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Energiforsk seminar:
Vibrations in nuclear applications 2019

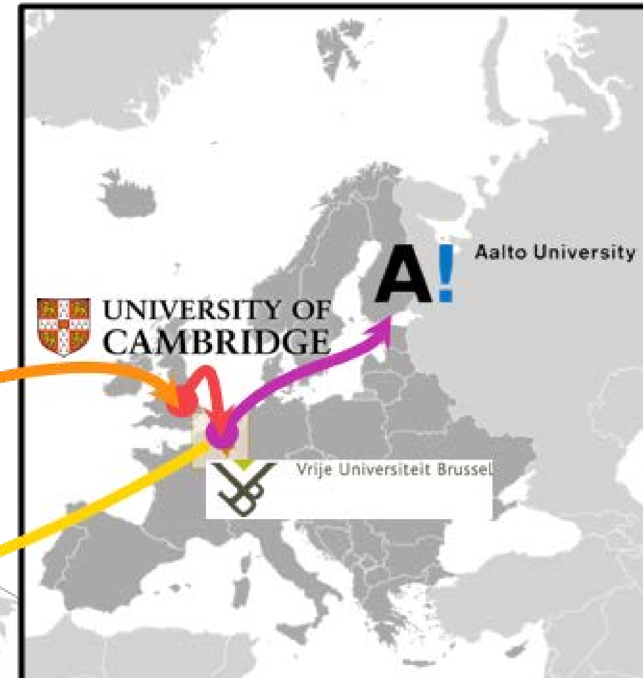
Sensitivity and accuracy of measuring small displacement amplitudes with Digital Image Correlation (DIC)

Espoo, November 14, 2019

Sven Bossuyt, Aalto University

Sven Bossuyt

- from Belgium originally
  - *Engineering degree in Materials Science*
- Ph.D. in Applied Physics from Caltech
  - *“Crystallization behavior of glass-forming alloys”*
- postdoc in Cambridge
  - *electrochemical de-oxygenation*
- return grant to Belgium
  - *mechanics of materials and constructions*
  - *inverse methods visit to TKK institute of mathematics*
- Academy Research Fellow at TKK/Aalto
  - *“Localization Phenomena in Experimental Mechanics Measured using Appropriate Assumptions”*
- Associate Professor at Aalto University
  - *M.Sc. Programme in Mechanical Engineering*
  - *Advanced Materials & Manufacturing Research Group*



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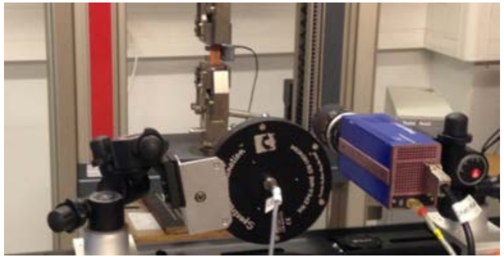
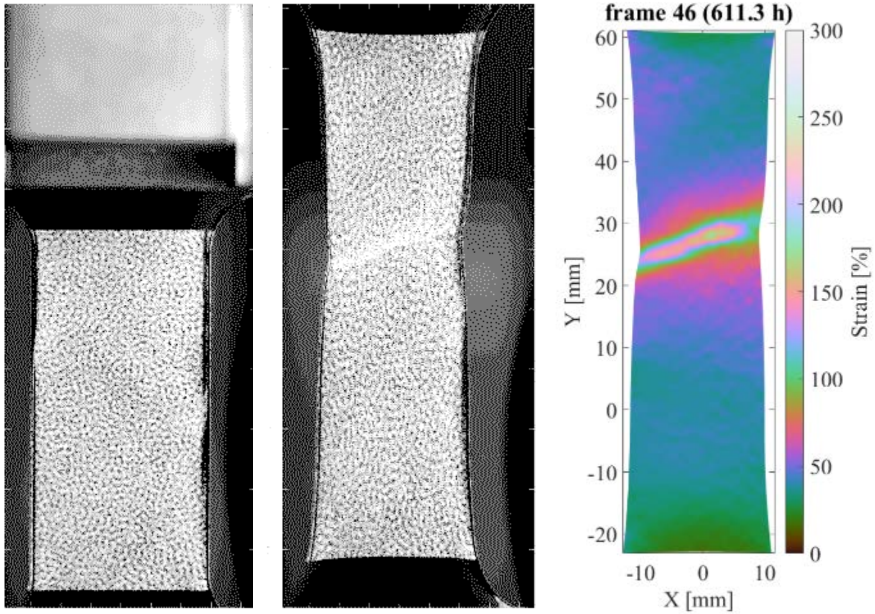
Introduction to DIC

Digital Image Correlation
to visualize and quantify strain localization



Advantages of DIC

- instantaneous non-contact optical full-field measurement
- leverage advances in digital cameras and computers
- sub-pixel resolution (due to peak fitting or interpolation)
- 3D displacements from stereo image pairs



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Antti Forsström, doctoral dissertation https://aaltodoc.aalto.fi/handle/123456789/40370

Digital Image Correlation
principle

Find sequence of displacement fields that map image of un-deformed material onto observed sequence of images of deformed material

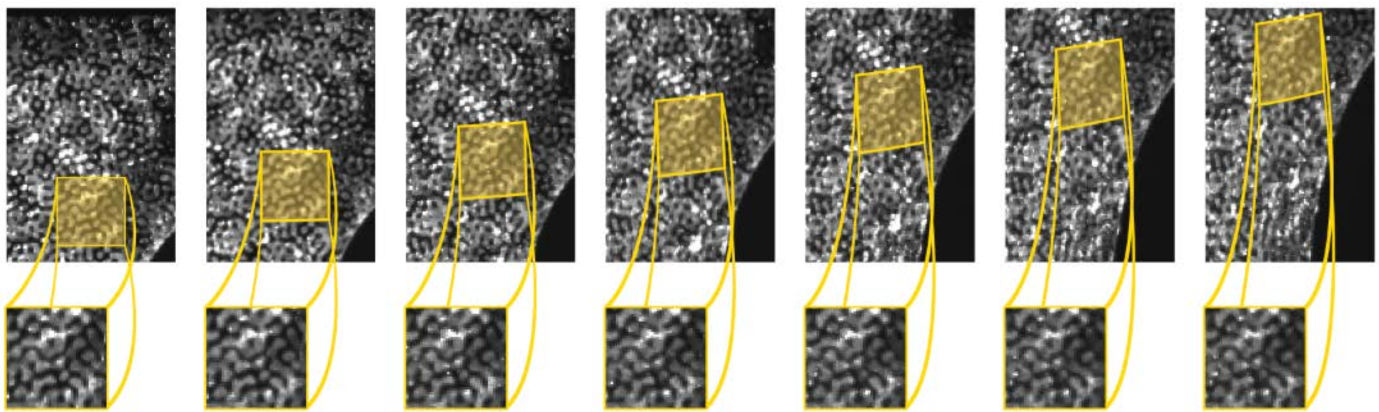


Image Requirements

for feasibility of digital image correlation

any image in digital format

- data must be converted to pixel array
- optical, electron, scanning probe, tomography...

predictable image of deformed object

- distinguishable from original
- uniform (or known) imaging geometry and illumination
- no new or missing features

Image Requirements

for quality of digital image correlation results

large pixel count, low noise

- maximize potential information content

small feature spacing

- allow small correlation window for best spatial resolution

high contrast, irregular, well-resolved features

- give significant changes in correlation for best accuracy and precision

Mathematical formulation

DIC as inverse problem

optical flow constraint

$$\vec{\nabla} I \cdot \frac{\mathrm{d}\vec{x}}{\mathrm{d}t} + \frac{\partial I}{\partial t} = 0$$

- image intensity "moves with object"

Lucas-Kanade

- approximate flow in a neighborhood of each point of interest
- "subset-based"

$$\begin{bmatrix} \Delta_x \\ \Delta_y \end{bmatrix} = \begin{bmatrix} \sum(\partial_x I_0)^2 & \sum \partial_x I_0 \partial_y I_0 \\ \sum \partial_x I_0 \partial_y I_0 & \sum(\partial_y I_0)^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum \partial_x I_0 (I - I_0) \\ \sum \partial_y I_0 (I - I_0) \end{bmatrix}$$

Horn & Schunck

- global minimization

$$\underset{\vec{u}(\vec{x})}{\arg\min} \iint_{\vec{x}} \left(I|_{\vec{x}} - I_0|_{\vec{x}-\vec{u}} \right)^2 + \alpha^2 \left\| \nabla^2 \vec{u} \right\|^2$$

Sub-pixel precision measurement of displacements with DIC

grayscale imaging

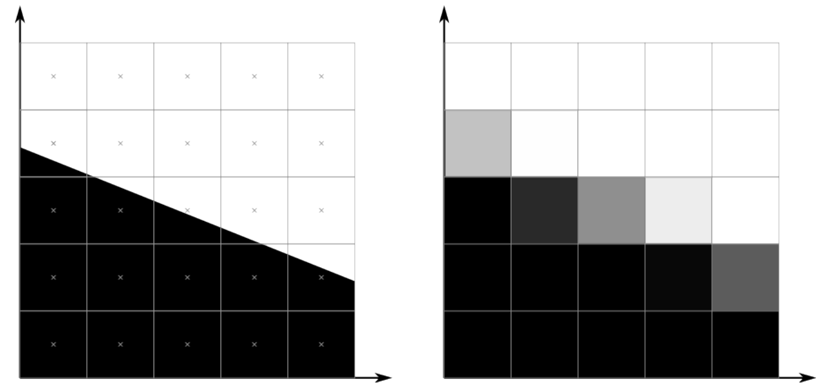
- displacement resolution = grayscale resolution / grayscale gradient

interpolation

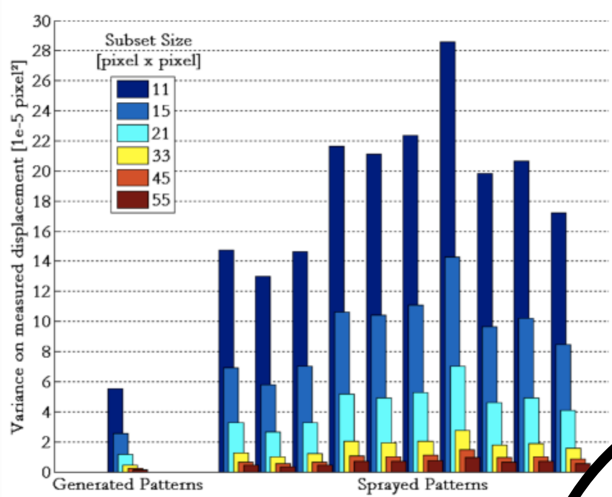
- needed to apply sub-pixel displacements
- average with neighbouring pixels

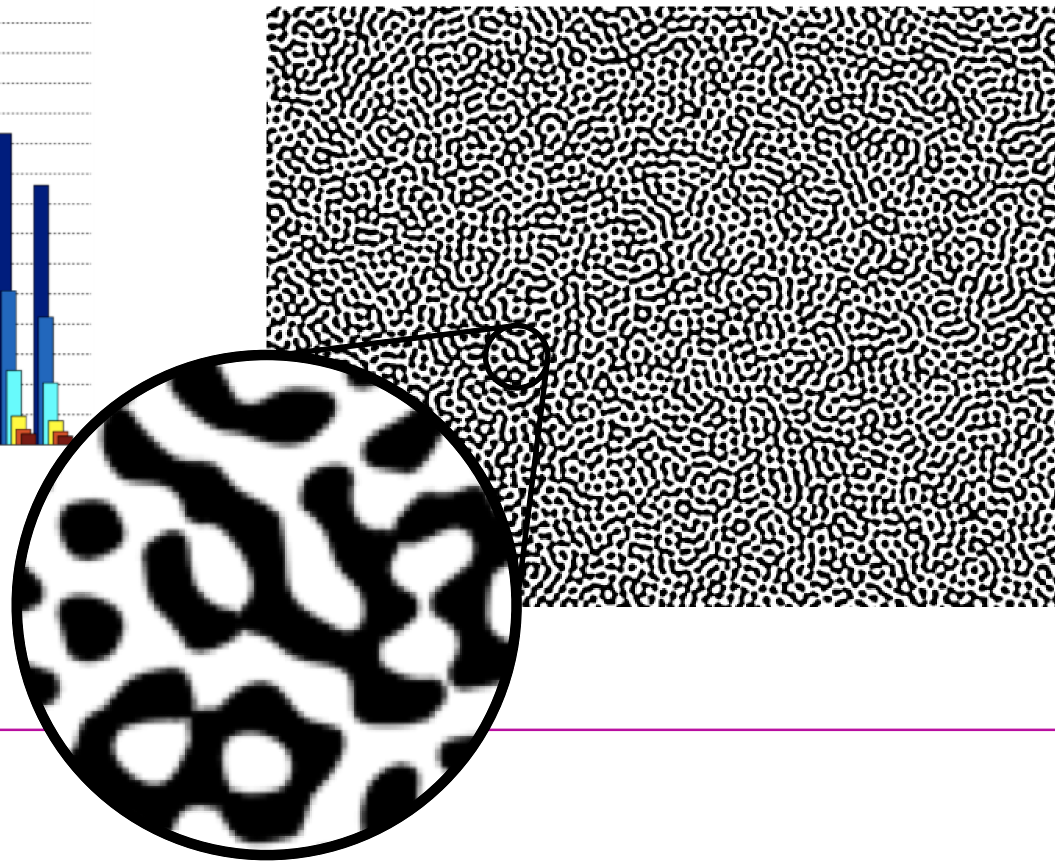
accuracy limited by calibration

- viewing geometry
- lens distortions



High-contrast patterns
optimized for correlation peak that is sharp





Depth from stereo images

Epipolar lines (as in ray-tracing)

= set of points in 3D space that are viewed at the same point in the 2D image

matching points in 2 images must lie on intersection of epipolar lines

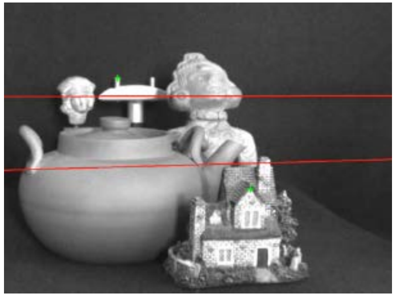
Stereo disparity (as in human vision)

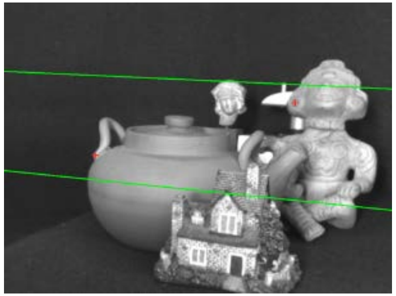
same image matching problem for stereo as for time sequence

Calibration

traditionally done with calibration grid

alternatively minimize strain measured in rigid body translations





Digital Image Correlation
in brief

match images of deformed object to reference image of that object

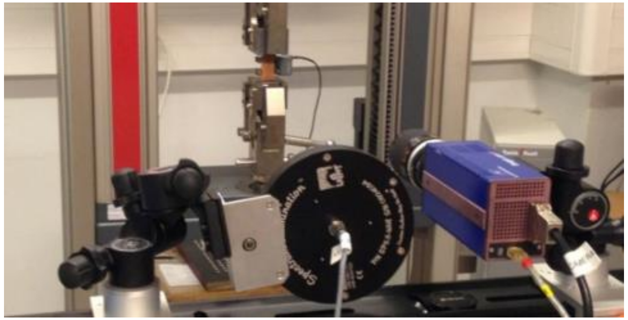
- cross-correlation via FFT
  - *peak amplitude indicates how well it matches*
- Lucas-Kanade
  - *deform reference image with hypothetical displacement fields, then interpolate and calculate sum of squared differences*
- find the displacement field that gives the best match with observed image

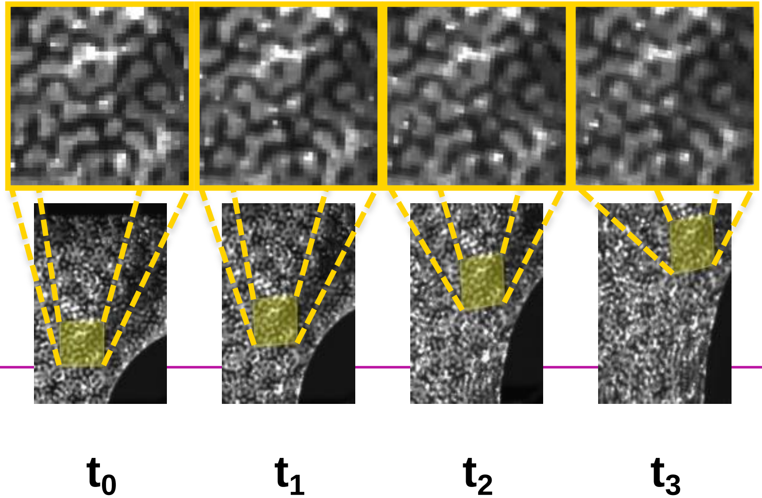
advantages:

- instantaneous non-contact optical full-field measurement
- leverage advances in digital cameras and computers
- sub-pixel resolution (due to peak fitting or interpolation)
- 3D displacements from stereo image pairs

issues:

- calibrating camera geometry and distortions
- contrast and feature spacing in image
- implicit assumptions in algorithm and in discretization method of fields
  - *e.g., cracks and shear bands replaced by unrealistically high but smooth localized strain*





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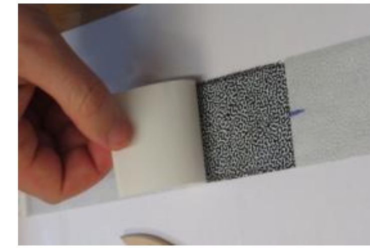
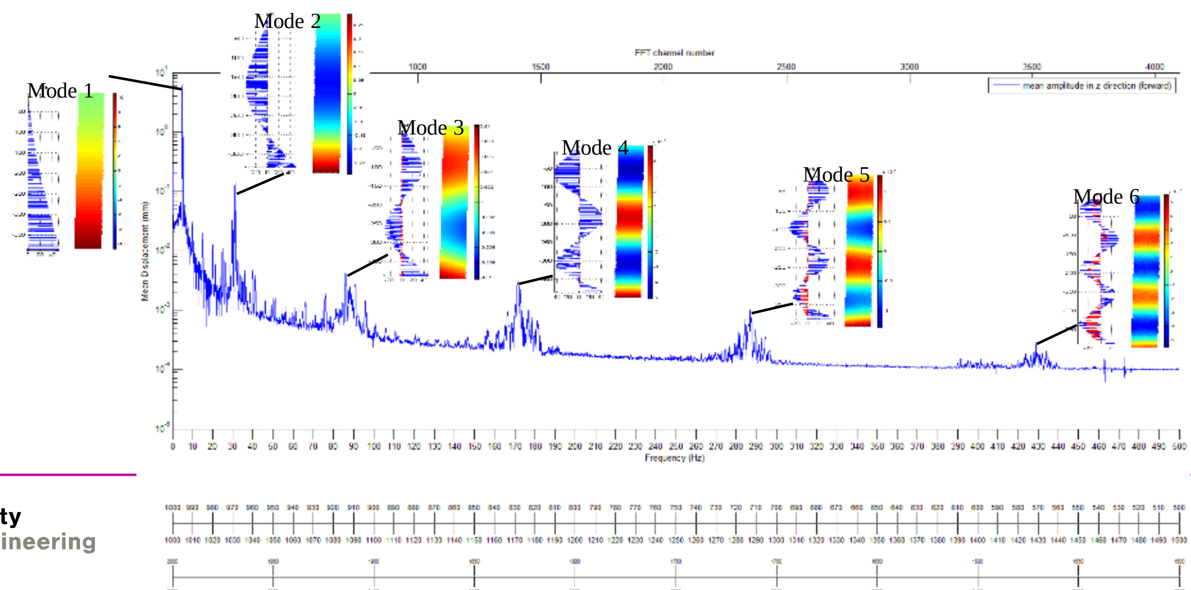
Applications of DIC

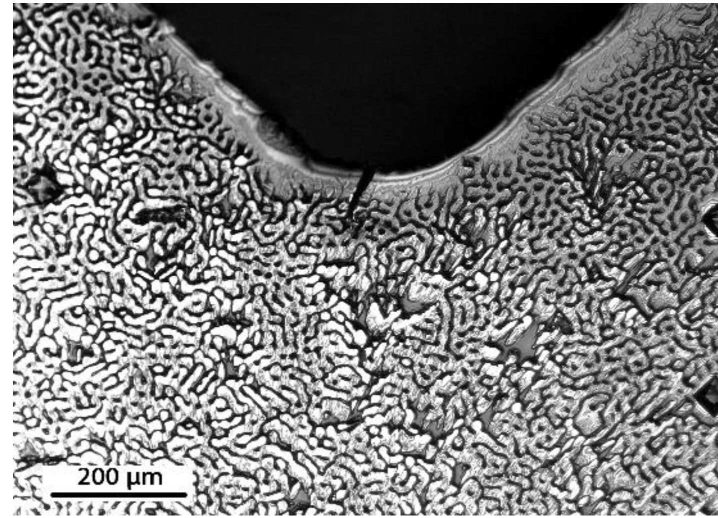
Dynamic Digital Image Correlation



- Synchronized high-speed digital camera pair
  - *Out-of-plane displacement from stereo disparity*
- Operational mode shapes separated by Fourier filtering
  - *Displacement sensitivity < 0.001 pixel*



Micro-mechanics of fatigue in polycrystalline metal

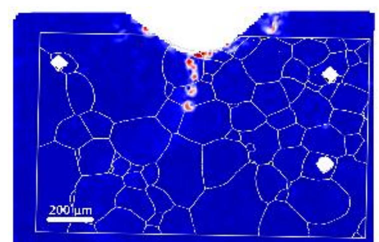


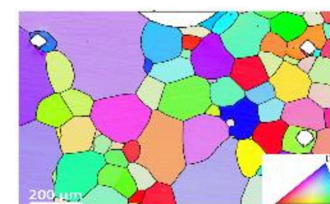
Full-field measurements

- Resolve displacement, strain, and strain rate inside grains
- Electron BackScatter Diffraction characterizes grain structure
- Non-contact measurement during fatigue test at typical rates (few Hz)

Limitations

- Surface measurements
- Optical imaging ≳ 1µm
- Sub-µm resolution much slower





E. Malitckii, H. Remes, & al. *Acta Mater.*, 144, p. 51-59, 2018.
DOI 10.1016/j.actamat.2017.10.038

Quasi-Static Elasticity Imaging

Integrated DIC

- Finite Element Model formulation of displacement field
- Optimization of model parameters to fit images directly

Spatially varying elastic constants

- Independent for each element
- Orthotropic linear elastic

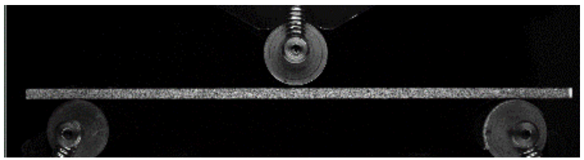
Regularization of spatial variations

- E.g. total variation

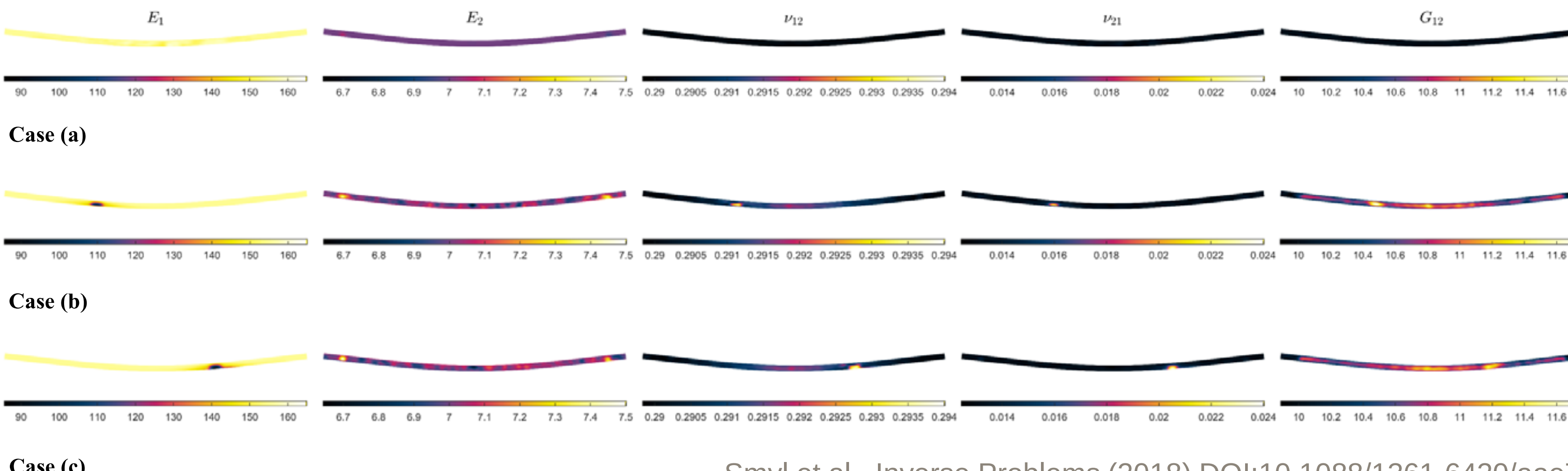
Smyl et al., SoftwareX (2019) DOI:10.1016/j.softx.2019.01.004

DIC-QSEI for structural health monitoring

- Undeformed CFRP beam



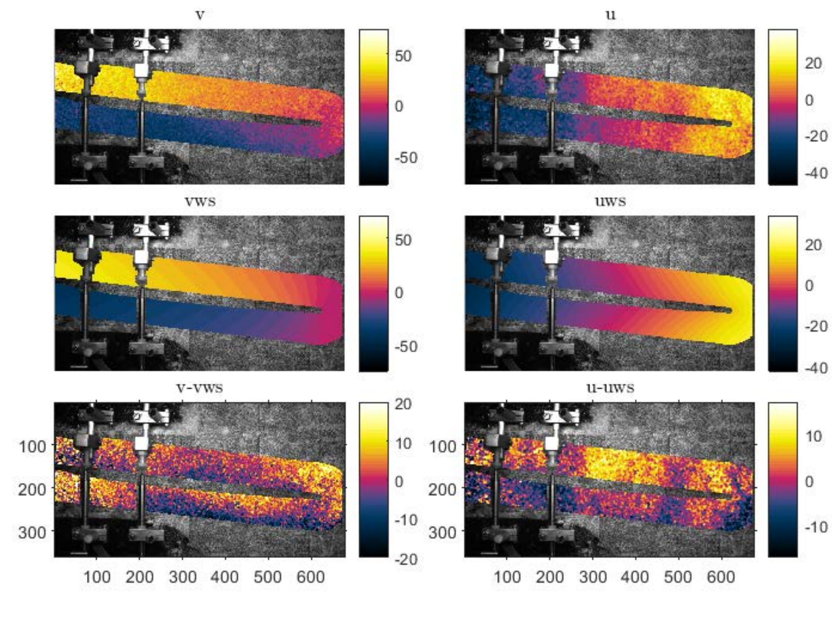
- Deformed CFRP beam



Smyl et al., Inverse Problems (2018) DOI:10.1088/1361-6420/aae793

DIC measurements in ice mechanics

- Challenging experiment conditions
  - *Wet and cold*
  - *Large field of view with short working distance*
  - *Unstable crack propagation*
- Crack opening displacement COD~ 1µm < ½ pixel
- Analytical LEFM solution for displacement field near crack tip
  - *Least squares fitting works well to determine crack tip location*



W. Ahmad, M.Sc. Thesis in progress

Summary of Challenges and Opportunities

Applying optimised patterns

- at different length scales on different materials
- convenience versus accuracy and reliability

Theoretical analysis for patterns with sharp edges

- level sets for location of edges
- additional information

Full understanding and characterisation of measurement accuracy

- calibration and standardisation
- spatial resolution
- "experimodelment"

Closer integration with numerical simulation

- finite element shape functions to make results more comparable
- initial guesses for displacement field

Numerical methods for highly localised deformations

- physics-based regularisation
- calculation speed and numerical stability
- pattern changes with very large deformations

Industrial internet

- monitor real-world performance (e.g. vibrations)
- strategies for archiving video data to be used for a posteriori analysis

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DIC for small displacement amplitudes

DIC for small displacement amplitudes

benefits

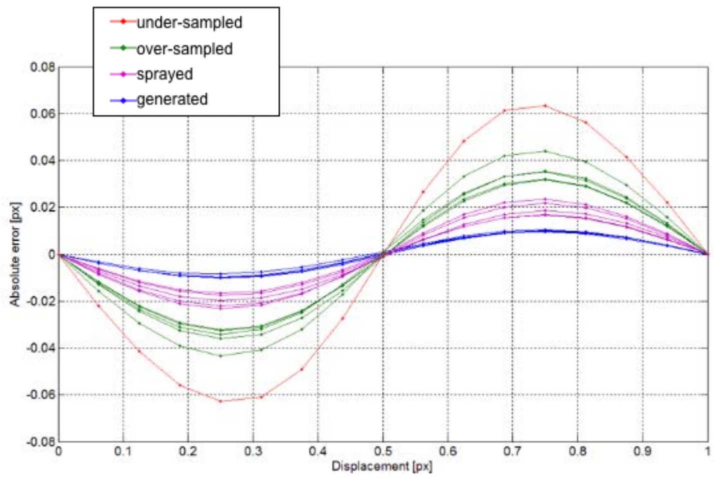
- Can pre-calculate sensitivities, simplifying calculations
- Time analysis can be done for pixel intensities
- Some sources of systematic error do not occur

Drawbacks

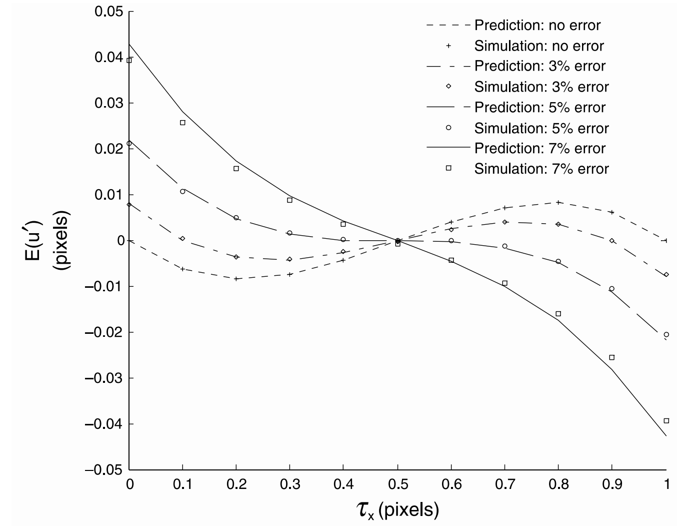
- Relative error is larger when value is small
- Some sources of systematic error that become uncorrelated for large displacement gradients remain correlated

Systematic errors in DIC

- Under-sampling (pixels too large) is worse than over-sampling (pixels too small)
- Synthetic patterns with well-defined feature size perform significantly better than sprayed patterns
- Depends on interpolation
- Error is small



DIC error analysis



interpolation error

- relative to “sinc” interpolation which is exact for band-limited signals

noise-induced bias

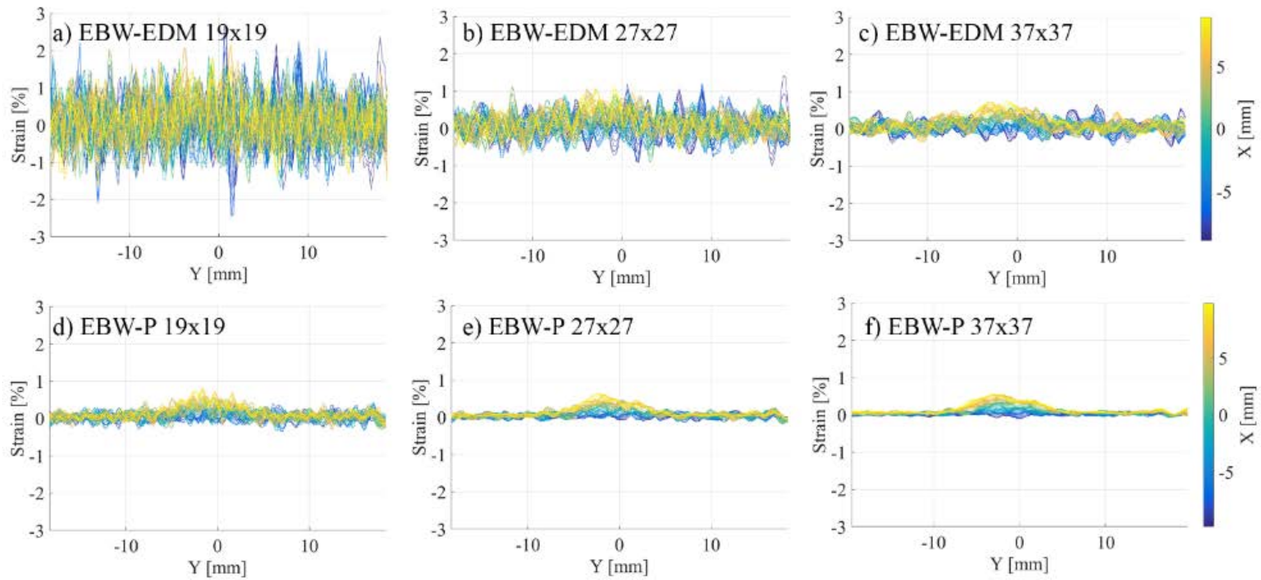
good patterns have high contrast

- better contrast reduces both variance and systematic errors
- highest-contrast images are not band-limited!

$$E(u') \cong u_0 - \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}[h(\mathbf{x}_{ij})\cdot\nabla T_x((\xi_{ij})_0)]}{\sum_{i=1}^{N}\sum_{j=1}^{N}[\nabla T_x((\xi_{ij})_0)]^2} + f_{rc}(\omega)\cdot\frac{N^2\sigma^2}{\sum_{i=1}^{N}\sum_{j=1}^{N}[\nabla T_x((\xi_{ij})_0)]^2}$$

Y. Q. Wang, M. A. Sutton, H. A. Bruck, and H. W. Schreier: *Strain*, **45**,160–178 (2009)

Effect of pattern on displacement variance



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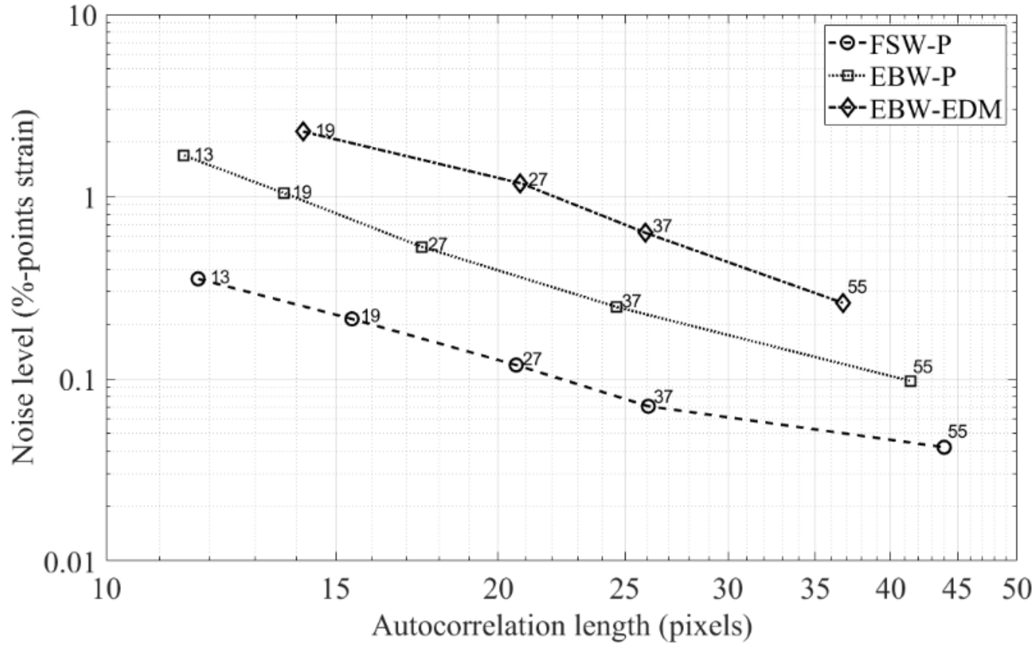
Variance and spatial resolution



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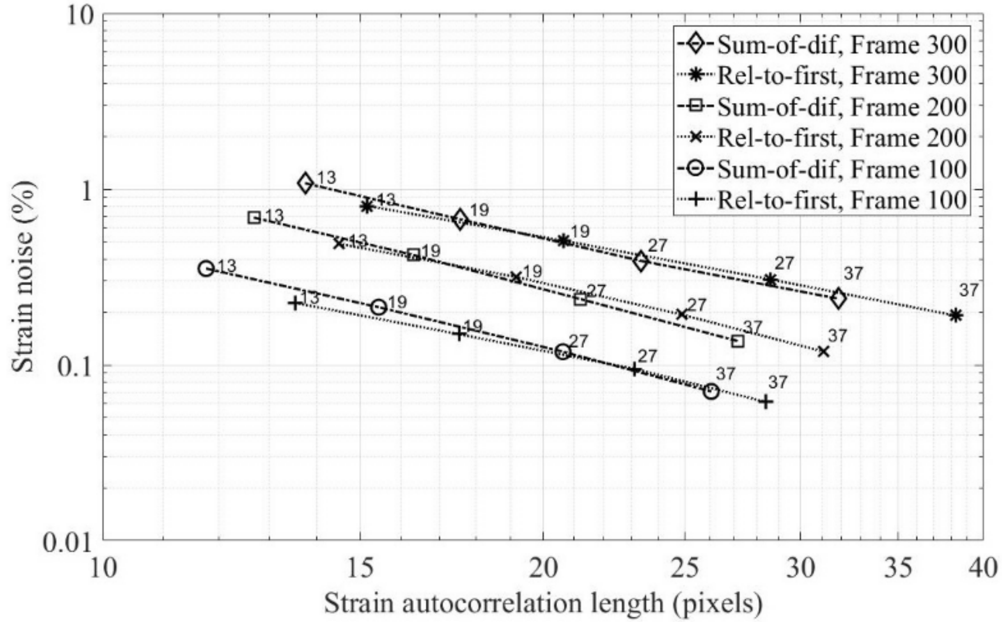
Variance and spatial resolution



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DIC compared with "Eulerian Video Magnification"

optical flow constraint

- image intensity “moves with object”

$$\vec{\nabla} I \cdot \frac{\mathrm{d}\vec{x}}{\mathrm{d}t} + \frac{\partial I}{\partial t} = 0$$

Lagrangian view

- *"Where is this material point now?"*
- typical for solids
- requires point tracking or optical flow
  - *most natural for large displacements with small deformations*
  - *same calculations for depth-from-stereo; 3D as a bonus*

Eulerian view

- *"Which material point is here now?"*
- typical for fluids
- simply gives results at fixed image coordinates
  - *for small amplitude motion, this is makes little difference, and it is much faster*

http://people.csail.mit.edu/mrub/vidmag/

Recommendations for DIC with small displacements

Optimized patterns

- feature size depends on imaging conditions

Prior knowledge

- average as many pixels in time and space as you know can be averaged
- small displacements allow to use methods that don't work for large displacements

Stereo imaging

- stereo disparity has systematic variation in fractional pixel displacement
  - *allows to detect and possibly to correct that*
- epipolar constraint is a built-in consistency check