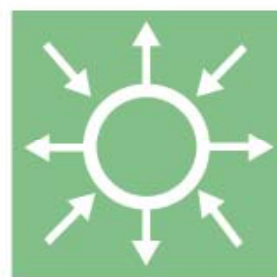




Market Power in Hydro– Thermal Supply

Elforsk rapport 06:86



Karl-Axel Edin

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ELFORSK

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Preface

Despite having had a deregulated electricity market in Sweden for over ten years we still need to increase our understanding as to how deregulated electricity markets actually work and how possible problems are to be solved.

One question that is always in focus is if the competition between generators in the Nordic electricity market really works the way it was intended. Many argue that the concentration in ownership of generation plants already has gone too far. Together with joint ownership in nuclear facilities and barriers for entrance, critics say that this has resulted in higher electricity prices than necessary.

In this report different methods to (ex ante) study potential possibilities for generating firms to influence the electricity price (market power) and (ex post) discover possible manipulation through analysing the spot price and other observed factors on the electricity market are analysed.

Elforsk (Electricity research) is owned by the Swedish electricity industry. Its corporate business idea is to carry out research and development in line with the interests of the owner companies and carry out these research projects in cooperation with other parties on the market. The Market Design program was initiated in 2000 for the purpose of increasing the knowledge of how deregulated electricity markets work. The program is financed by Svensk Energi, EBL-Kompetanse in Norway and Sweden's Energy Agency.

More information on the program, our reports and current activities is available on program's home page, www.marketdesign.se.

Stockholm, December 2006



Peter Fritz
Program Secretary, Market Design
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1 Introduction

1.1 The electricity market

The properties of the power market are closely linked to storage of energy. If electricity could be stored inexpensively like ordinary goods the production structure and the market would be much different. There would be one or a few types of thermal power plants operating at near full capacity all the time. The system marginal cost function would be nearly flat. There would be little room for market power since new producers could easily enter to share excess profits if incumbent producers tried to exercise their market power.

Because in reality storage of electricity is expensive and consumption varies over time the optimal production structure is a mixture of different production technologies with diverse capital and operating costs. The result is a production system with rising marginal cost depending on which type of production is needed on the margin to meet demand at a certain moment. From a competition point of view there is, unlike other types of markets, not one homogenous market but several separate markets. Each type of power in a marginal cost range constitutes a separate market with possibilities to exert market power.

The availability of storage is, therefore, of key importance when assessing market power in the electricity market. With ample storage the electricity will function more like an ordinary market. With little storage the conditions may be very different from ordinary commodity markets.

Another feature of today's deregulated electricity markets is the use of auction systems to organize sales and purchases. From a competition point of view auction markets can have quite different properties compared to other models for competition, for instance the Cournot model.

A third feature of the electricity market is the degree of contestability. There is relatively ease of entry of new producers threatening incumbent producers if they charge high prices.

A fourth feature of the electricity market is its transparency, especially where auction systems are used. Typically hourly auction prices and aggregate quantities are published. It may even be possible to get information on bid functions. Production plant properties are also often well known since there are rather few standard production technologies with well known operating costs. Therefore, electricity markets are ideally suited to test different theories.

1.2 Purpose of paper

The purpose of the longer underlying paper is to give a comprehensive treatment of the electricity market with storage, i.e. hydro power, with an auction market organisation and to test the models on the Nordic market in order to explore the explanatory power of auction market theory and the

theory of contestable market. The main theoretical effort in the paper concerns auction theory with inventories.

The paper develops an inter-temporal auction model of a thermal-hydro power market. Parallel to the derivation of the basic equations a numerical model is developed in order to illustrate the results of the model.

Section 2 of the present paper summarizes the basic equations (derived in the longer paper) for an inter-temporal auction thermal-hydro market.

Section 3 contains the illustrations of solutions to equations for some stylized markets.

In section 4 the auction model is tested on the Nordic market.

2 The equations

2.1 Optimal bid functions

The first step in modelling the auction market is to establish the equations for the optimal bid functions of the auction agents. The general equations for the bid functions are established in the longer paper. The following notations are used:

$m=1...N$	agent index
s	store (water reservoir) quantity
i_t	water inflow rate
d_t	consumption rate
h	hydro power production rate
f	thermal power production rate
$p_m^c(x_m)$	marginal cost function for agent m
$x_m^c(p)$	inverse marginal cost function for agent m
$R_m(x_m)$	profit function for agent m
$p_m(x_m)$	bid function for agent m
$x_m(p)$	inverse bid function for agent m

The basic optimal bid function equations can be written

$$[1.1] \quad \frac{dR_m(x_m)}{dx_m} \stackrel{\text{def}}{=} p_m(x_m) - p_m^c(x_m) - \frac{1}{\sum_{n \neq m}^N x_{n,p}(p_n(x_n))} \cdot x_m \equiv 0$$

Consumer agents are represented by negative x values. $x_{n,p}(p)$ is the derivative of $x_n(p)$ with respect to p . With an infinite number of agents for which each $x_{n,p} > 0$ the optimal bid function equals the marginal cost, $p_m(x_m) = p_m^c(x_m)$ because the last term is zero. These equations form a system of N first order differential equations.

This paper deals mostly with groups of identical agents, thermal producers (F), hydro producers (H) and consumers (D), where only one group has market power, the other groups bidding their marginal cost. If this group is F then the optimal aggregate bid function equation for this group has the following "simple" form.

[1.2]

$$p^F(f) - p^{cF}(f) - \frac{f}{(N^F - 1) \cdot \frac{1}{p_f^F(f)} + N^F \cdot [h_p^c(p^F(f)) - d_p^c(p^F(f))]} \equiv 0^1$$

Here N^F is the number of identical thermal producers, $h^c(p)$ the aggregate inverse hydro marginal cost function and $h_p^c(p)$ its derivative with respect to p . $d^c(p)$ is the aggregate inverse hydro marginal cost function and $d_p^c(p)$ its derivative with respect to p . The same equation can be used for the other groups by permutating F , H and D . Because of the presence of $p_f^F(f)$ it is a differential equation if $N^F \neq 1$. For $N^F = 1$ it is an ordinary equation.

Since the hydro power marginal cost and bid functions at time t depend on the store level s the bid functions depend on s for all groups, not only hydro power producers. The dependence of the bid functions on s is not explicitly shown in the following.

2.2 Pure thermal system

2.2.1 Identical producers

In a pure thermal system with N identical producers and consumers bidding their marginal cost equation [1.2] (dropping superscript F) takes the form

$$[1.3] \quad p(f) - p^c(f) - \frac{f}{\frac{N-1}{p_f(f)} - N \cdot d_p^c(p(f))} \equiv 0$$

With inelastic consumption, $d_p(p)=0$ the bid function equation becomes

$$[1.4] \quad p(f) - p^c(f) - \frac{1}{N-1} \cdot f \cdot p_f(f) \equiv 0$$

It has the following integral solution

$$[1.5] \quad p(f) \equiv (N-1) \cdot f^{N-1} \cdot \int_f^a dg \cdot g^{-N} \cdot p^c(g)$$

where a is the integration constant. For a CE marginal cost function $p^c(f) = f^{1/\epsilon}$ and with the requirement that $p(f) > 0$ for all f (implying $a = \infty$) the optimal bid function is

¹ Here the identity $f_p(p) \equiv \frac{1}{p_f(f(p))}$, where $p(f)$ is the inverse of $f(p)$, is used

$$[1.6] \quad p(f) \equiv \frac{1}{1 - \frac{1}{(N-1) \cdot \varepsilon}} \cdot f^{\frac{1}{\varepsilon}}$$

This is just a constant scale factor times the marginal cost.

2.2.2 One big producer

With one big producer^B and many small producers^S and inelastic consumers bidding their marginal cost equation[1.1] can be written

$$p^B(f) - p^{cB}(f) - \frac{f}{f_p^{cS}(p^B(f))} \equiv 0$$

Here $p^B(f)$ is the big producer's bid function and $p^{cB}(f)$ its marginal cost function. $p^{cS}(f)$ is the aggregate small producers' marginal cost with inverse $f_p^{cS}(p)$ (the aggregate supply function).

Since this is not a differential equation one can solve it directly at the clearing point. If one denotes the small producers' aggregate production by g , then at clearing $p^B(d-g) = p^{cS}(g)$. So

$$p^{cS}(g) - p^{cB}(d-g) - (d-g) \cdot p_g^{cS}(g) \equiv 0^2$$

The last equation determines the clearing quantity $g^{clear}(d)$ and the clearing price as function of consumption $p^{clear}(d) = p^{cS}(g^{clear})$. As an illustration assume that the producers share the same aggregate marginal CE cost function $p^c(f) = f^{d/\varepsilon}$ so that

$$f^{cB}(p) \equiv a \cdot f^c(p) \equiv a \cdot p^\varepsilon \Rightarrow p^{cB}(f) \equiv p^c\left(\frac{f}{a}\right) \equiv \left(\frac{f}{a}\right)^{\frac{1}{\varepsilon}}$$

$$f^{cS}(p) \equiv (1-a) \cdot f^c(p) \equiv (1-a) \cdot p^\varepsilon \Rightarrow p^{cS}(f) \equiv p^c\left(\frac{f}{1-a}\right) \equiv \left(\frac{f}{1-a}\right)^{\frac{1}{\varepsilon}}$$

Here a is the big producer's share of the aggregate thermal supply function. The clearing equation can then, with the big producer's market share denoted by $x = (d-g)/d$, be cast in the form

² Since $f^{cS}(p)$ and $p^{cS}(f)$ are inverse functions $f_p^{cS}(p^{cS}(g)) \equiv \frac{1}{p_g^{cS}(g)}$

$$[1.7] \quad \frac{\frac{1}{x} - 1}{\frac{1}{a} - 1} \equiv \left[1 + \left(\frac{1}{x} - 1 \right) \cdot \frac{1}{\varepsilon} \right]^{\varepsilon}$$

It is seen that the big producer's market share x is less than its share a of the supply, since $\varepsilon > 0$. The clearing price as function of a and d is given by

$$p^{clear}(d, a) \equiv p^c \left(\frac{g}{1-a} \right) \equiv \left(\frac{d \cdot (1-x)}{1-a} \right)^{\frac{1}{\varepsilon}}$$

$$\pi \equiv \frac{p^{clear}(d, a)}{p^{clear}(d, 0)} \equiv \left(\frac{1-x}{1-a} \right)^{\frac{1}{\varepsilon}}$$

The last line gives the ratio of price to the marginal cost (represented by $a=0$). This ratio is independent of d like in the previous case with identical producers.

The following diagram shows the ratio for one bid producer compared with the ratio for identical producers from equation [1.6] as function of the Herfindahl index with $\varepsilon=0.5$ (quadratic marginal cost).

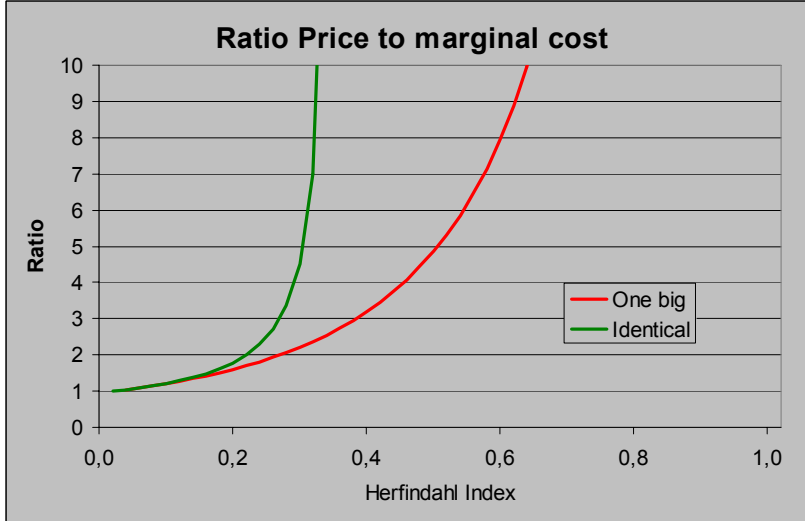


Figure 1: Ratio of price to marginal cost as function of the Herfindahl index for a quadratic marginal cost function in the case with one big producer and the case with identical producers.

The figure shows that for a given Herfindahl index a production structure with identical producers is more prone to market power than a system with one big producer and many small.

2.3 Hydro power

In the equation [1.2] and its permutations the marginal cost function $p^{cF}(f)$ and $p^{cD}(f)$ are exogenous. In an inter-temporal model with storage of water in reservoirs, however, the hydro marginal production cost function $p^{cH}(h)$ is not exogenous but is determined within the model. The exogenous properties of hydro power are storage capacity (TWh) and generating capacity (GW). The hydro marginal cost is given by future power prices, which are determined by the inter-temporal equations of the model. It is closely related to the marginal store value.

The hydro marginal cost function is denoted by $u_t(s)$ where t is the time and s the water energy in the store. This is a decreasing function of s , illustrated in the following diagram

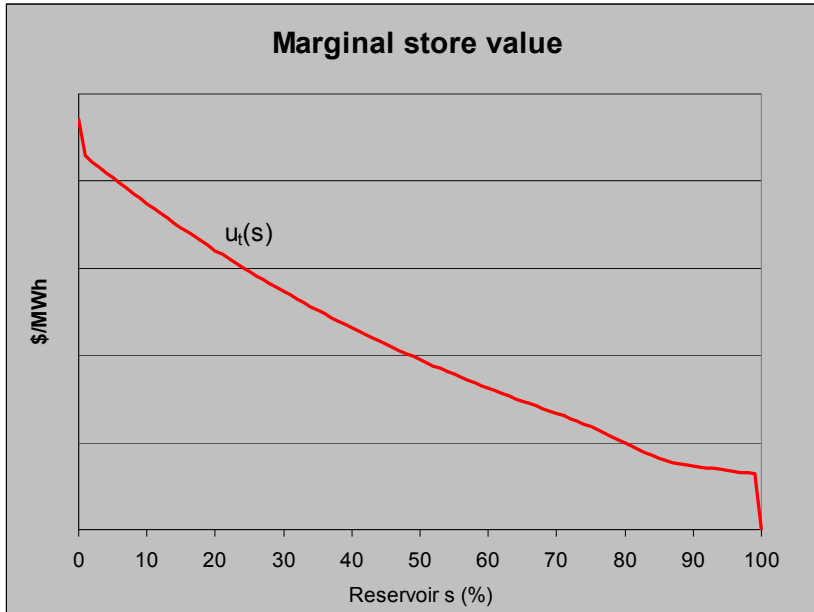


Figure 2: Typical marginal store value function.

With the restriction that hydro production cannot exceed the generating capacity h^{max} or the store content s , the marginal cost of production at time t is given by

$$[1.8] \quad p^{cH}(h) \equiv \begin{cases} r \cdot u_{t+1}(s + i_t - h) & h \leq h^{high} \\ \infty & h^{high} < h \end{cases}$$

$$h^{high} \equiv \min(h^{max}, s)$$

Here the discount rate factor r is included because it is assumed that the bids are submitted at the beginning of price period t and production of hydro power takes place at the end of time period t . The inflow i_t is included for the same reason.

In case the price is determined by an exogenous marginal cost function $p^c(f)$ a hydro producer will sell so much hydro power at a certain moment that the marginal store value is equal to the clearing price within the capacity constraints.

$$[1.9] \quad \begin{aligned} r \cdot u_{t+1}(s + i_t - h) &\equiv p^c(d_t - h) \\ 0 \leq h &\leq h^{high} \end{aligned}$$

2.3.1 Auctions

In an auction market the optimal hydro power production is determined by the bid function. The basic bid function equation [1.2] can be used also for hydro power with hydro marginal cost function given by [1.8]. In the case with N^H identical hydro producers the aggregate hydro bid function equation takes the form.

$$[1.10] \quad p^H(h) - p^{cH}(h) - \frac{h}{(N^H - 1) \cdot \frac{1}{p_h^H(h)} + N^H \cdot [f_p^c(p^H(h)) - d_p^c(p^H(h))]} \equiv 0$$

If one knows the marginal store value at $t+1$ one can thus determine the optimal bid function at time t . This will then determine the clearing price h_t at time t and the optimal hydro production h_t in period t .

How can one determine the store value? Denote the store value (not the *marginal* store value) as function of store level s at the start of period t by $U_t(s)$. The value of the store at the start of period t is equal to the net revenue during period t plus the value of the store at the start of period $t+1$. The inter-temporal equation is thus

$$U_t(s) = h_t \cdot p_t + r \cdot U_{t+1}(s + i_t - h_t)$$

where h_t is hydro production in period t , p_t the market price and $u_t(s) = dU_t(s)/ds$. With $h_t(s)$ determined the store value at time t is given by

$$U_t(s) = h_t(s) \cdot p_t + r \cdot U_{t+1}(s + i_t - h_t(s))$$

In terms of the marginal store value function the inter-temporal equation is given by derivation of the above equation with respect to s . It can be shown that if hydro producers have no market power (see longer paper)

$$[1.11] \quad u_t(s) \equiv \begin{cases} p_t(s) & h_t(s) \neq h^{\max} \\ r \cdot u_{t+1}(s + i_t - h_t(s)) & h_t(s) = h^{\max} \end{cases}$$

This is the basic inter-temporal equation that can be used iteratively starting far in the future with some assumption or guess of the store value at some final time. The equation holds also in cases where a producer has both hydro and thermal production since the producer will then try to equalize the marginal cost of production in all its production plants, including hydro power.

However, with few hydro producers that can affect the price the solution is more complicated. In the case with N identical pure hydro producers the inter-temporal equation reads (see the longer paper)

$$[1.12] \quad u_t(s) \equiv \begin{cases} p_t(s) - \frac{1}{(N-1) \cdot h_p(p_t(s)) + N \cdot g_p^c(p_t(s))} \cdot h_t(s) & h_t(s) \neq h^{\max} \\ r \cdot u_{t+1}(s + i_t - h_t(s)) & h_t(s) = h^{\max} \end{cases}$$

$$g^c(p) \stackrel{\text{def}}{=} f^c(p) - d^c(p)$$

Equations [1.10] and [1.12] are used to determine the time development of the system

The procedure used to solve the inter-temporal equation is the Bellman method³. One starts far in the future at $t=T$ and assumes, for instance, that the store is closed so that the marginal store value $u_{T+1}=0$. Then one determines the optimal bid function for the auction in period T for all s . This will give the clearing price $p_T(s)$ and the clearing hydro production $h_T(s)$ in time period T . Put $p_T(s)$ and $h_T(s)$ into equation [1.12] to get the marginal store value $u_T(s)$ at the start of period T , determine the bid functions for $t=T-1$ and the clearing price $p_{T-1}(s)$ and $h_{T-1}(s)$. This is repeated until one comes to the present time. Then one is ready to construct the optimal hydro bid function for the coming auction, time $t=0$.

Solving equations [1.10] and [1.12] gives the optimal hydro and thermal bid functions, future store levels s_t , future hydro production h_t and future prices p_t through the equations.

$$[1.13] \quad \begin{aligned} s_{t+1} &\equiv s_t + i_t - h_t(s_t) \\ h_t &\equiv h_t(s_t) \\ p_t &= p^{cF}(d_t - h_t) \end{aligned}$$

In the last line it is for simplicity assumed that demand is inelastic. In the longer paper stochastic water inflow is included in the model.

2.4 Pure thermal producers

In the case with few identical pure thermal producers with hydro producers and consumers bidding their marginal cost equation [1.2] can be used directly

³ The true merit of the Bellman method is concerned with stochastic variations, in the present case of stochastic water inflow and consumption.

to determine the optimal bid function $p^F(f)$. For hydro power producers the bid function is the marginal cost and the inter-temporal equation is given by [1.11].

2.5 Integrated thermal and hydro producers

In the case of one group of N identical producers with integrated thermal and hydro production equation [1.2] takes the form

$$[1.14] \quad p(x) - p^c(x) - \frac{x}{\frac{N-1}{p_x(x)} - N \cdot d_p^c(p(x))} \equiv 0$$

Here x is the bidden quantity, $p^c(x)$ the marginal cost for a mixed thermal-hydro producer and $d^c(p)$ is the marginal aggregate consumption function. The equation is formally the same as for the pure thermal system equation [1.3]. With inelastic consumption, $d_p(p)=0$, the bid function equation has the same integral solution as [1.5]

$$[1.15] \quad p(x) \equiv (N-1) \cdot x^{N-1} \cdot \int_x^a dz \cdot z^{-N} \cdot p^c(z)$$

where a is the integration constant.

Since each producer internally will try to portion out production so that the marginal cost of thermal and hydro power plants are equal the optimal hydro power production equation is

$$[1.16] \quad \begin{aligned} r \cdot u_{t+1}(s + i_t - h) &\equiv p^c(x - h) \\ 0 \leq h &\leq h^{high} \end{aligned}$$

The inter-temporal equation is the same as [1.11]

$$[1.17] \quad u_t(s) \equiv \begin{cases} p_t(s) & h_t(s) \neq h^{\max} \\ r \cdot u_{t+1}(s + i_t - h_t(s)) & h_t(s) = h^{\max} \end{cases}$$

Equations [1.14], [1.16] and [1.17] are used to determine future prices and hydro production.

3 Illustrations

The results in the previous section are illustrated with an example of a simple physical system. The marginal cost function is quadratic and constant over time. Consumption and water inflow is periodic with one year period, shown in the following diagram.

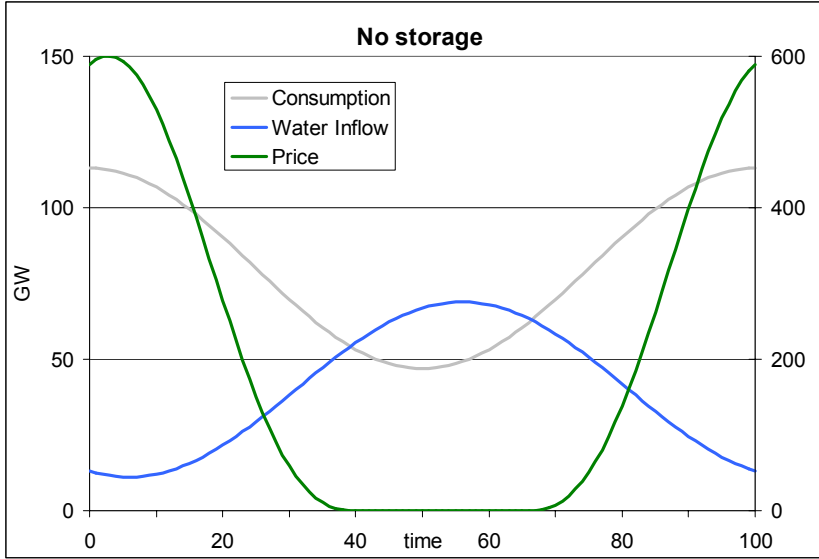


Figure 3: Reference diagram, with no storage and quadratic marginal cost. The average price is 200.

The year is divided in 100 periods. Consumption is inelastic, $d_p^c(p)=0$. The technical properties of the power system are the same in all illustrations.

The physical system is constructed so that there is an equal amount of thermal power and hydro power produced during one year in order to simplify the comparison between the market power of thermal and hydro producers. The physical properties are purposely chosen so that the effect of hydro power generating capacity (h^{max}) and limited storage capacity (s^{max}) affect the result.

Before showing the result of the numerical calculations it must be mentioned that the solution of the equations are sometimes very sensitive to the precision of the calculation. This is mainly due to the difficult nature of the bid function equation. The numerical methods used in this paper are not sophisticated. Small changes in parameters sometimes result in large changes in the result. In interpreting the illustration this should be born in mind.

3.1 Minimum cost

The first illustration is the minimum cost case where the system is optimized with the goal of minimizing future cost of thermal production. This is

equivalent to having an auction market with an infinite number of both thermal and hydro producers. Hydro power producers will try to use so much h of their water reservoir that the marginal store value equals the price. The price is given by the marginal cost of thermal production $p^{cF}(d-h)$, where d is consumption. The optimal hydro power equation is then

$$[1.18] \quad \begin{aligned} r \cdot u_{t+1}(s+i-h) &\equiv p^{cF}(d-h) \\ 0 \leq h &\leq h^{high} \end{aligned}$$

This gives the function $h_t(s)$, the optimal hydro production as function of the store quantity s , at time t . The inter-temporal equation is

$$[1.19] \quad u_t(s) \equiv \begin{cases} p_t(s) & h_t(s) \neq h^{\max} \\ r \cdot u_{t+1}(s+i-h_t(s)) & h_t(s) = h^{\max} \end{cases}$$

The temporal solution is shown in the following diagram for the given physical system.

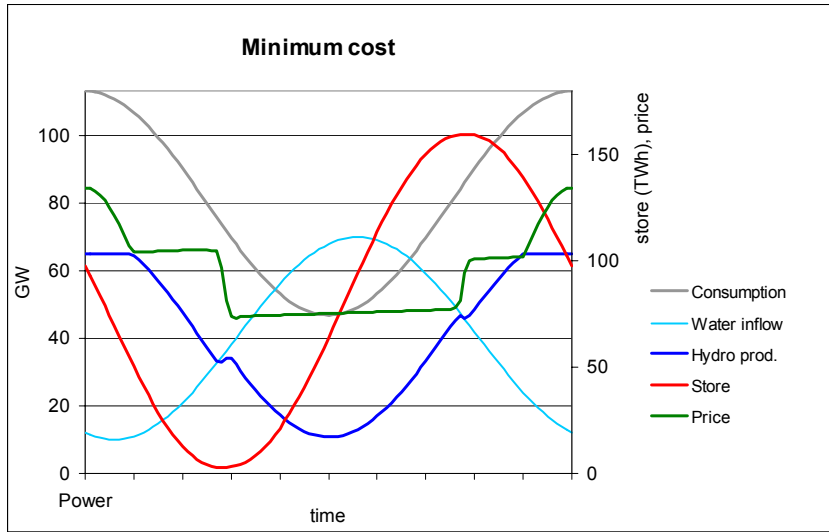


Figure 4: The diagram shows the main temporal characteristics of a thermal-hydro power system that is cost minimized. The price jumps down when the store bottoms and jumps up when the store has become full. The price has a bump when the hydro generating capacity (65 GW) is fully used (the flat part of the blue curve). The top of the store curve reaches the storage capacity, 160 TWh.

3.2 Few hydro producers

In this example there are N^H identical hydro producers and an infinite number of thermal producers. Thermal producers bid their marginal cost. For hydro

power the optimal bid function equation [1.10] with inelastic consumption takes the form

$$p^H(h) - p^{cH}(h) - \frac{h}{(N^H - 1) \cdot \frac{1}{p_h^H(h)} + N^H \cdot f_p^c(p^H(h))} \equiv 0$$

Here $p^H(h)$ is the aggregate hydro bid function, $p^{cH}(h)$ the hydro marginal cost function and $f^c(p)$ the aggregate thermal supply function. It can be noted that the equation takes a simpler form in the case with just one hydro producer, $N^H=1$, since the equation then becomes an ordinary equation (not a differential equation).

The hydro marginal cost function $p^{cH}(h)$ corresponds to the marginal store value function $u_t(s)$ and is given by equation [1.8].

The inter-temporal equation linking u_t to u_{t+1} is given by [1.12] (modified for inelastic consumption)

$$u_t(s) \equiv \begin{cases} p_t(s) - \frac{1}{(N^H - 1) \cdot h_p(p_t(s)) + N^H \cdot f_p^c(p_t(s))} \cdot h_t(s) & h_t(s) \neq h^{\max} \\ r \cdot u_{t+1}(s + i_t - h_t(s)) & h_t(s) = h^{\max} \end{cases}$$

The first diagram shows typical bid functions.

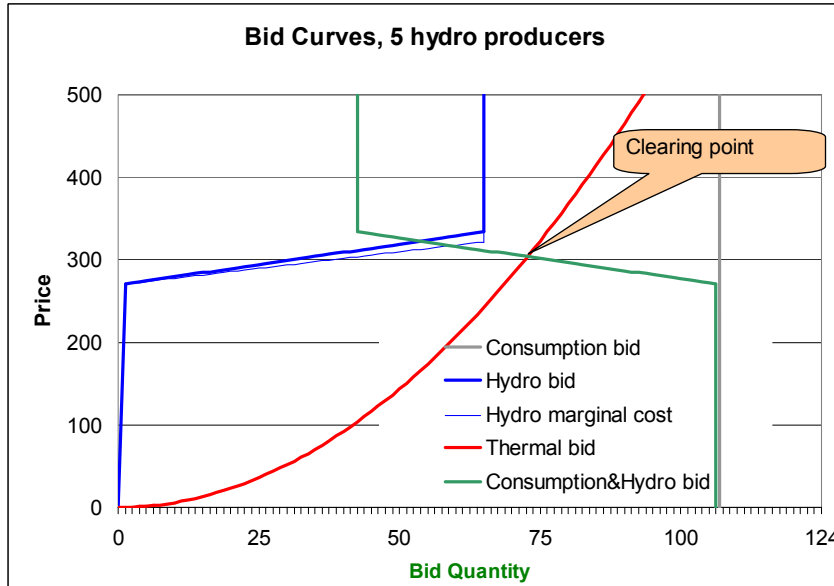


Figure 5: Hydro bid function with 5 hydro producers and an infinite number of thermal producers bidding their marginal cost. The thin blue line represents the hydro marginal cost, i.e. the marginal store value. It is seen that the bid curve is just noticeably above

the hydro marginal cost. The green line is the combined hydro and consumption bid function.

The effect of market power on the hydro bid function is to make it steeper than the hydro marginal cost function. Since in reality only a small part of the available store can be used because of generating capacity restrictions the hydro marginal cost function will often be a rather straight line. See Figure 2.

Using the inter-temporal equation the time-development can be determined. The next diagram shows the inter-temporal solution with 7 hydro producers

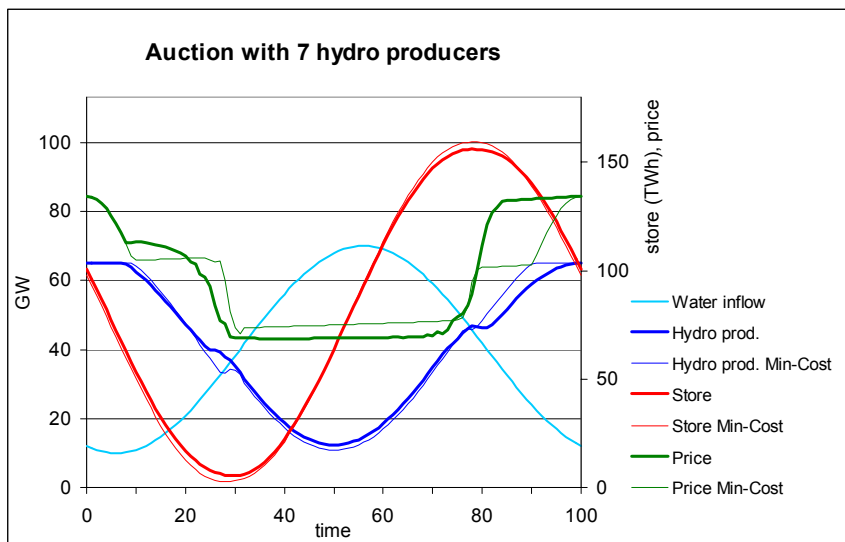


Figure 6: Temporal solution with 7 identical hydro producers and an infinite number of thermal producers. Thin lines show the minimum-cost case. The hydro power generating capacity is 65 GW as is seen in the flat part of the blue hydro power generating curve.

The price is higher in the winter and lower in summer, but on average roughly the same as in the minimum-cost case. Going from five to three hydro producers does not change the curves much. A general feature of hydro power market power is also that the store capacity is not fully utilised (in the diagram the red store curve does not reach the top of the minimum-cost store curve).

Although the average price is rather unaffected by market power the distribution of income is changed, since consumption is higher in winter than in summer. The effect of market power in hydro production is thus to shift the price time profile so that hydro producers get a higher price when their sales are high at the expense of losing somewhat in the low season.

3.3 Few thermal producers

In the next example there are few thermal producers and an infinite number of hydro producers bidding according to their marginal store value.

3.3.1 Identical producers

In the case with N^F identical thermal producers we can use [1.2] directly setting $d_p^c = 0$.

$$[1.20] \quad p^F(f) - p^{cF}(f) - \frac{f}{(N^F - 1) \cdot \frac{1}{p_f^F(f)} + N^F \cdot h_p^c(p^F(f))} \equiv 0$$

The equation for optimal hydro power production is the same as in the minimum-cost case and also the inter-temporal equation for the marginal store value $u_i(s)$.

The solution of [1.20] is illustrated in the following diagram.

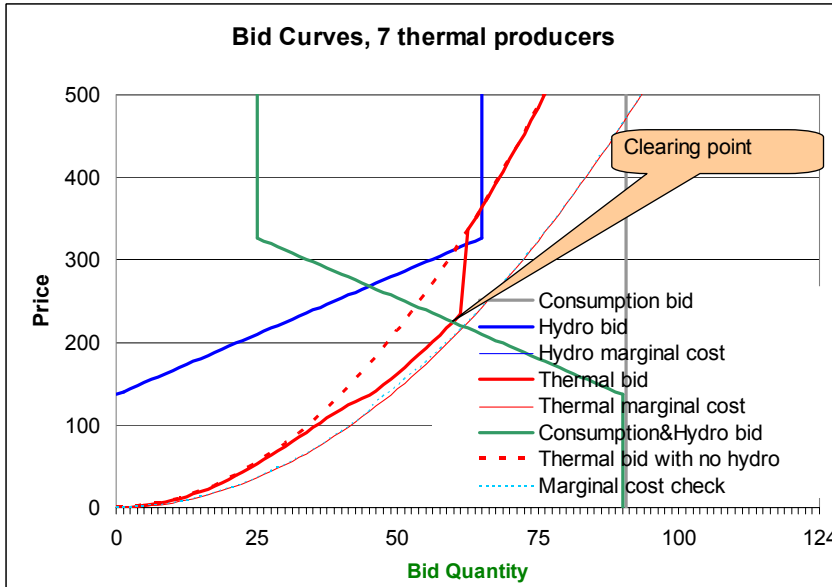


Figure 7: Thermal bid function, marginal thermal production cost and hydro bid curve. The dotted red line is the optimal producer bid curve if there were no hydro bids.⁴

The dotted red line shows the optimal producer bid curve if there are no hydro bids. With competition from hydro power the producer bid curve (thick solid red line) shifts from the dotted red line closer to the marginal cost curve (thin red line) in the price range where hydro bids are effective ($140 < p < 320$) in the diagram. There is a discontinuity of the thermal bid curve when it jumps from near the marginal cost curve to the dotted curve (in this example around 1.5 times the marginal cost curve). This discontinuity is caused by the discontinuity of the hydro bid curve in combination with inelastic demand.

⁴ The thin dotted black line ("Marginal cost check") is the marginal cost function reconstructed from the producer bid function. This is inserted to show the precision of the solution for the producer bid function. If the solution is exact the dotted black line should be equal to the marginal cost line (thin red line). In this case the solution is very accurate. This means that, if the thermal marginal cost were equal to the dotted curve, the solution for the optimal bid curve (thick red line) would be exact.

3.3.2 One big producer

Another example of market concentration in thermal power is the case with one big thermal producer with many small thermal producers and many hydro producers. In this case equation [1.1] with inelastic consumption ($d_p^c=0$) takes the form

$$p^B(f) - p^{cB}(f) - \frac{f}{f_p^{cS}(p^B(f)) + h_p^c(p^B(f))} \equiv 0$$

Here $p^B(f)$ is the big producer's bid function, $p^{cB}(f)$ its marginal cost function, $f^{cS}(p)$ the small thermal producers' aggregate inverse bid function and $h^c(p)$ the hydro producers' aggregate inverse bid function. The two last groups bid their marginal cost. Since with $N^B=1$ the equation is not a differential equation it is easy to solve numerically.

As an illustration take the example with the one big and the small thermal producers sharing the same aggregate marginal cost function $p^c(f)$ so that

$$f^{cB}(p) \equiv a \cdot f^c(p) \Rightarrow p^{cB}(f) \equiv p^c\left(\frac{f}{a}\right)$$

$$f^{cS}(p) \equiv (1-a) \cdot f^c(p) \Rightarrow p^{cS}(f) \equiv p^c\left(\frac{f}{1-a}\right)$$

Here a is the big producer's share of the aggregate thermal supply function.⁵ The bid function equation then takes the form

$$[1.21] \quad \boxed{p^B(f) - p^c\left(\frac{f}{a}\right) - \frac{f}{g_p^c(p^B(f))} \equiv 0}$$

$$g^c(p) \stackrel{def}{=} (1-a) \cdot f^c(p) + h^c(p)$$

Here $g^c(p)$ is the (exogenous at time t) combined hydro and small thermal producer supply function with inverse $p^G(g)$. At clearing we have $g+f=d$ and $p^B(d-g)=p^G(g)$. Putting this into [1.21] gives the following equation for the clearing quantity g .

$$p^G(g) - p^c\left(\frac{d-g}{a}\right) - (d-g) \cdot p_g^G(g) \equiv 0$$

The solution for g determines the clearing price, $p^{clear} = p^G(g)$, the big thermal producer's sale, $d-g$, the small thermal producers' aggregate sales, $f^{cS}(p^{clear})$,

⁵ Note that the actual market share of the bid thermal producer will be less than a since the producer will withhold supply.

and the aggregate hydro sales $h^c(p^{clear})$. Using the inter-temporal equation future clearing prices can be determined.

The following diagram shows the temporal solution for $a=0.38$, which gives the same Herfindahl index as for the other illustration.

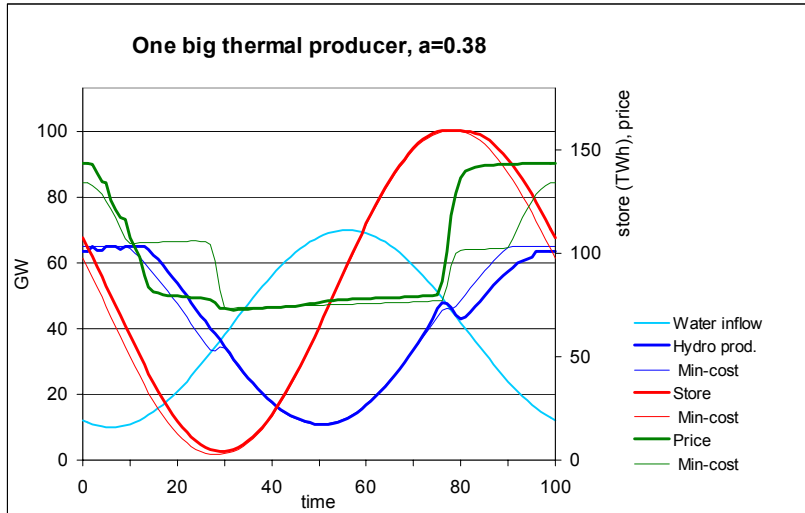


Figure 8: Time development of price and water reservoir with one big thermal producer with 38 percent of thermal production (19 percent of total production).

The price swing is bigger than in the minimum-cost case and also the average price, but less pronounced.

The degree of market power for pure thermal producers depends crucially on the store capacity and the hydro power generating capacity. The more ample these capacities are the less market power thermal producers have. This is illustrated in the following figure where the hydro power generating capacity is 80 GW instead of 65 GW used in Figure 8

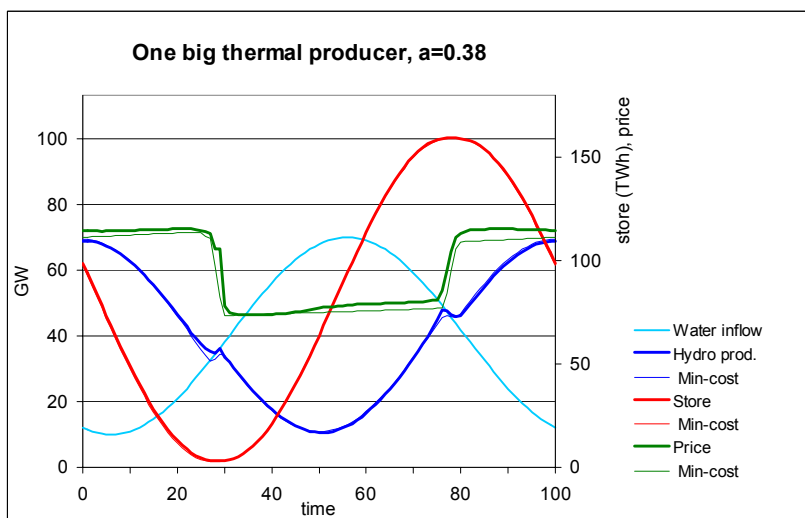


Figure 9: Time development with higher hydro (80 GW) power generating capacity.

In this case the price curve is almost identical to the minimum-cost case with 80 GW.

3.4 Combined hydro and thermal producers

In this example there are N identical producers having both hydro and thermal power. The bid function equation is given by [1.14] which with inelastic consumption has the integral solution [1.15]

$$p(x) \equiv (N-1) \cdot x^{N-1} \cdot \int_x^a dz \cdot z^{-N} \cdot p^c(z)$$

Here $p^c(x)$ is the aggregate marginal cost of the combined thermal and hydro production. It is defined by its inverse $x^c(p)$ through

$$x^c(p) \equiv f^c(p) + h^c(p)$$

where $f^c(p)$ and $h^c(p)$ are the producers' thermal supply and hydro supply functions, respectively.

The optimal hydro power equation and the inter-temporal equation are the same as [1.18] and [1.19], respectively, in the minimum-cost case.

The first diagram shows a typical bid curve.

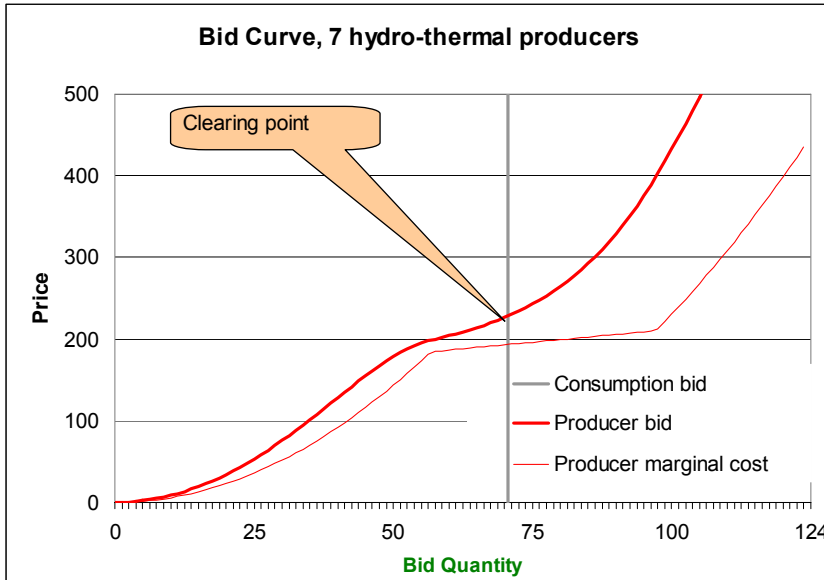


Figure 10: Bid curve for integrated hydro and thermal producers. Thin line shows the marginal production cost curve.

The thin line, representing the marginal cost function, is the sum of the (quadratic) thermal power supply function and the nearly linear hydro supply function. In this figure the marginal store value at the present store level s is around 200. The effect of having few producers is that the bid function shifts

to the left and upward and smoothes compared with the marginal cost function.

The following diagram shows the time development in the case of 28 identical integrated producers. This number is chosen because it has the same Herfindahl index as the previous cases with 7 pure hydro and 7 pure thermal producers respectively.

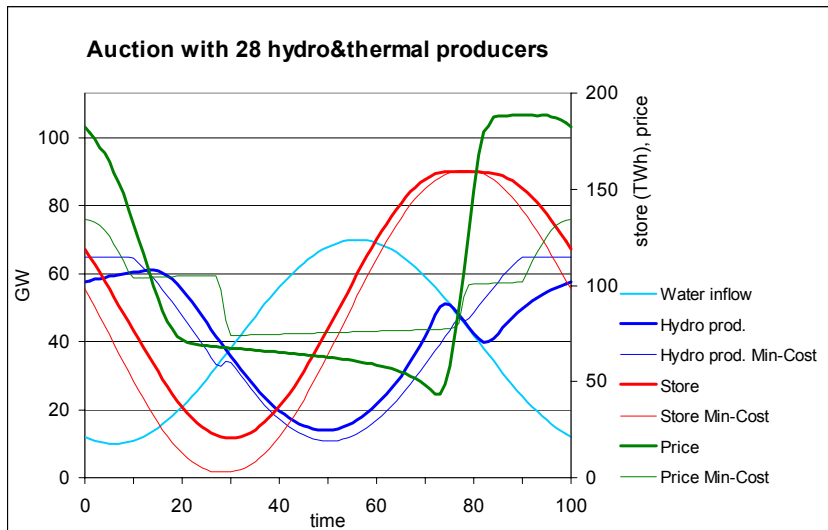


Figure 11: Thick green line shows the price development. It is quite different from the minimum cost case (thin green line)

The time average price is not much higher than in the minimum-cost case shown in thin lines but has a much different time profile. Considering that the number of producers is nearly 30 the effect of market power in the model is considerable, although it does not manifest itself so much in the average price. It is also to be noted that the store curve is compressed upwards so that the store is kept full for a longer period.

The solutions in this case with integrated thermal and hydro production are rather stable. The reason is that the marginal cost function is rather smooth.

3.5 Summary of illustrations

The following table shows time averages for the illustrations.

	Market organisation			
	Cost minimum	Auction		
		Hydro	Thermal	Thermal-hydro
No. of producers→		7	7	28
Price	94	94	98	102
Income Thermal	41	42	44	50
Income Hydro	44	45	46	49
Cost Consumer	85	88	90	99

Table 1: Summary of time averages for the illustrations. The “Thermal” column shows the result for one big thermal producer with a 38% share of the aggregate thermal supply function. It represents roughly the same Herfindahl index as the other columns.

We see that for both the pure-thermal and thermal-hydro cases the average price is higher than in the pure-hydro case. In the pure-hydro case the average price is almost the same as in the minimum-cost case. But because of the change in the time profile there is a loss for the consumers. In the thermal-hydro cases the time profile is very different from the minimum cost case.

One conclusion is that with pure hydro producers the organisation of hydro power has little effect on the result for the average price.

The numerical values and shape of the curves should be considered as rough estimates. However, numerous checks indicate that the qualitative differences between the illustrated cases are correct.

4 Test on real markets

The key question in this section is how the models discussed in this paper agree with reality. The test is done on the Nordic power market, with a large share of hydro power. Here there are other observables beside price that can be tested against models, for instance the development of the water reservoir over the year.

4.1 The Nordic power supply

The main characteristics of the Nordic market is total production of around 400 TWh per year of which around half is hydro power, hydro power generating capacity of around 30 GW and water reservoir capacity of around 110 TWh. Demand (Norway+Sweden) and water inflow is shown in the following diagram

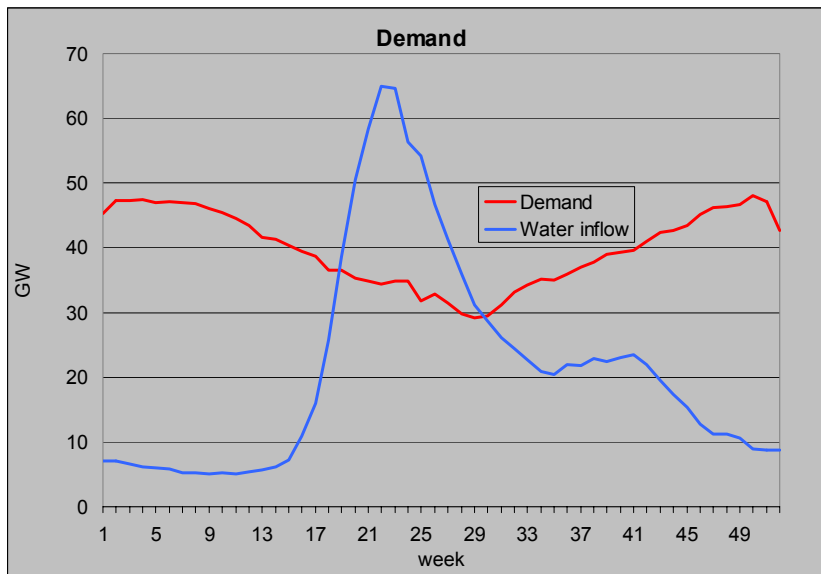


Figure 12: Consumption of electricity and water inflow to the reservoirs.

A typical marginal aggregate thermal production cost function $p^c(f)$ for a certain week (34) is shown in the following diagram

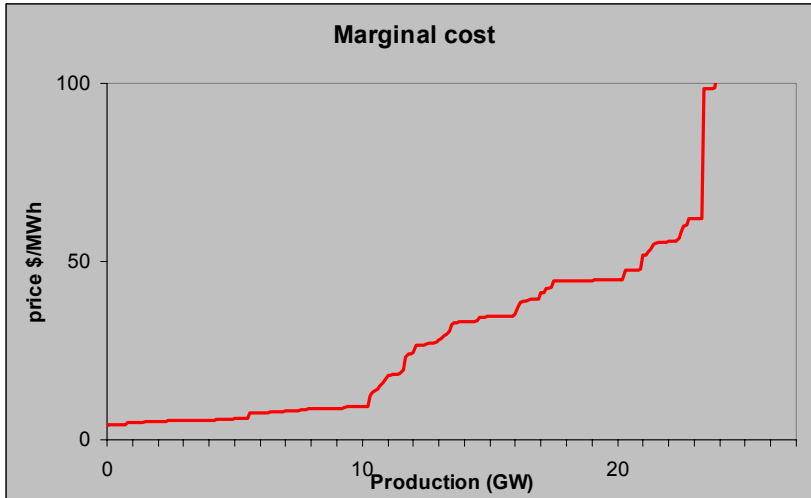


Figure 13: Typical shape of the marginal thermal production cost in the Nordic power supply.

The market is organized as an auction market, with daily auctions for each hour the following day.

4.2 Pure thermal system

Before testing on the Nordic market let us look at pure thermal power system (non-temporal) with inelastic consumption. In a power system without storage electricity must be produced each moment to exactly match consumption.

Identical producers

Assuming there are N identical producers sharing the above marginal cost curve $p^c(f)$ and that consumption is inelastic the optimal bid curve $p(f)$ equation is given by [1.5]

$$p(f) \equiv (N-1) \cdot f^{N-1} \cdot \int_f^{\infty} dg \cdot g^{-N} \cdot p^c(g)$$

Since we have assumed inelastic demand we can set $f=d$ in this equation to get the clearing price.

With the above marginal cost function we get the effective market price as function of consumption shown in the following diagram

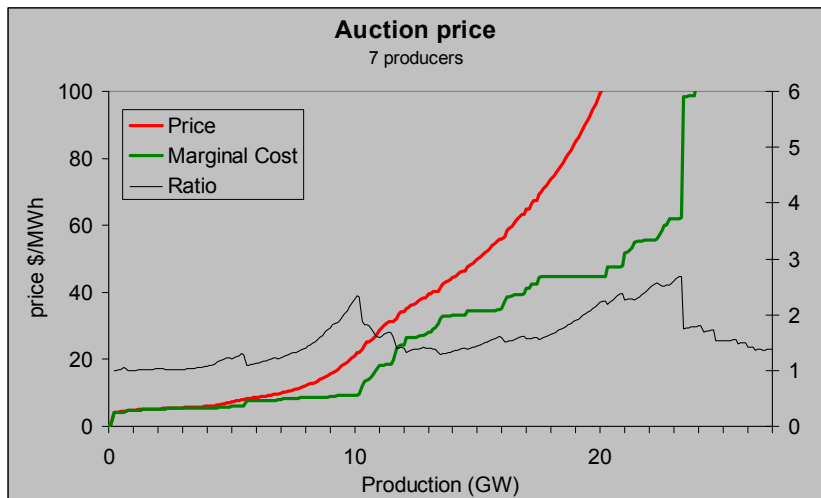


Figure 14: The effective clearing price as function of consumption (red curve) and the marginal cost function. The thin line

The price is much higher than marginal cost. Consumption would fall in the range 10-20 GW in a pure thermal production system like this giving a price-to-marginal-cost ratio nearly 2.

4.2.1 One big producer

This case is dealt with in section 2.2.2. Applied to the marginal cost function in Figure 13 with a 1/7 market share for the big producer gives the result in the following diagram.

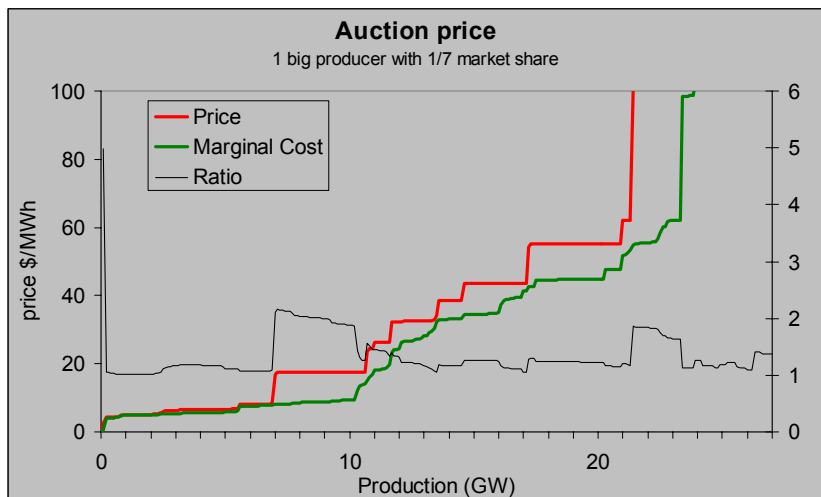


Figure 15: Clearing prices in a pure thermal system with one big producer with 1/7 market share.

The price to marginal cost ratio is less than in the previous illustrations, around 1.5. With 3 producers (33% market share each) the price to marginal cost ratio is around 2.

Given general knowledge of prices in some pure thermal markets and the fact that producer concentration is rather high, it is unlikely that one would find much explanatory power in the in the pure thermal models discussed here.

4.3 The Nordic market

The model used to test the Nordic market is based on a market model, PoMo, developed for commercial use. This model has cost minimisation as the goal function and finds the optimal use of hydro power and thermal power for each week 5 years ahead. It is a stochastic model with water inflow to the water reservoirs as the main stochastic variable and uses the Bellman inter-temporal solution method.

The following diagram shows the time profile of the price (green thick line) of the original cost minimizing PoMo model. This is taken from a simulation for the year 2005 with the actual fuel prices and normal reservoir levels at the start of the year. This price profile is compared with the average historical time profile during the last 4 years. The historic price level for each year is scaled so that the average price each year equals the average price of the model.

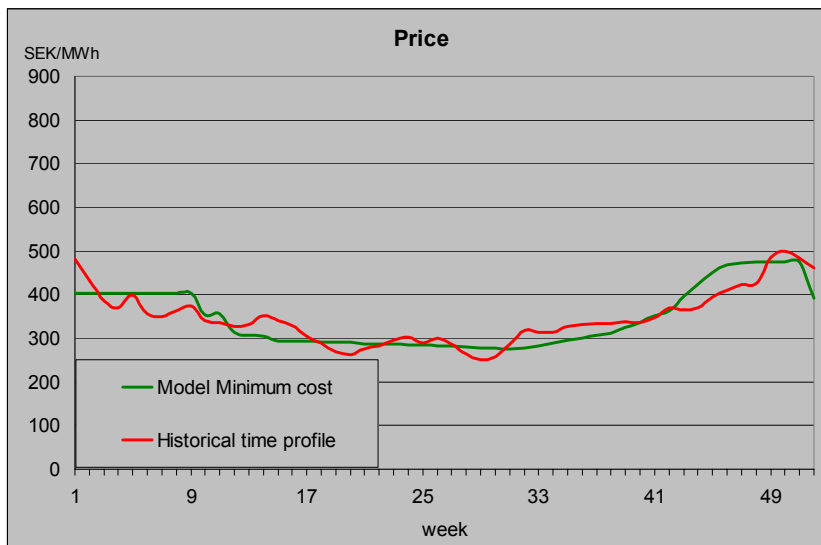


Figure 16: Simulation of a model where price equals marginal cost (minimum-cost). The green line represents the historical price profile in the last 4 years.

There is good overall agreement between model and reality in the time profile. Other cost-minimizing models of the Nordic market give the same overall result.

The model has been modified from a cost minimizing model to an auction market model where producers try to maximize the discounted value of their future net income as discussed before without regard to the possibility of entry of contestant producers.

For computational reasons the producer organisation chosen is where all producers have both hydro and thermal power. It is assumed that there are N identical producers sharing the real weekly marginal thermal cost functions

and the hydro power resources in the Nordic supply system. The following diagram shows the result for 20 power producers ([blue thick line](#)).

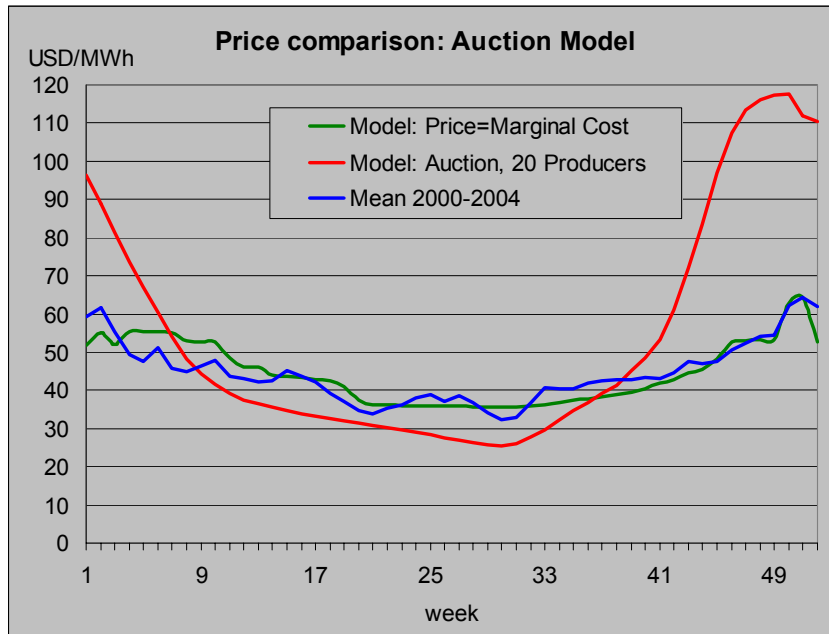


Figure 17: Auction model results for the Nordic market (blue line).

There is a significant difference in the time profile between the cost minimizing case (green) and the profit maximisation case (blue). The time average price is also different but not as pronounced. The intuitive explanation is that producers will try raising prices in the high season in order to increase revenue at the expense of somewhat lower revenues in the low season.

It should be observed that a producer structure with identical production resources gives each producer the possibility to affect all parts of the bid curve and, therefore, represents an organisation particularly prone to market power.

The time development of the water reservoirs is also an observable that can be compared with the model. As we have seen previously one effect of having few producers in a thermal-hydro system is that the reservoir is full over a longer period the fewer producers there are. This is confirmed in the following diagram showing a comparison between the cost minimizing PoMo and the auction model with 10 producers.

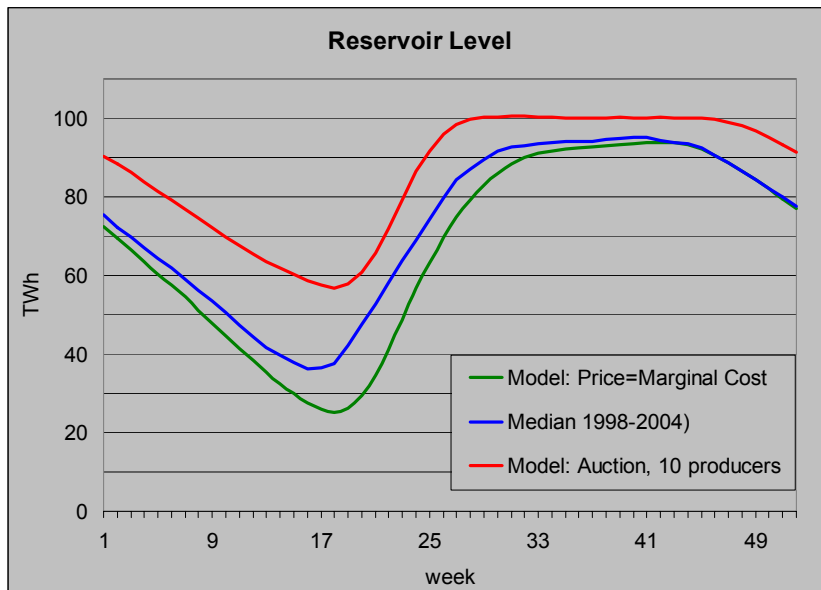


Figure 18

In the auction model producers would keep their reservoirs full for almost 5 months in the autumn. The time profile and the level are very different from the cost minimizing case. The profile is also very different from the historical reservoir filling shown in the thin black line in the diagram.

This producer structure chosen is rather unlike the situation for the Nordic market. So it is difficult to see how the basic model agrees with to reality. Of the producer organisations illustrated earlier it may be that the structure with few pure thermal producers and many pure hydro power producers represents the Nordic market better. This structure is manageable within the general model described but has not yet been analysed. It is on the agenda for further work. For a certain Herfindahl index market power is expected to be less for this producer structure. See Figure 1.

However, given the result for the stylized markets in section 3 it seems that also a more realistic assumption for the Nordic producer structure will produce results rather different from reality. In particular the price swing is expected to be much greater.

4.4 Conclusion

The general conclusion from the application of the model on a pure thermal supply and on the Nordic market seems to be that the model does not agree well with reality, especially for pure thermal market.

There can be several explanations for this even within the assumption that producers try to maximize their profits.

- The power market is contestable. Producers can be restrained by the threat that new producers will enter the market in order to share excess profits of incumbent producers. It is only if the price is just above the

marginal cost of production that there will be equilibrium.⁶ The result of the market tests does not contradict the theory that the power market is highly contestable. If this is true power producers have little market power, even if they are few. In this case it is not the number of producers but the fundamental character of the power market that limits producers' market power.

- Old conventions. The producers in the Nordic power market have traditionally used models based on cost minimization for hydro production planning. This has continued after the deregulation. It is possible that this is a self-fulfilling prophesy in which case the price would be close to the marginal cost.
- Transition from overcapacity. The deregulation of the Nordic market took place in a situation of power production overcapacity. It is possible that remaining overcapacity prevents producers to fully take advantage of their market power. The counter argument is that the marginal cost used in the model simulations are based on the existing capacity representing potential overcapacity.
- The threat of government. The power market is under close scrutiny by the media and by the government. Excess profits could, therefore, trigger political counter actions. This could constrain producers in taking advantage of their market power.
- One potential explanation is that in reality demand elasticity is high. This could be tested by using the equations with elastic consumption. Since the supply elasticity is rather high in the relevant interval (mostly >1), the demand elasticity must be very large if the models are to come close to reality. In the auction model the difference in time profile between the model and reality is also difficult to correct by assuming higher demand elasticity.
- Industrial concern. A special form of demand elasticity concerns industry. In the Nordic market electricity intensive industry accounts for a large share of consumption. The long term elasticity is rather high. So it could be in the interest of power producers to keep prices low in order to secure future sales volumes. One difficulty with this explanation with regard optimal behaviour in the auction system is that the main effect of profit maximisation is a change of the time profile of the price with higher winter and lower summer prices. The gain in net revenues comes mostly because consumption is higher in winter. But this is not the case for electricity intensive industry, with very flat consumption profile. So a change of the time profile of the price would not hurt industry much.
- Ignorance. The last explanation offered is the simplest, namely that power producers don't know how to optimize the behaviour on an auction market. After all the electricity auction market in the Nordic area is quite new and the mathematical theory and computational methods for determining optimal auction bids in a market with storage are rather complicated.

⁶ Except for peak power plants

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