that only one quality of concrete have been used in all parts of the monolith, corresponding to concrete C25/30. The effect of steel reinforcement was included assuming steel reinforcement of standard ribbed steel bars of Swedish type Ks40, all with a diameter of $\phi 18$ mm. Two layers of horizontal and vertical reinforcement bars were placed in the entire monolith, as shown in Figure 4.2. In the construction joint, the segment between the front-plate and the buttress, two additional layers of bars were added perpendicular to the front-plate. Physical and mechanical material parameters for steel, rock and concrete included in the analysis are given by Ansell and Malm (2008) and Malm and Ansell (2010).

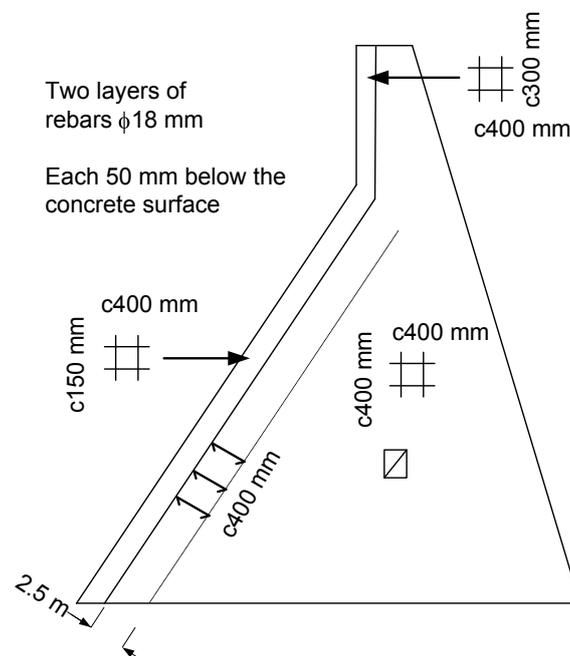


Figure 4.2 Arrangement of steel reinforcement in the analysed concrete monolith.

4.3 Temperature loads

The maximum and minimum outside air temperatures that the dam-structure experience also give rise to the largest temperature differences, since the temperature variation in the water is much less than in the air. This means that the maximum stresses in the concrete structure induced by temperature occur under high summer or mid-winter, when the outside temperatures reach the maximum and minimum of the year. Stresses in the concrete occur due to restraint deformations caused by the bond to the underground and temperature gradients within the structure. The concrete is assumed to be unstrained at its initial temperature $+10^{\circ}$ C, corresponding to the zero stress condition at casting.

The two maximum temperature states during summer and winter are shown in Figures 4.3–4, based on measured temperature variations at the Storfinnforsen dam presented by Ansell et al. (2008). The 8.5 m of water closest to the surface is assumed to have small temperature changes with season while water at greater depth is assigned a constant temperature over the year. The temperature variation of the rock is identical to that of the surface water on the summer but colder during the winter. Two different cases are studied, with and without an insulating wall that divides the area of the buttress in contact with the outside air into two separate zones. The space created between the front-plate and the wall will have temperatures much closer to that of the surface water. The part facing downstream, i.e. away from the front-plate, is given high positive and negative temperatures, depending on season. The numerical calculations used to simulate the two seasonal extreme temperature states are performed as steady-state analyses where the temperature fields are reached step-wise from the initial temperature of the concrete.

In addition to the maximum seasonal temperature cases were also the effect of seasonal variations examined (Malm and Ansell, 2010). This finite element analysis of the dam covers five full temperature cycles, i.e. a time span of five years. After the first two years the assumed temperature distributions were changed, thereby accounting for the installation of an insulating wall as described above. For each year, two extreme temperature conditions were considered, i.e. summer and winter. Between the winter and the summer season the thermal strains are increased linearly, and vice versa. The calculated results thus come from one initial step with water pressure and gravity load and thereafter a sequence of ten steady-state calculations beginning with a crack-free concrete structure. Each season, summer or winter, then contributes to the cracking of the concrete.

4.4 Other loads

In addition to temperature loads, gravity and water pressure are considered, but no other loads such as for example ice-loads and seismic loads. During the last years equipment have been used for circulation of the surface water, preventing the formation of ice within 3–4 m from the front-plate. The water is the most important of the loads and one monolith is assumed to be designed to carry the load from 38.5 m of water acting on the 8 m width of the front-plate. The water pressure gives a resulting force of approximately 58 MN in the horizontal direction.

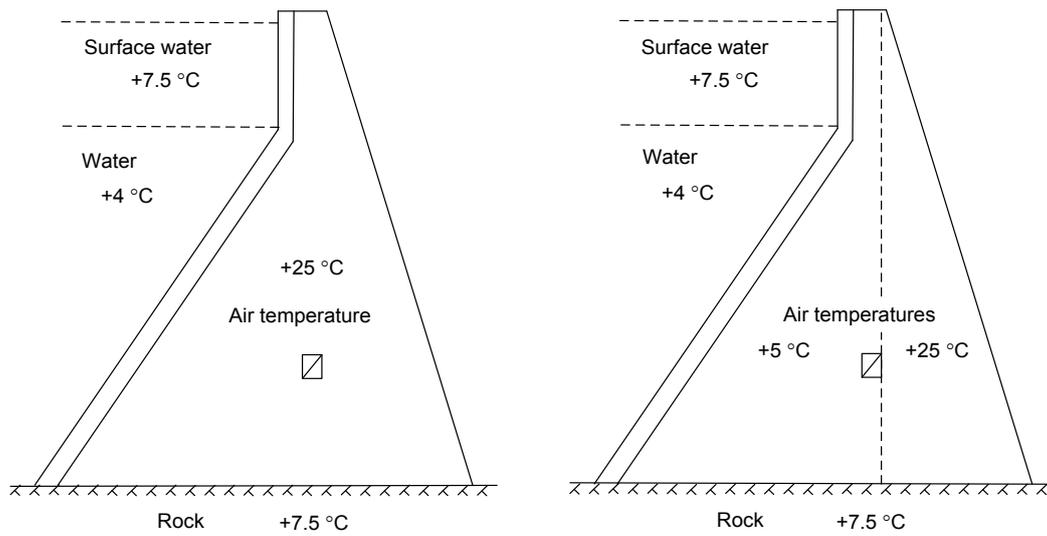


Figure 4.3 Maximum temperatures in air, water and rock during summer for a monolith with and without an insulating wall.

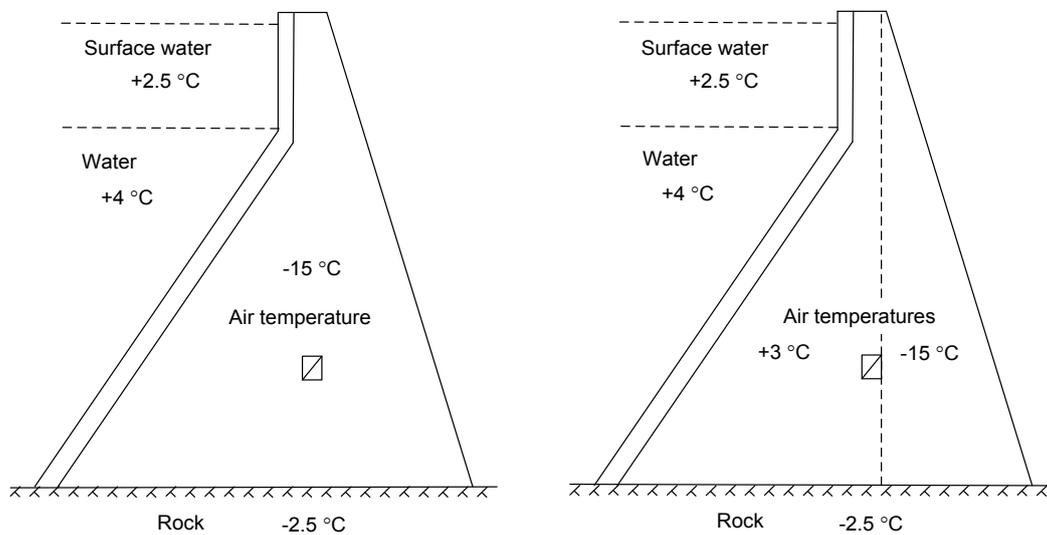


Figure 4.4 Maximum temperatures in air, water and rock during winter for a monolith with and without an insulating wall.

4.5 Non-linear material model

The studied concrete dam has been modelled using a non-linear material formulation for the behaviour of concrete. The material model used in the analysis is based on plasticity theory and called concrete damaged plasticity. With this model it is possible to define the material degradation of compression as well as tension, where the tensile softening behaviour is based on a crack-opening law and fracture energy. Damage is associated with the failure mechanisms of concrete (cracking and crushing) and therefore results in a reduction in the elastic stiffness. The damaged plasticity model is intended for analysis of reinforced concrete structures but can also be used for plain concrete. Concrete reinforcement can be modelled as discrete rebars or as reinforcement layers embedded in concrete. The model is also suitable for the analysis of other quasi-brittle materials such as rock, mortar and ceramics. No visco-elastic effects, such as for example creep, are included in the model.

Material properties, numerical parameters and short summaries of theoretical relations associated with the concrete damaged plasticity model are given by Ansell and Malm (2008) and Malm and Ansell (2010). A more detailed description of the model and its implementation is given by Malm (2006). Further information regarding damage and/or plasticity theory can be found in e.g. the paper by Karihaloo (2003) and in the book by Chen and Han (1995).

4.6 Finite element model

The studied concrete dam has been modelled using the Abaqus/Standard (Abaqus, 2006) finite element software. A numerical analysis with the program is divided in steps, each corresponding to a load change from one magnitude to another. For the analysis of a dam the first steps can be the addition of gravity load and water pressure, followed by one or more temperature loads. For each step a procedure is chosen that corresponds to the type of analysis to be performed, and a full analysis may thus include various numerical procedures. The state of the model in terms of stress, strain, temperature, etc. is updated throughout all analysis steps. The effects of previous steps are always included in each new step. It is therefore important to use a sequence of steps that accurately describes the actual load situation for the structure analysed.

The finite element analysis was here performed to determine temperature distributions from each maximum summer and winter temperature case. The temperature distributions were thereafter used as input for a calculation of displacements and stresses in the monolith, based on non-linear material properties. For solving the non-linear equilibrium equations Newton's method is the general numerical technique due to its convergence rate for the often studied types of problems. It should be remembered that solving non-linear problems are much more computationally demanding than linear problems and the objective in such cases is often to obtain a convergent solution at a minimum effort.

An accurate description of the geometry is especially important when detailed predictions of strains are required. Modelling of curved, intersecting plates and shells is possible with shell elements in a three-dimensional analysis and

the concrete monolith was for this analysis modelled as shells, also including the effect of steel reinforcement. The approach makes it possible to assign temperatures to each surface of the shells, describing temperature distributions over all surfaces, as well as for the centre of the shells.

The three-dimensional model was built up with approximately 9,400 triangular shell elements with 33,000 degrees of freedom, as seen in Figure 4.5. The rock is here modelled as an elastic 2 m thick plate, with material properties as given by Ansell and Malm (2008) and Malm and Ansell (2010). This will lead to less stress build-up at the concrete-rock interface, compared to a case where the rock is un-deformable. In this way, an elastic coupling to the ground is obtained, which approximates the function of anchorage of dam structures in situ. Reinforcement bars were included in the model, coupled to the shells as embedded elements.

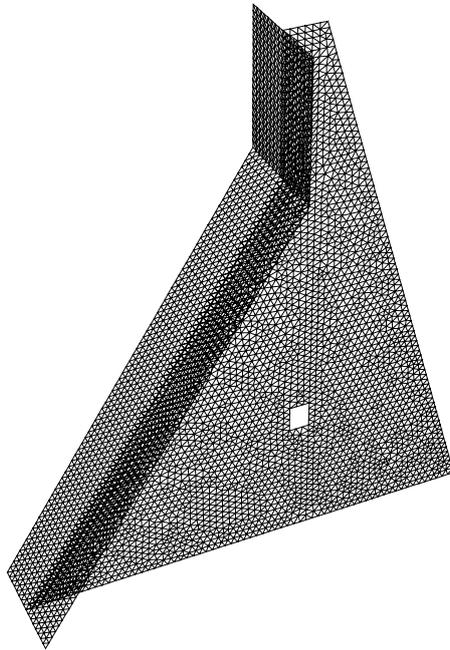


Figure 4.5 Finite element model with 9,418 triangular shell elements.

5 Numerical results

5.1 General structural behaviour

The deformed shapes due to extreme temperatures are similar to those of the two-dimensional model, with the buttress bending towards the coldest side. The sizes of the deformations are also comparable and the deformed shapes of the shell model are therefore not shown here.

5.2 From summer temperatures

Tensile stresses and deformations due to maximum summer temperatures as given to the left in Figure 4.3 are shown in Figure 5.1. The summer temperatures give rise to high tensile stresses in the front-plate but only minor in the buttress, as seen in Figure 5.1(a). The plate is cracked (b) at the lower 15 m, with a vertical crack along the connection to the buttress, ending with a horizontal crack across the plate. There are also large tensile stresses at the connection between the upper vertical and the lower parts of front-plate. The maximum crack width (crack opening displacement, COD) for the horizontal crack in the front-plate is approximately 0.75 mm and its development with the outside air temperature can be seen in Figure 5.2.

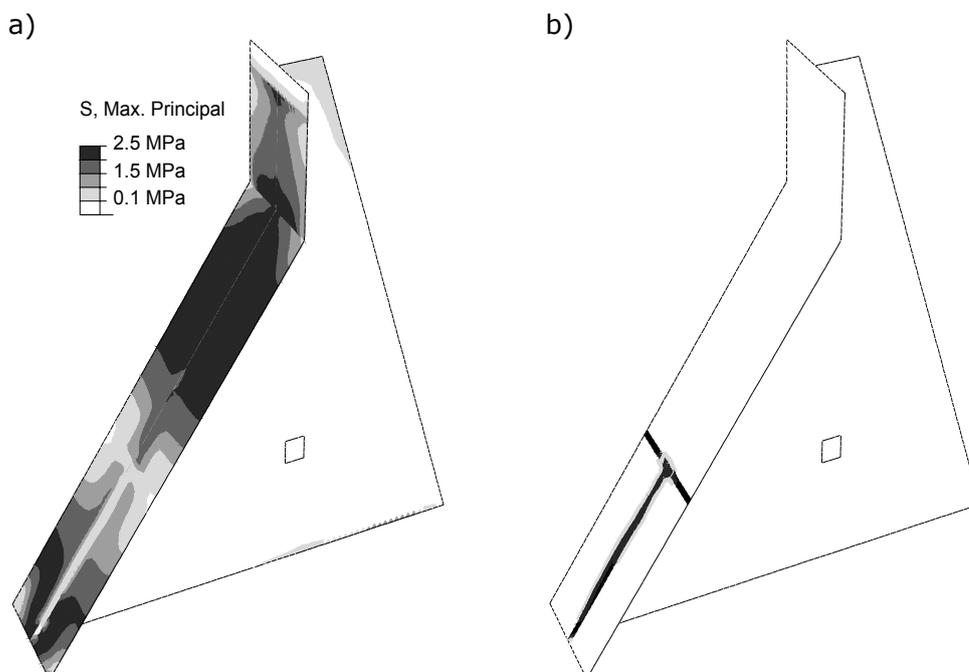


Figure 5.1 Maximum principal tensile stresses (a) and damaged concrete (b), due to summer temperatures.

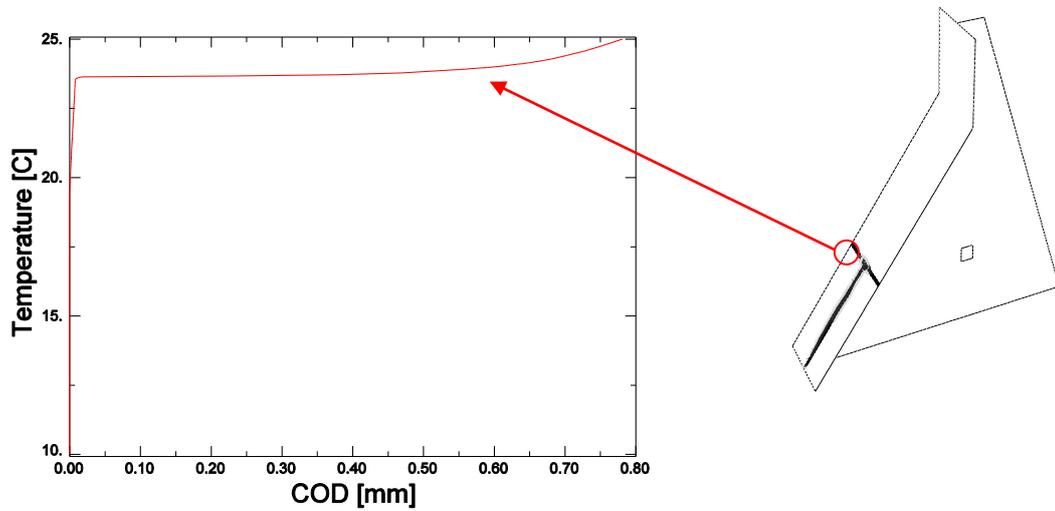


Figure 5.2 Temperature vs. crack opening displacement for the horizontal crack in the front-plate due to summer temperatures.

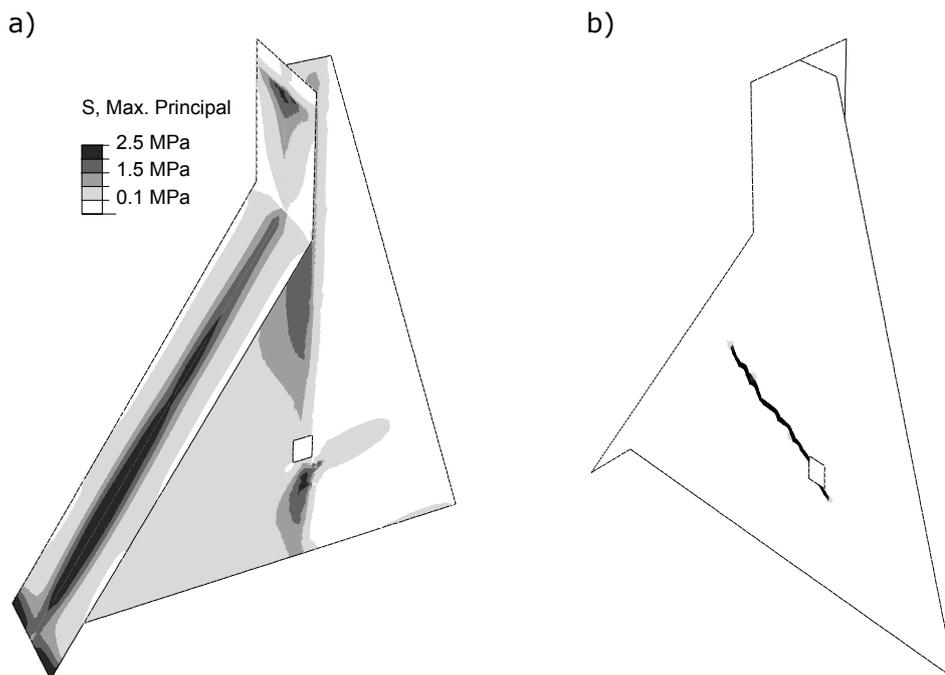


Figure 5.3 Maximum principal tensile stresses (a) and damaged concrete (b) due to summer temperatures, with insulating wall.

The inclusion of the insulating wall results in a division of the surfaces of the buttress into two temperature zones, as shown to the right in Figure 4.3. The sharp limit between the moderate temperatures between the wall and the front-plate and high outside temperatures result in large tensile stresses in the buttress, as seen in Figure 5.3. In this case the summer temperatures will also initiate cracks at the rectangular opening which propagates diagonally towards the front-plate and in the opposite direction, as seen in Figure 5.3(b).

The cracks are initiated when the outside air temperature is about $+20\text{ }^{\circ}\text{C}$, but will propagate first when this has reached approximately $+25\text{ }^{\circ}\text{C}$.

5.3 From winter temperatures

The stresses due to winter temperatures show less stress in the front-plate but high stress fields in the lower part of the buttress, at the heel of the dam, as seen in Figure 5.4(a).

There are low stresses in the front-plate but high stress fields in the lower part of the buttress. There are also in this case large tensile stresses at the centre of the front-plate. These stresses do not initiate cracking, apart from the edges of the concrete at the concrete-rock interface as shown in Figure 5.4(b). The maximum crack width (COD) for the inclined crack in the buttress is approximately 0.17 mm , as can be seen in Figure 5.5.

The inclusion of the insulating wall here only results in micro-cracks in the front-plate, as shown in Figure 5.6. The cracks at the concrete-rock interface do not, however, appear in this case. The sharp line between the moderate temperatures between the wall and the front-plate and low outside temperature give rise to large stresses in the buttress, in the zone with the lowest temperatures.

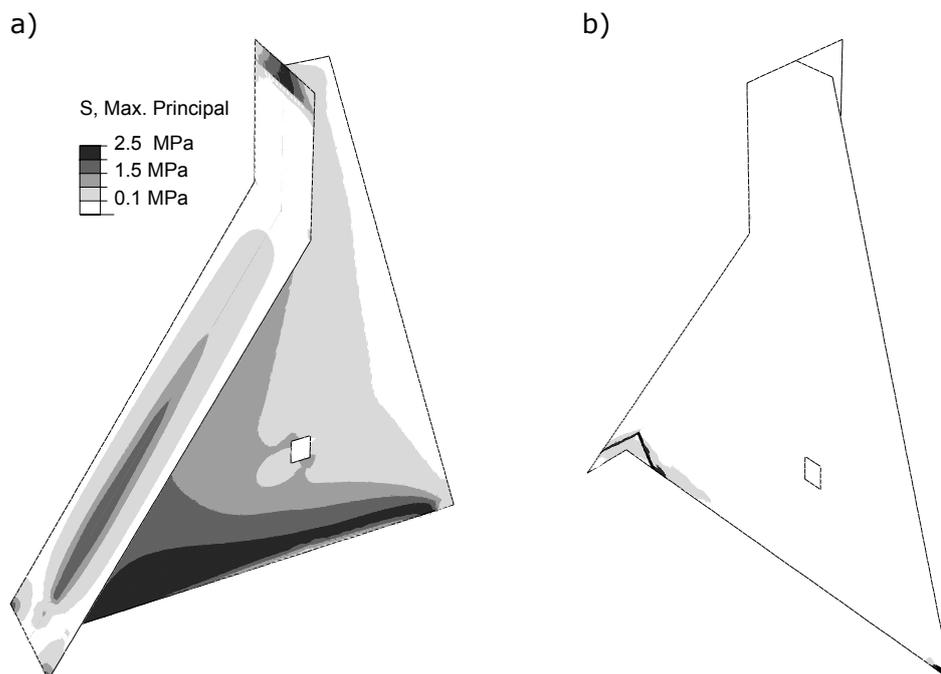


Figure 5.4 Maximum principal stresses (a) and damaged concrete (b) due to winter temperatures.

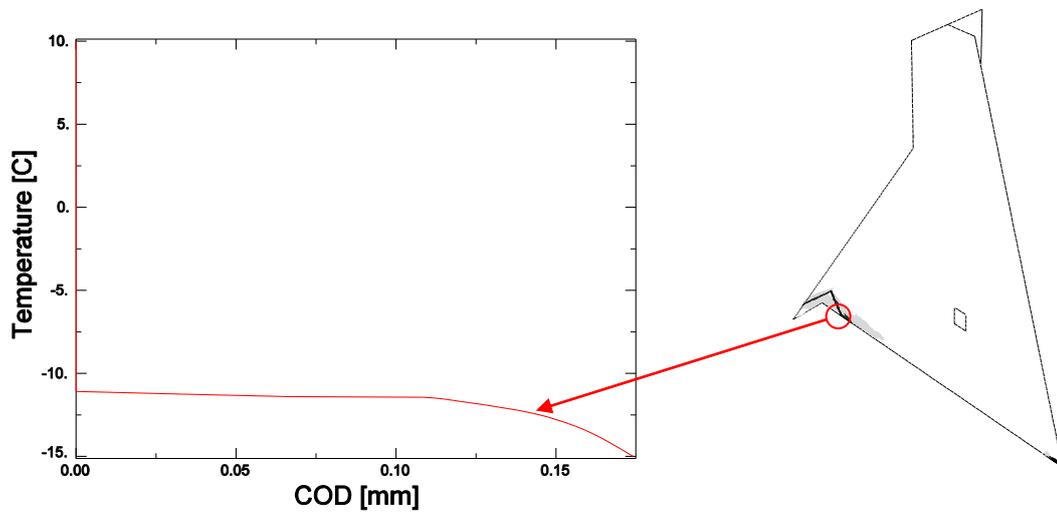


Figure 5.5 Temperature vs. crack opening displacement for the inclined crack in the buttress due to winter temperatures.

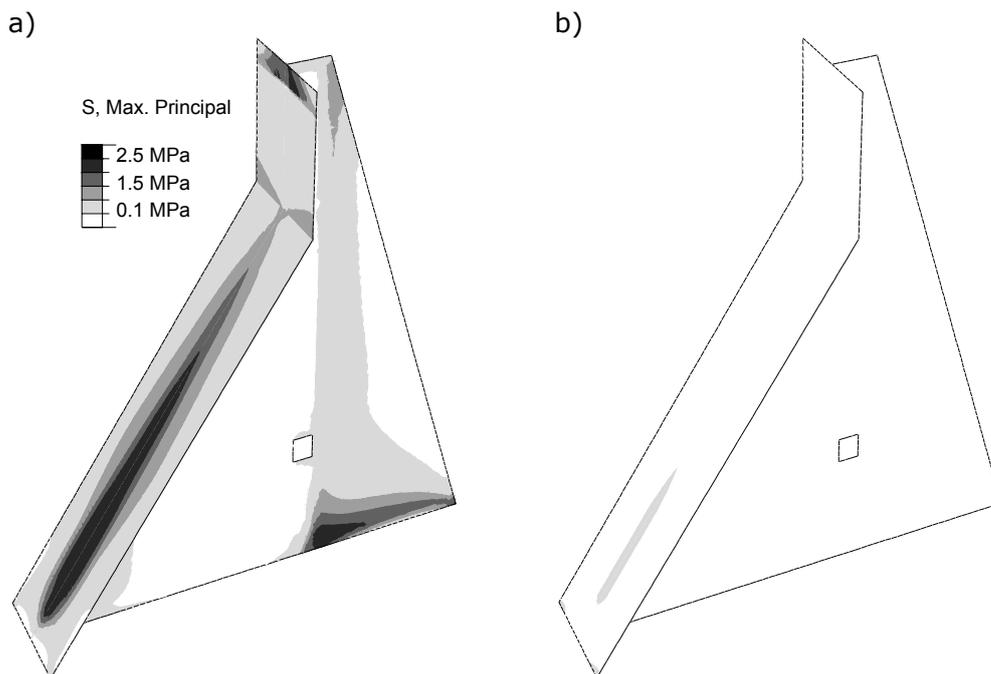


Figure 5.6 Maximum principal tensile stresses (a) and damaged concrete (b) due to winter temperatures, with insulating wall.

5.4 Seasonal effects on a dam - without insulating wall

The formation of concrete cracks depends on whether the first season experienced by the un-cracked dam is summer or winter. Subsequent steady-state calculations, with alternating summer or winter temperatures, will be affected by the previous since a more cracked structure is weaker which may affect the initiation of new cracks. After two seasons with cyclic seasonal temperature the dam reaches a constant crack pattern where new cracks only form if the extreme temperatures increase. The crack pattern after two years becomes identical irrespective of which season the analyses start with. The cracks that occur during the two first years in the analysis, without the heat insulation wall, are shown in Figures 5.7–10, starting with winter conditions that lead to some cracking in the lower part of the buttress. As seen, the front-plate is severely cracked already during the first summer season. The maximum principal tensile stresses are also shown in Figures 5.7–10. It can be seen that large tensile stresses occur on the downstream side of almost the whole front-plate. During the winter season large tensile stresses occur in the lower part of the buttress. If the cyclic seasonal analysis is started with summer conditions, one additional year is needed to obtain the results in Figures 5.9–10.

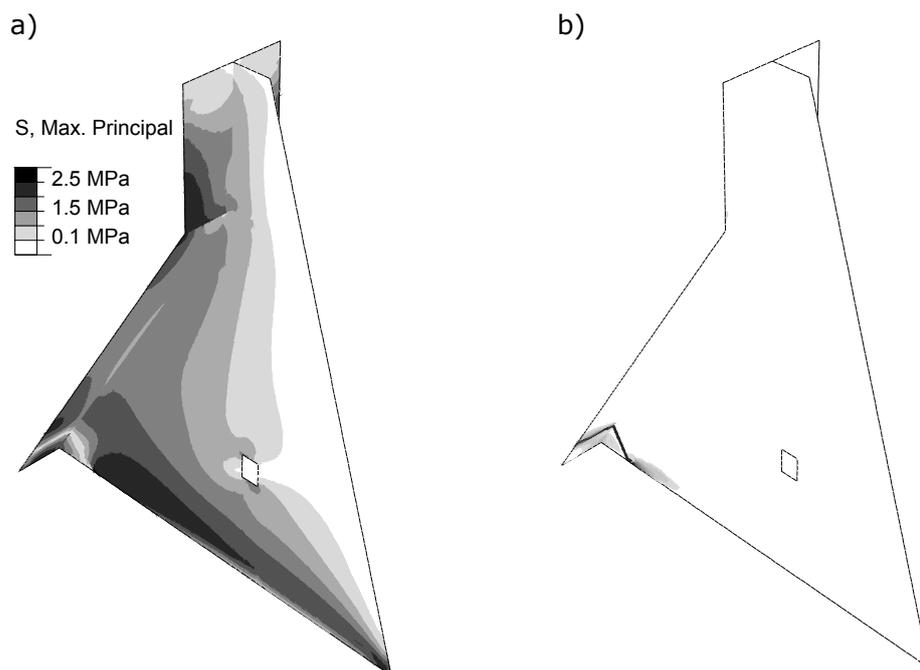


Figure 5.7 Year 1: Maximum principal tensile stresses (a) and damaged concrete (b) due to winter temperatures, without insulating wall.

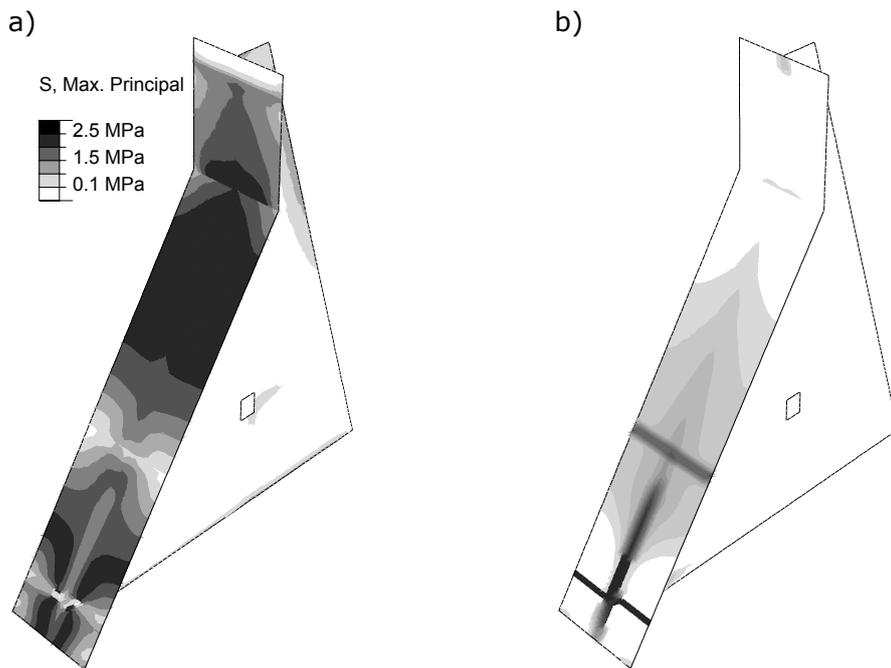


Figure 5.8 Year 1: Maximum principal tensile stresses (a) and damaged concrete (b) due to summer temperatures, without insulating wall.

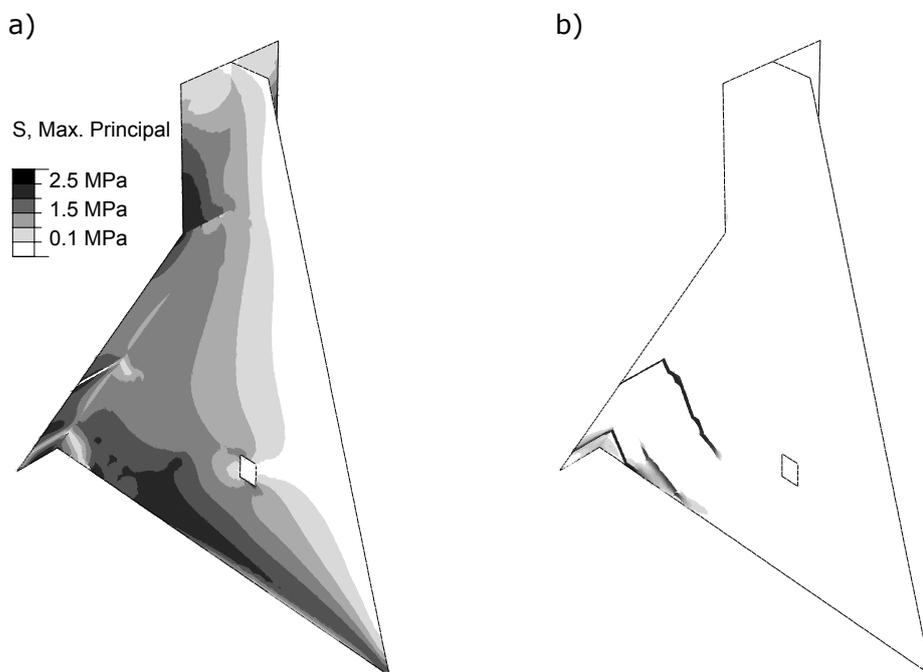


Figure 5.9 Year 2: Maximum principal tensile stresses (a) and damaged concrete (b) due to winter temperatures, without insulating wall.

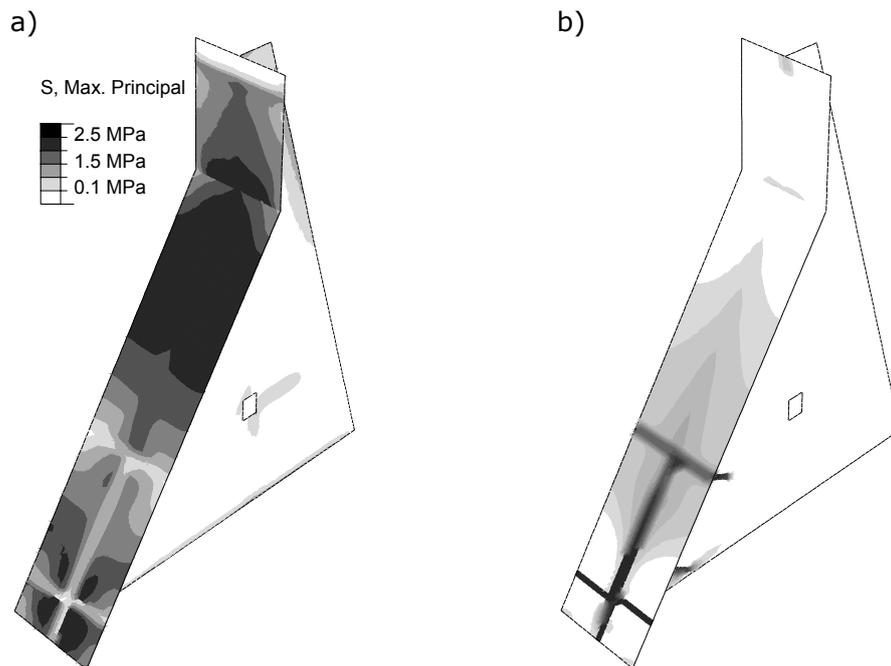


Figure 5.10 Year 2: Maximum principal tensile stresses (a) and damaged concrete (b) due to summer temperatures, without insulating wall.

5.5 Seasonal effects on a dam - with insulating wall

From the third year of the analysis the presence of an insulating wall is accounted for, resulting in the concrete damage shown in Figures 5.11–12. The results from the fourth and the fifth year are presented in Figures 5.13–16. These results, from the last three years of the analyzed time span, also show possible damage caused by this modification of the original structure. The first new cracks developed during the summer, an inclined crack that initiated at the inspection passage and propagated swiftly towards the front-plate. During the following winter analysis step, year 4, this crack propagated from the opposite side of the inspection passage towards the downstream base of the buttress. After this point the dam with the insulating wall has reached a constant, stabile crack pattern where additional cracks or crack propagation will not occur unless higher extreme temperatures appear. It can be seen when comparing Figures 5.7–16 that the tensile stresses in the front-plate are reduced after inclusion of the insulating wall. Large stresses do, however, appear in the buttress during the summer, as can be seen in Figure 5.12. The insulating wall results in a division of the surfaces of the buttress into two temperature zones, resulting in a distinct line in the buttress in the stress figures, where high stresses occur on the colder side.

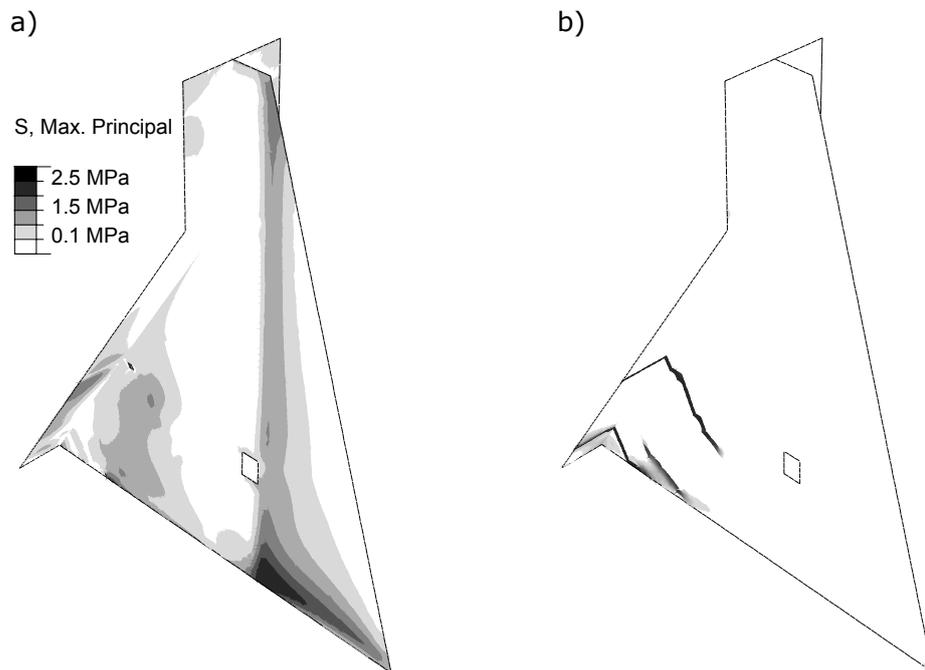


Figure 5.11 Year 3: Maximum principal tensile stresses (a) and damaged concrete (b) due to winter temperatures, with insulating wall.

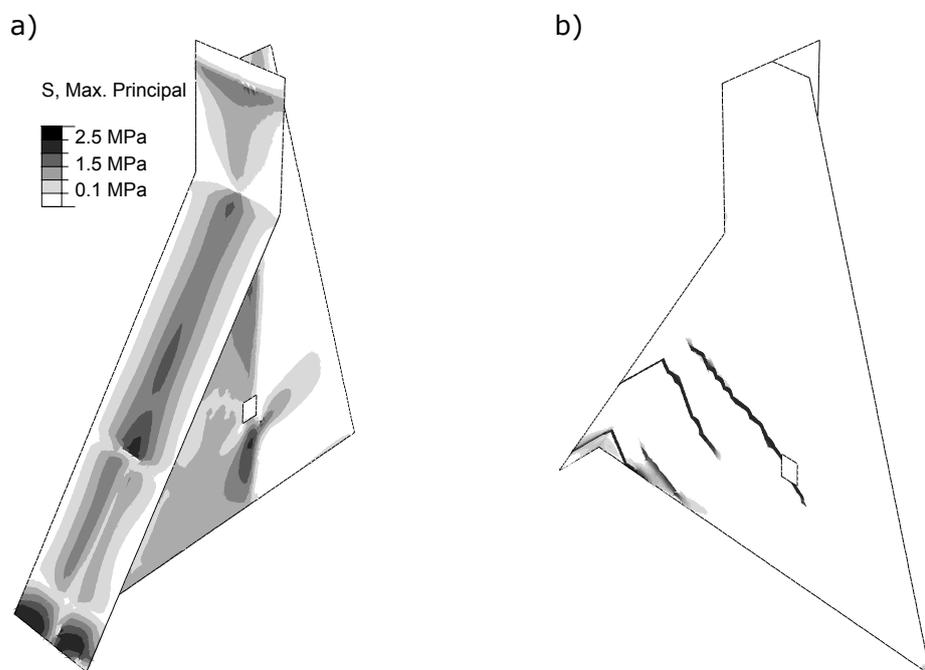


Figure 5.12 Year 3: Maximum principal tensile stresses (a) and damaged concrete (b) due to summer temperatures, with insulating wall.

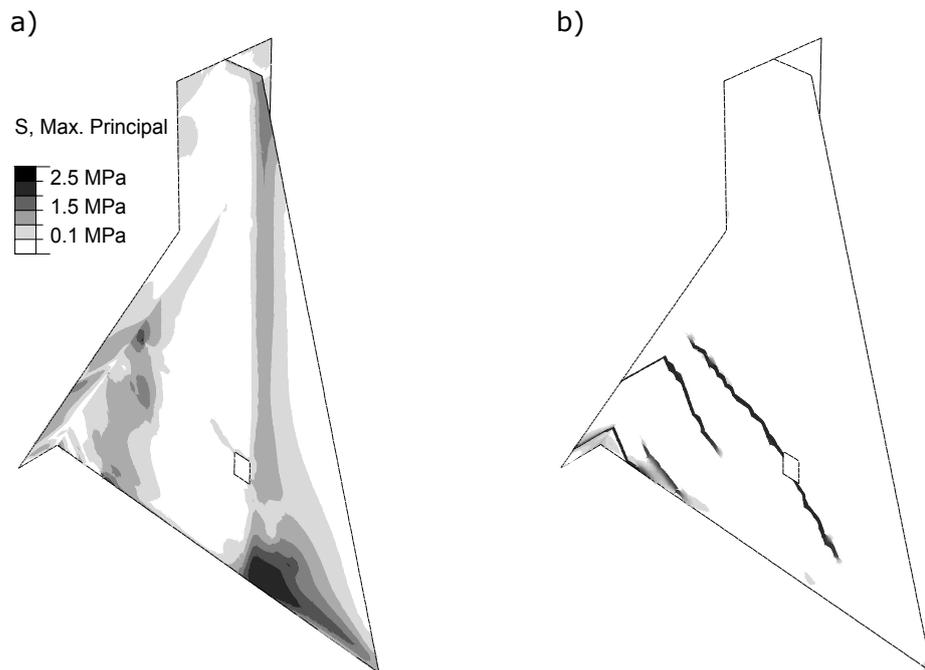


Figure 5.13 Year 4: Maximum principal tensile stresses (a) and damaged concrete (b) due to winter temperatures, with insulating wall.

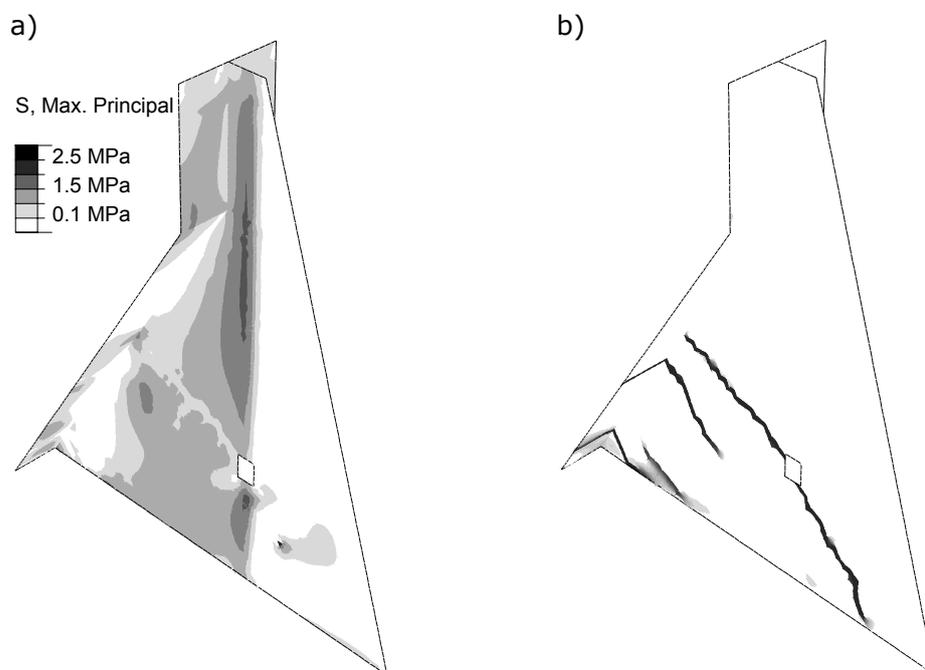


Figure 5.14 Year 4: Maximum principal tensile stresses (a) and damaged concrete (b) due to summer temperatures, with insulating wall.

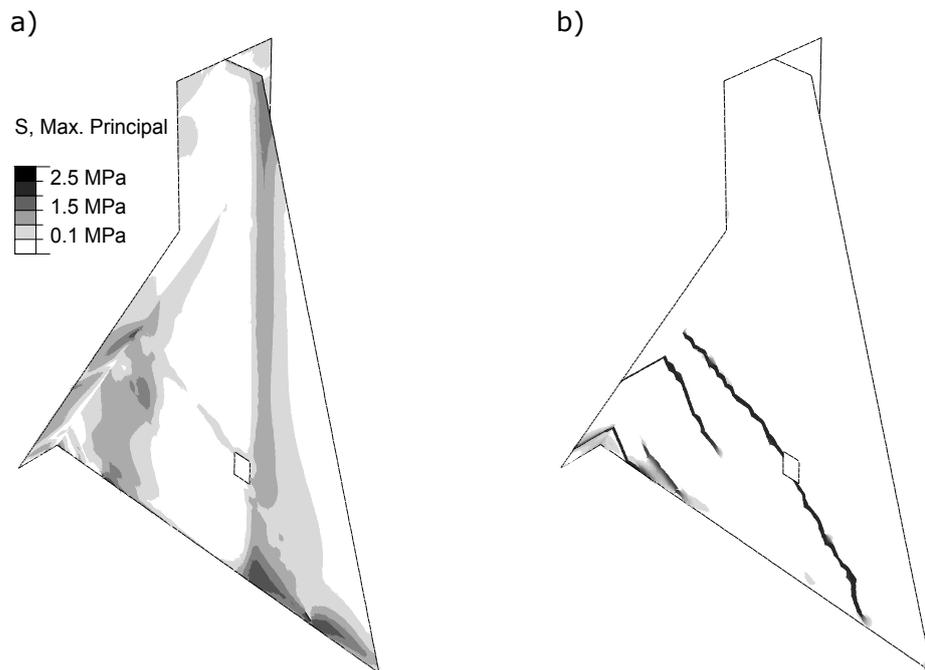


Figure 5.15 Year 5: Maximum principal tensile stresses (a) and damaged concrete (b) due to winter temperatures, with insulating wall.

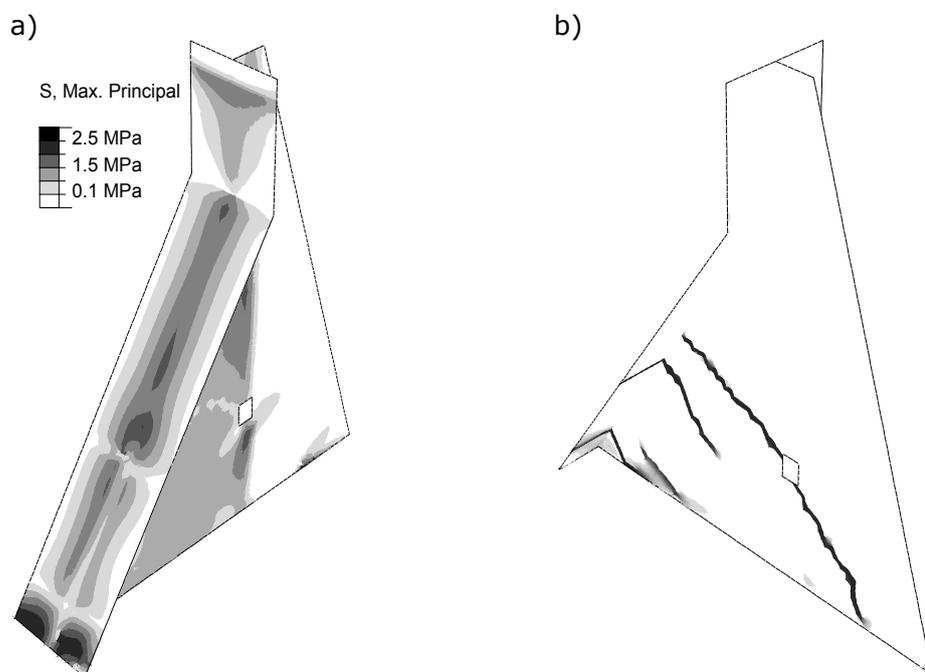


Figure 5.16 Year 5: Maximum principal tensile stresses (a) and damaged concrete (b) due to summer temperatures, with insulating wall.

The variations of crack widths during the analysis are presented in Figure 5.17. The two largest cracks in the dam are shown, a horizontal crack in the front-plate (type 1) and the inclined crack in the buttress starting from the inspection passage (type 3). The variation of the displacement of the dam crest is shown in Figure 5.18. The displacements have been normalized to remove the permanent displacement from the gravity load and the external water pressure. Movement upward is defined as a positive vertical displacement and a horizontal movement in the downstream direction is defined as a positive horizontal displacement.

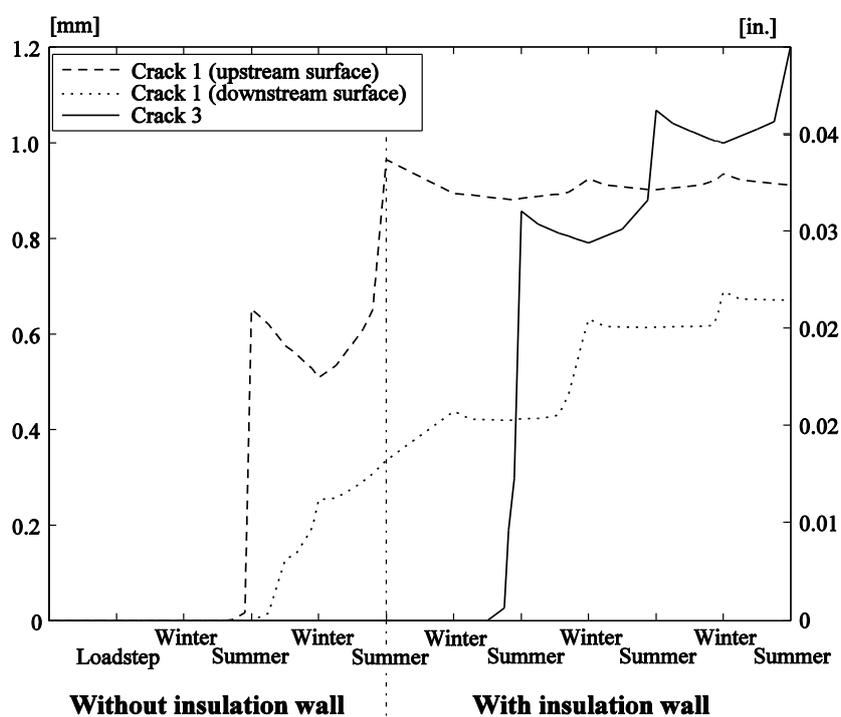


Figure 5.17 Seasonal variation in crack width. From Malm and Ansell (2010).

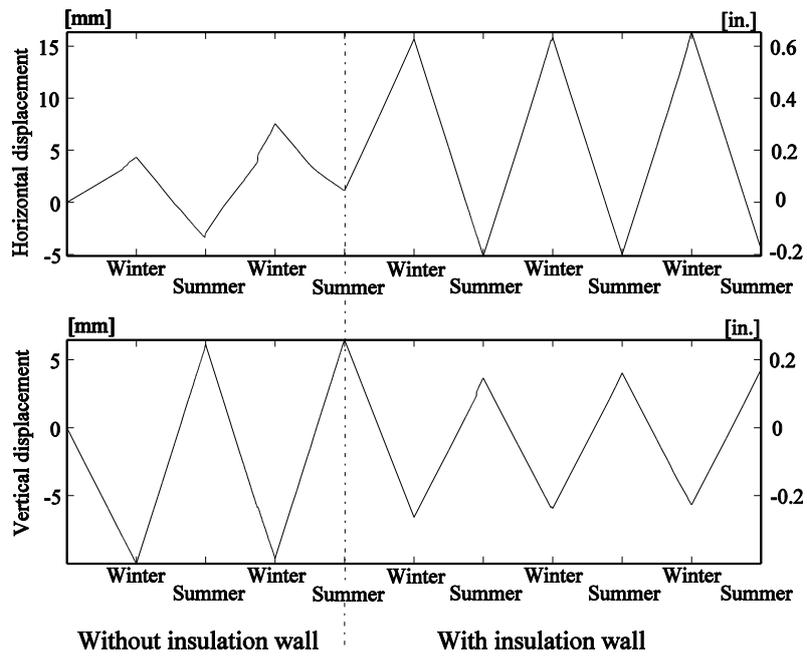


Figure 5.18 Seasonal variation of horizontal and vertical displacement. From Malm and Ansell (2010).

6 Discussion

6.1 Comparison with observations

The calculated results show that all four types of cracks observed in situ were obtained in the non-linear analyses. Horizontal cracks in the front-plate, type (1) according to Figure 2.4, occur due to high tensile stresses during the summer, which can be seen in Figure 5.1. The winter temperatures results in cracking of type (1) and (2) on the downstream side of the front-plate and in the lower part of the buttress respectively, as can be seen in Figure 5.4. These cracks of type (2) are probably caused by a combination of static load from the water and restrained shrinkage due to low temperatures. The third type of cracks (3) goes diagonally from the rectangular opening orthogonally towards the front-plate. Such a crack is clearly visible in Figure 5.3 and a crack propagating in the opposite direction is also found, caused by extreme summer temperatures following the instalment of the insulating wall. This can be seen when comparing Figures 5.1 and 5.3. A detailed analysis showed that the diagonal cracks propagate when the temperature within the space enclosed by the front-plate and the insulating wall is colder than approximately $+10^{\circ}$ C at the same time as the outside temperature exceeds approximately $+25^{\circ}$ C. Cracks from the concrete-rock interface, type (4), are caused by restrained movement due to low temperatures, and possibly also concrete shrinkage. High tensile stresses and possible concrete damage in this area are visible in the winter temperature-results in Figures 5.4 and 5.6. The magnitude of the stresses within the area close to the concrete-rock interface depends on the rigidity of the rock underground.

The simulation of five years of seasonal variation shows that cracks of type (1) and (2) appear already during the first year, as seen in Figures 5.7–10. Diagonal cracks in the buttress are visible from the second winter. A crack of type (3) appears during the first summer following the instalment of the insulating wall and this crack was also found to propagate in the opposite direction during the subsequent winter season, as seen in Figures 5.11–12. A crack of type (4) appeared during the fifth winter season where the inclined crack from the inspection passage has propagated into a vertical crack at the concrete rock interface, which can be seen in Figures 5.13–16.

It can be seen in the Figure 5.17 that the crack width on the upstream side of the crack in the front-plate continuously increases until the insulating wall is inserted. This crack stabilizes thereafter, and for the remaining seasons in the analysis this crack has a maximum width of 1.0 mm. However, on the downstream side of the front-plate this crack continues to increase. The crack with the largest width is the inclined crack of type (3) that originates from the inspection passage, with a maximum width of 1.2 mm. The maximum observed crack width observed on the monitored Storfinnforsen dam is about 2.0 mm (Fahlén and Näslund, 1991), but most of the observed cracks have widths within 0.3 to 1.0 mm.

The vertical displacements, shown in Figure 5.18, have reduced due to the insertion of the insulating wall. The calculated horizontal displacement has on

the other hand increased after the insulating wall was included. The analysis showed that both the horizontal and the vertical displacement are slowly increasing due to the crack propagation. The calculated crack patterns, crack types and crack widths shows good agreement with the ocular observations on the Storfinnforsen dam. Some cracks were most probably also formed, during the cooling phase when the dam was built, in the contraction joint between pillars and front-plates and vertically in the pillars near the rock. These first cracks have facilitated the further formation of the cracks described above.

6.2 Comparison with previous results

Two-dimensional models will always be computationally more efficient than three-dimensional equivalents, with respect to cost in time and computer capacity. A three-dimensional model will, however, always give a more accurate description of a structure such as the analysed dam when compared to a two-dimensional model, for example of the type previously used by Ansell et al (2008). It is important to note that deformations and cracking due to transverse bending of the front-plate of the type of analysed dams can not be described with a two-dimensional model. The previously investigations by Björnström et al (2006) showed that a three-dimensional model based on solid elements of brick-type will yield large models that give long calculation times, most often followed by numerical difficulties.

The tested model based on shell elements has much fewer degrees of freedoms than a comparable model with solid elements but still describes the full geometry of the dam. It is possible to accurately model the two plates of the analysed monolith as plane shells, thereby also facilitating the application of surface temperatures as input for the analysis. With shell elements it is also possible to obtain information on the temperature variation through the thickness. Another important feature is that steel reinforcement can be included as embedded bars within the shells, which increases the accuracy of the analysis. The method of two-step analysis proved to be efficient. In a first step the temperature distribution within the structure is calculated on basis of the surface temperature input. This relatively fast analysis step is then followed by a relatively slow non-linear calculation of stresses and displacements.

6.3 Numerical modelling

The results show that an analysis using non-linear material properties for the concrete is a powerful tool for describing and follow the initiation and propagation of cracks found in situ on the studied type of concrete buttress dams. A non-linear material model makes it possible to describe the post-peak behaviour of the concrete compared to a strictly elastic analysis, but it is considerably more computationally demanding. The convergence of a non-linear analysis must be ensured by carefully selecting the type of finite elements, mesh density and numerical tolerances. These choices are also significantly more important when using a three-dimensional model compared to one in two dimensions.

A static analysis of cracking in concrete with Abaqus/Standard (Abaqus, 2006) can in some cases result in convergence difficulties due to local instabilities. These difficulties can some times be avoided by using a visco-plastic regularization of the constitutive equations. Visco-plastic regularization temporarily permits stresses to be outside the yield surface. This usually increases the chance of convergence without compromising the results, if the viscosity parameter is given a small value compared to the time increment. Another way to overcome the convergence difficulties when using a static method is to introduce artificial damping. This can be done by introducing discrete dashpots to all nodes in all directions or by using the built-in automatic stabilization function. In the performed analyses the tolerances were reduced to a level several times lower than the default values. An increased number of iterations were allowed in this case. After this, a low value of artificial damping was introduced in combination with small visco-plastic regularization.

It was found by Malm and Ansell (2010) through observation of the variation in strain energy that the stabilizing energy content was kept at a low level during the presented analysis and henceforth did not compromise the results. Analyses were also made with higher content of stabilizing energy which showed that despite this it was still possible to get an accurate crack pattern. Models were also tested where the total stabilizing energy at the end of the analysis was equal or slightly higher than the total strain energy during the summer case and still resulted in a crack pattern similar to the one presented here.

6.4 Conclusions

The use of three-dimensional shell models proved to be more accurate than two-dimensional models. Although the shell models were computationally more demanding than the simpler two-dimensional models, they were still much more efficient than three-dimensional solid element models.

The numerical results show that it is possible, by means of numerical non-linear analysis, to describe and follow the initiation and propagation of cracks found in situ on the studied type of concrete buttress dams. The use of three-dimensional shell models proved to very accurate although rather computationally demanding with a CPU time of approximately 7.5 hours for the seasonal analysis.

The results show that the seasonal temperature variation may cause high tensile stresses at different locations on the dam, and that the cracks can be initiated from at least four locations on the dam. All types of cracks can also be initiated and propagate simultaneously. The analysis indicates that the instalment of an insulating wall, that together with the front-plate encloses a space that includes the rectangular opening, may be one of the causes for the inclined cracking of the dam. The insulation wall prevents the ice-formation on the front-plate and protects it against frost damage. However, a more suitable placement of the insulation wall, at the downstream end of the buttresses, could have prevented the cracking of the pillars.

6.5 Further research

The future research will aim at calculation of dam safety of existing cracked dams, where different possible failure modes and loads must be taken into account. Combinations of various temperatures, water pressure and ice-pressure should be used with models also accounting for the long term effect of concrete creep. The effects of water pressure within concrete cracks are also of interest.

The importance of using non-linear material models for the steel reinforcement must be studied further.

The temperature distribution and propagation within the concrete must be studied in detail and verified with measurements and observations in situ. The movement of the dam crest due to seasonal temperature changes must be measured on a full size dam and used for verification of the finite element models. It is also of great interest to first predict the effect of possible repair methods for cracked dams by calculations and then evaluate this through in situ observations.

It is also very important to be able to predict the final, global failure of a monolith. To do this a global load-deformation-curve as in Figure 6.1 has to be calculated. The deformation of the dam crest is a suitable measure. After appearance of the first crack the monolith will behave non-linearly, with the formation of more cracks leading to global failure. The load-deformation curve is however not deterministic, but probabilistic due to variations in basic variables, such as for the Young modulus of elasticity, strength, etc. The model may therefore be refined also with probabilistic calculations.

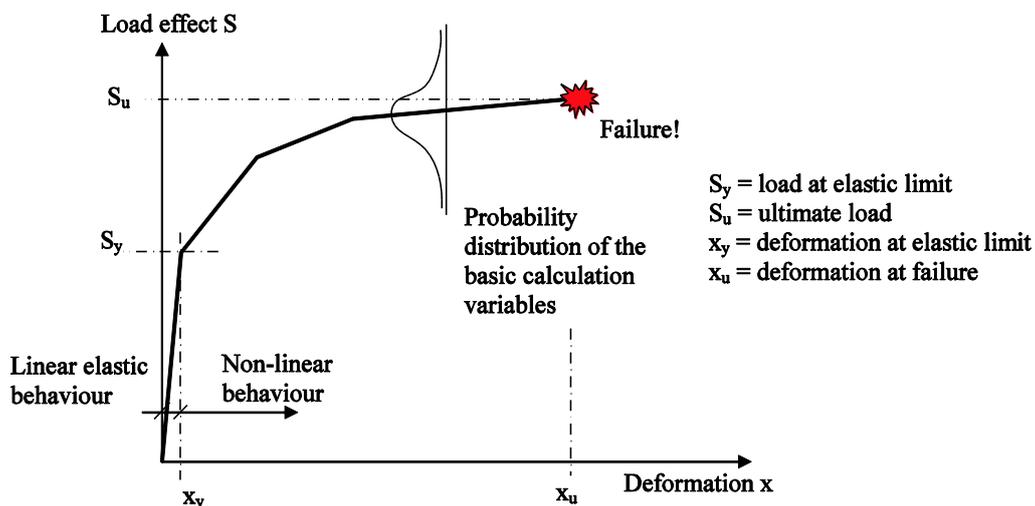


Figure 6.1 Principles of a load-deformation curve showing failure of a concrete monolith.

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