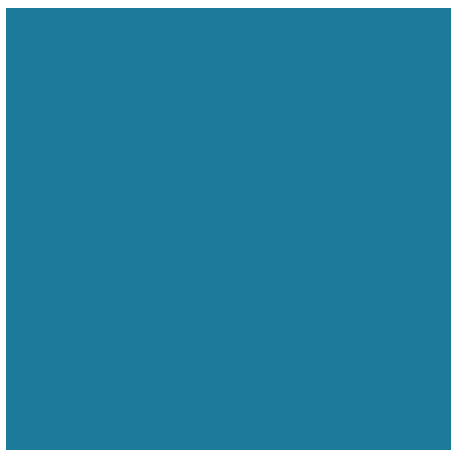
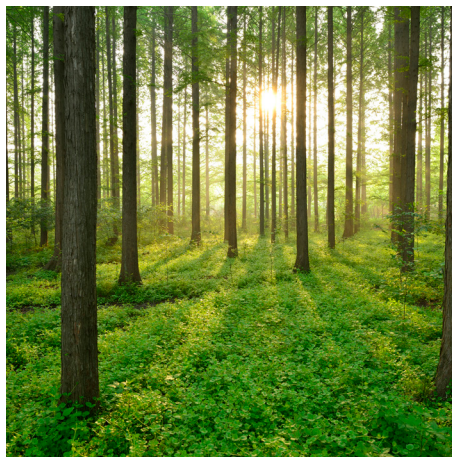


DEVELOPMENT OF A DIGITAL TWIN FOR TORSIONAL VIBRATIONS OF TURBOGENERATORS

REPORT 2025:1084



Development of a Digital Twin for Torsional Vibrations of Turbogenerators

SVEN HEROLD, HENDRIK HOLZMANN, RAINER NORDMANN,
FRAUNHOFER INSTITUTE LBF DARMSTADT

ISBN 978-91-89919-84-6 | © Energiforsk February 2025. Image: Gui Yong Nian, Adobe Stock.

Energiforsk AB | Phone: 08-677 25 30 | E-mail: kontakt@energiforsk.se | www.energiforsk.se

Foreword

This report forms the results of a project performed within the Energiforsk Vibrations in Nuclear Applications Program. The Vibrations Program aims to increase the knowledge of causes, monitoring and mitigation of vibrations, thereby contributing to the safety, maintenance and development of a diverse range of machinery in the Nordic nuclear power plants.

The turbogenerator, consisting of steam turbine and generator, is a key piece of machinery in the nuclear power plant. The long shaft train connecting steam turbine and generator is susceptible for torsional vibrations. Torsional vibrations are hard to detect but can cause damage to the machinery resulting in costs and unplanned production stops.

With this study, the Vibrations Program wanted to explore if a so-called Digital Twin could be applied to this specific problem. The results of the study show that with the right choice of concept and input data, a Digital Twin can be developed to help monitor and provide early-stage detection of torsional vibrations.

The study was carried out by Dr. Herold, Dr. Holzmann and Dr. Nordmann, Fraunhofer Institute LBF Darmstadt. The study was performed within the Energiforsk Vibrations Program, which is financed by Vattenfall, Uniper, Fortum, TVO, Skellefteå Kraft and Karlstads Energi.

These are the results and conclusions of a project, which is part of a research Program run by Energiforsk. The author/authors are responsible for the content.

These are the results and conclusions of a project, which is part of a research programme run by Energiforsk. The author/authors are responsible for the content.

Summary

Turbogenerators, consisting of steam turbines and generators are important components in nuclear power plants. The steam turbines convert thermal energy into mechanical energy causing the rotor to spin. And the generator produces electrical energy by electro-mechanical interactions in the air gap of the generator where the total energy is transferred. In case of disturbances in the electrical generator-grid-system (short circuits, unsymmetric grid loads, Sub Synchronous Resonance, ...) transient torsional vibrations of the shaft train will be excited, which may become very large due to the low damping in the system. For a safe and reliable operation, the electro-mechanical interaction processes and the torsional vibrations of the shaft train have to be observed very careful. A Digital Twin for monitoring and evaluating the torsional vibrations can be a well suited and valuable tool to solve this task.

By means of the engineering tools theoretical modelling, numerical analysis and experimental analysis the main tasks of simulation, validation and identification can be performed to operate a Digital Twin with success. With a Finite Element Model (FEM), as an important part of the Digital Twin, torsional vibrations can be calculated either as natural vibrations with natural frequencies and mode shapes or as forced vibrations due to the electro-mechanical air gap excitation. During operation, the real torsional vibrations of the turbogenerator are determined at specified locations, where sensors measure torsional displacements or shear stresses. By comparison of the measured and the corresponding calculated torsional vibrations the vibration difference is introduced into a calibration loop of the Digital Twin, in which the actual system parameters of the turbogenerator as well as the actual air gap torque can be identified and adjusted. The developed Digital Twin can be used as an online system, running permanently parallel to the real turbogenerator

- as a monitoring system, which determines continuously the torsional vibration state of the Turbogenerator at each location of the shaft train in the time domain as well as in the frequency domain
- as a detection system for the identification and diagnosis in case of system failures, disturbances and parameter changes (mass, stiffness, damping)

This report describes the development of the Digital Twin and its application, especially for the Olkiluoto turbogenerator OL3. By changing the Finite Element Model, the Digital Twin can be applied for other units as well.

Keywords

Digital Twin, Turbogenerator, Torsional Vibrations, Air Gap Torque, Monitoring

Sammanfattning

Turbogeneratorer, som består av ångturbiner och generatorer, är viktiga komponenter i kärnkraftverk. Ångturbinerna omvandlar värmeenergi till mekanisk energi som får rotorn att snurra. Generatoren producerar elektrisk energi genom elektromekaniska interaktioner i generatorns luftgap där den totala energin överförs. Vid störningar i det elektriska generator-nät-systemet (kortslutningar, osymmetriska nätbelastningar, subsynkron resonans, ...) kommer transienta vridningsvibrationer i axeltåget att uppstå, vilka kan bli mycket stora på grund av den låga dämpningen i systemet. För en säker och tillförlitlig drift måste de elektromekaniska interaktionsprocesserna och axeltågets torsionsvibrationer observeras mycket noggrant. En digital tvilling för övervakning och utvärdering av torsionsvibrationerna kan vara ett väl lämpat och värdefullt verktyg för att lösa denna uppgift.

Med hjälp av de tekniska verktygen teoretisk modellering, numerisk analys och experimentell analys kan de viktigaste uppgifterna simulering, validering och identifiering utföras för att driva en Digital Twin på ett framgångsrikt sätt. Med en Finite Element Model (FEM), som är en viktig del av Digital Twin, kan torsionsvibrationer beräknas antingen som naturliga vibrationer med egenfrekvenser och modformer eller som påtvingade vibrationer på grund av den elektromekaniska luftgapsexcitationen. Under drift bestäms turbogeneratorns verkliga torsionsvibrationer på angivna platser, där sensorer mäter torsionsförskjutningar eller skjuvspänningar. Genom att jämföra de uppmätta och motsvarande beräknade torsionsvibrationerna förs vibrationsskillnaden in i en kalibreringsslinga i den digitala tvillingen, där turbogeneratorns faktiska systemparametrar och det faktiska luftgapsmomentet kan identifieras och justeras. Den utvecklade digitala tvillingen kan användas som ett onlinesystem som körs permanent parallellt med den verkliga turbogeneratoren

- som ett övervakningssystem, som kontinuerligt bestämmer turbogeneratorns torsionsvibrationstillstånd vid varje plats i axeltåget i tidsdomänen såväl som i frekvensdomänen
- som ett detekteringssystem för identifiering och diagnos vid systemfel, störningar och parameterförändringar (massa, styvhet, dämpning)

Denna rapport beskriver utvecklingen av den digitala tvillingen och dess tillämpning, särskilt för Olkiluotos turbogenerator OL3. Genom att ändra Finite Element-modellen kan Digital Twin tillämpas även på andra enheter.

Nyckelord

Digital Twin, Turbogenerator, Torsionsvibrationer, Övervakning av luftgapsmoment

List of content

1	Torsional Vibrations of Turbogenerators	8
1.1	Excitation Sources of Torsional Vibrations	8
1.2	Character of Torsional Vibrations of Turbogenerators	9
1.3	Torsional Natural Frequencies and Mode Shapes	10
1.4	Torsional Vibrations due to Air Gap Torques – Design Cases	11
2	Digital Twin for Torsional Vibrations of Turbogenerators	13
2.1	The Digital Twin – a Digital Copy of the Real System	13
2.2	The Tools for the Development of the Digital Twin	13
2.3	Concept of the Digital Twin for Torsional Vibrations	14
3	Finite Element Model of the Digital Twin (Component 1)	16
3.1	The Finite Element Model for Torsional Vibrations	16
3.2	Mass and Stiffness of the Turbogenerator Shaft Train OL3	17
3.3	Modelling of the Turbogenerator with Last Stage Blades	18
3.4	Natural Frequencies and Mode Shapes of the Base Model	21
3.5	Comparison with Results from the Manufacturer	21
3.6	Comparison with Results from Measurements	22
3.7	Damping of the Base Model	23
3.8	Forced Torsional Vibrations with the Base Model	26
4	The Comparison Point of the Digital Twin (Component 2)	29
4.1	The Measured Torsional Vibrations of the Shaft Train	29
4.2	The Calculated Torsional Vibrations of the Shaft Train	31
4.3	Difference of Measured and Calculated Vibrations	31
5	The Calibration of the Mass and Stiffness Matrices of the Shaft Train (Component 3)	32
5.1	Sensitivity Analysis for the Torsional Natural Frequencies	32
5.2	The Calibration Procedure for Mass and Stiffness	34
6	The Calibration of the Modal Damping (Component 3)	35
6.1	The Half-Power-Bandwidth Method	35
6.2	The Calibration for the Modal Damping at Operation	35
7	Calibration of the Air Gap Torques (Component 4)	37
7.1	Comparison of Calculated and Measured Shaft Stresses	37
7.2	The Calibration Procedure for the Air Gap Torque	37
8	Digital Twin - a Continuous Monitoring System for the Turbogenerator	39
8.1	Vibrations and Stresses at all Locations- Virtual Sensors	39
9	Identification of System Parameter Changes	40
9.1	Example 1: Change of one Modal Damping, SSR-Event	40
9.2	Example 2: Change of Coupling Torsional Stiffness	41
10	Identification of the Air Gap Torque	43

10.1	Example 3: Change Air Gap Torque due to 2-Ph. Short Circuit	43
10.2	Example 4: Change Air Gap Torque due to unsymm. Grid	44
11	Conclusions and Outlook	45
11.1	Conclusions	45
11.2	Outlook	45

1 Torsional Vibrations of Turbogenerators

Chapter 1 introduces the research area of Torsional Vibrations of Turbogenerators in Nuclear Power Plants (NPP). Due to the fact, that the Digital Twin in this research project has been developed especially for the Turbogenerator of the Olkiluoto unit OL3, this introduction is partly also related to this unit.

1.1 EXCITATION SOURCES OF TORSIONAL VIBRATIONS

Turbogenerators, consisting of steam turbines and generators are important components in nuclear power plants. The steam turbines convert thermal energy into mechanical energy causing the rotor to spin. And the generator produces electrical energy by electro-mechanical interactions in the air gap of the generator where the total energy is transferred and converted. In case of disturbances in the electrical generator-grid system besides the nominal air gap torques additional torques due to electro-mechanical interactions will appear. They depend on electrical quantities (currents in the rotor- and stator-windings, coupling inductances) and on mechanical quantities (torsional displacements and velocities), see figure 1 and formula (1.1).

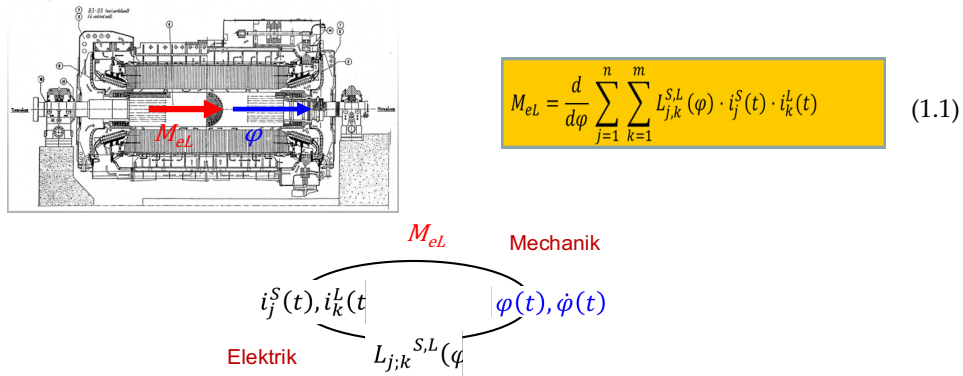


Figure 1: Air Gap Torque due to Electro-Mechanical Interaction [Source: TVO]

The disturbance related air gap torques usually excite high transient torsional vibrations in the shaft train, which have a relatively low decay rate due to the weak system damping. Disturbance cases are 2-phase and 3-phase short circuits, unsymmetrical grid loads (negative sequence current) or also dangerous sub synchronous resonances. They will be discussed in more detail in chapter 1.4. For a safe and reliable operation the electro-mechanical interaction processes and the torsional vibrations of the shaft train should be controlled permanently. To solve this task, a Digital Twin for monitoring the torsional vibrations can be a well suited and valuable tool.

1.2 CHARACTER OF TORSIONAL VIBRATIONS OF TURBOGENERATORS

Torsional vibrations of turbogenerators are described by time dependent torsional deformations $\mathbf{q}(t)$ along the shaft line. The torsional vibrations can be determined by equations of motion, which express the dynamic equilibrium of the time dependent air gap torques and the inertia, damping and stiffness torques of the different system components. The linear equations of motion are described by a model with mass damping and stiffness matrices $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$ and an excitation vector $\mathbf{M}_{el}$ for the air gap torque (Figure 2 and formula 1.2).

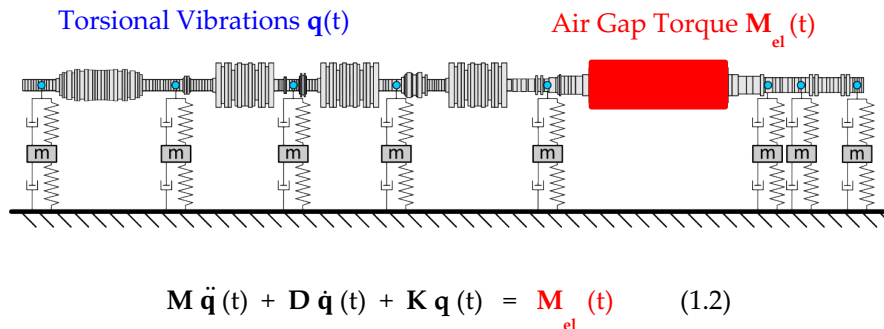


Figure 2: Model of Turbogenerator and Equations of Motion for Torsional Vibrations

The components to set up the equations of motion are mainly the cylindrical elements of the shaft train and the attached turbine blades. The shaft elements influence the dynamic behaviour with their moments of inertia and their torsional stiffness. The turbine blades with large moments of inertia contribute mainly to the inertia terms in the mass matrix. However, the last stage blades in the Low-Pressure Turbines (LPT) have to be considered as flexible beam elements at higher frequencies. In this case, the blades form a dynamic rotor-blade interaction system with the shaft. Furthermore the dynamic behaviour of the blades may depend on the rotational speed of the shaft train due to stiffening effects by centrifugal forces.

The dynamic behaviour of a turbogenerator can be characterized by modal parameters, that means by the different torsional natural frequencies with their corresponding mode shapes. To each torsional natural frequency belongs a damping value, which is called modal damping. If the modal parameters are known, the torsional vibration response of the turbogenerator can be presented by a sum of single degree of freedom (SDOF) systems for an arbitrary air gap excitation. This type of presentation is known as Modal Analysis.

The torsional damping of turbogenerators is in general very small. The sources are in the mechanical as well as in the electrical system. However, the most significant sources of the damping seem to come from the electrical parts of the generator and the grid, while the contributions from the mechanical turbine shaft (material damping, friction, steam,...) are very low. The torsional damping is usually considered as modal damping, which means that each modal damping is related to one of the torsional natural frequencies. It is important to note, that the modal damping of the torsional modes increases significantly with the electrical load when the generator is connected to the grid.

1.3 TORSIONAL NATURAL FREQUENCIES AND MODE SHAPES

If the excitation air gap torques in equation (1.2) are set to zero, we obtain the homogenous equations of motion for the natural vibrations of the turbogenerator. Due to the fact, that the damping is very small, we can neglect the damping term. This leads to the homogenous equations (1.4), in which only the inertia and stiffness terms M and K determine the free vibrations.

$$M \ddot{q}(t) + D \dot{q}(t) + K q(t) = M_{el}(t) \quad (1.3)$$

$$M \ddot{q}(t) + K q(t) = 0 \quad (1.4)$$

With the mathematical approach (1.5) for the free natural vibrations of the system and the derivative (1.6) for the acceleration we obtain the Eigenvalue problem (1.7)

$$q(t) = \varphi \sin \omega t \quad (1.5)$$

$$\ddot{q}(t) = -\omega^2 \varphi \sin \omega t \quad (1.6)$$

$$(K - \omega^2 \cdot M) \cdot \varphi = 0 \quad (1.7)$$

The Eigenvalue analysis leads to the torsional natural frequencies ω_j and to the corresponding mode shapes φ_j . As an example figure 3 shows the four lowest natural frequencies of the OL3 turbogenerator in Olkiluoto.

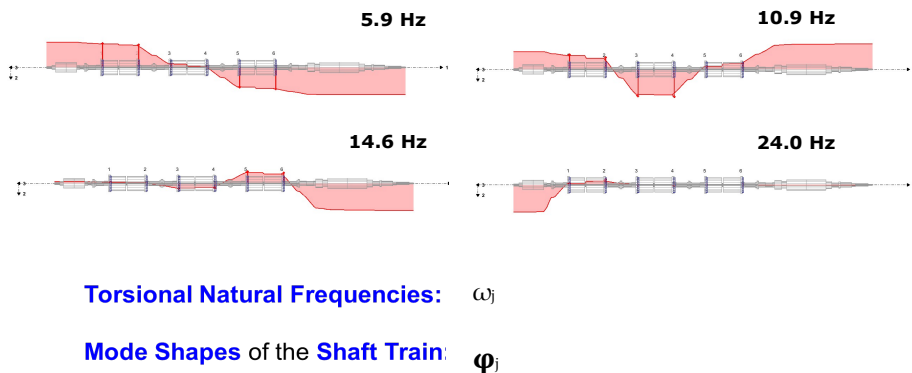


Figure 3: Lowest Torsional Natural Frequencies and Modes of the OL3 Turbogenerator

1.4 TORSIONAL VIBRATIONS DUE TO AIR GAP TORQUES – DESIGN CASES

In the design process of large turbogenerators, it is common practice to express the air gap torque due to electrical disturbances by fixed formulas instead of using the more complicated formula (1.1). This is a conservative approach to the real vibration behaviour. Experience shows that it is a useful approximation, which is often also confirmed by experimental results. Usually, the following electrical fault cases are considered during the design, which have the following excitation frequencies:

2-phase short circuit	1x grid and 2x grid frequency
3-phase short circuit	1x grid frequency
Faulty synchronization	1x grid frequency
Negative sequence current	2x grid frequency

In the following we consider the simplified air gap torques for the cases of a 2-phase short circuit and the negative sequence current (unsymmetric grid loads), which will later also be the test cases for the Digital Twin.

For **simplification** the presented formula for the Electrical Air gap torque of a 2 Phase Short Circuit does not consider the strong **electromechanical Interaction** as previously shown in formula (1). It is however a good **approximation** based on experience for this kind of disturbance.

$$M_e(t) = M_0 + \frac{M_0}{\cos \varphi} \cdot \frac{1}{x_d'' + x_{TR}} \cdot \{ \sin \Omega(t - t_0) - 0.5 \cdot \sin 2\Omega(t - t_0) \}$$

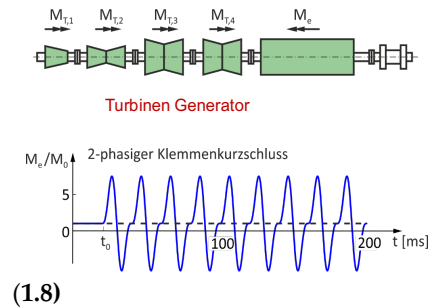


Figure 4 Assessment of Torsional Vibrations during the Design: 2-Phase Short Circuit

The simplified formula (1.8) shows the electrical air gap torque for a non decaying 2-phase short circuit, which is a conservative assumption. Excitation frequencies are the single grid frequency Ω and the double grid frequency 2Ω . The formula contains also some generator characteristics. In reality the air gap torque is decaying. In figure 5 the 2-phase air gap torques with and without a decay are presented, see figure 5, left side. The system response for the two cases is shown on the right side of figure 5. By comparison the conservative assumption for the case without decay seems obvious. Figure 5 also demonstrates the low decay rate of the torsional vibrations.

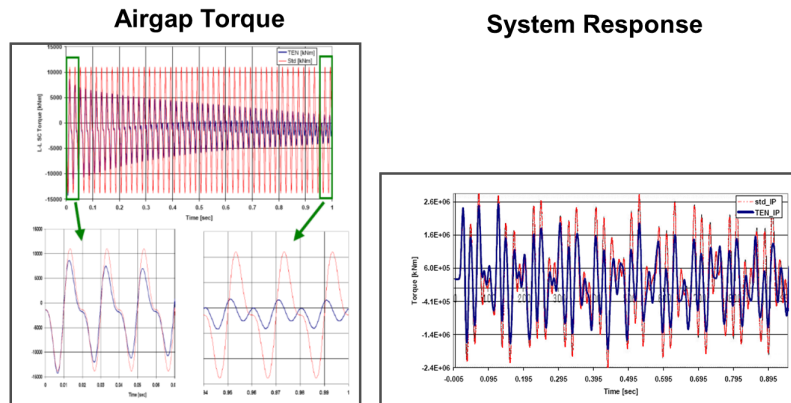


Figure 5: Air Gap Torque and Torsional Response with and without decay [Source: R. Nordmann]

Due to unequal loads in the electrical grid, unsymmetry will occur in the 3-phase electrical system. By electrical derivatives it can be shown, that due to this fact, the nominal air gap torque is superimposed by a pulsating torque with double grid frequency. In a 50 Hz grid system the air gap torque therefore excites the shaft train with a frequency of 100 Hz, which is a steady state excitation (Figure 6).

Due to **unequal loads** in the **electrical grid** and possible failures **unsymmetry** may occur in the 3 Phase Electrical system.

Due to this fact the Air gap torque is superimposed by a **pulsating torque** with **double grid frequency**. In a 50 Hz grid the air gap torque therefore excites the Shaft Train with a frequency of 100 Hz.

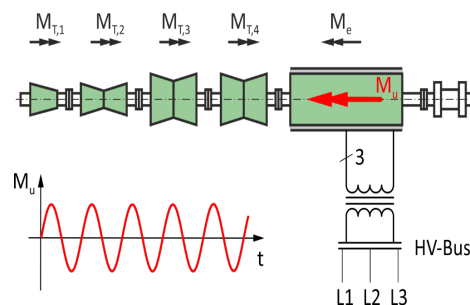


Figure 6: Assessment of Torsional Vibrations during the Design: Unsymmetric Load

In general, it has to be considered, that the system response always has two solution parts: the inhomogeneous solution due to the excitation air gap torque, which is superimposed by the homogeneous solution with the natural torsional frequencies. The homogeneous solution (natural vibrations) decays in case of damping, however in the case of low damping, the decay rate may be very small.

2 Digital Twin for Torsional Vibrations of Turbogenerators

In this chapter 2, the basic idea of a Digital Twin will be presented. After a more general description, the application of a Digital Twin for torsional vibrations of turbogenerators will be pointed out.

2.1 THE DIGITAL TWIN – A DIGITAL COPY OF THE REAL SYSTEM

A Digital Twin is a digital copy of a real system. The basic idea of such a Digital Twin is shown in figure 7 for a general system at operation. The deviations between sensor observations at the real system and predictions from the model of the Digital Twin are the base for an identification and update of the system parameters and the operational conditions. The identified system changes can for example be an indication for internal system failures or external disturbances. The deviations can also be used to generate signals for an active system control. In addition, recommendations for experts can be derived from the deviations for a possible change of operation conditions.

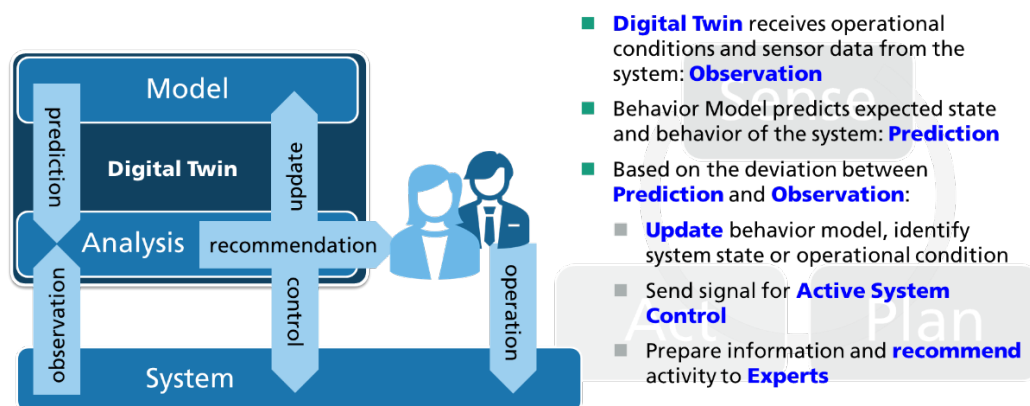


Figure 7: Digital Twin for a System at Operation

From figure 7 it can be recognized, that for the function of the Digital Twin, a relevant model and analysis procedures are needed. With respect to the development of a Digital Twin for the torsional vibrations of a turbogenerator the three engineering tools: theoretical modelling, numerical analysis and experimental analysis are necessary. These tools will be discussed in the next chapter together with the tasks for the function of the Digital Twin.

2.2 THE TOOLS FOR THE DEVELOPMENT OF THE DIGITAL TWIN

Figure 8 shows the three tools, which are usually applied to solve vibration problems in mechanical engineering. These tools are besides theoretical modelling the numerical and the experimental analysis. The theoretical modelling is based on

physical laws, particularly from mechanics. They lead to equations of motion, which express the dynamic behaviour of mechanical systems. By means of numerical analysis, equations of motion can be solved for natural as well as for forced vibrations. And vibrations can be measured by experimental analysis with sensors, actuators and devices for signal processing. By different combinations of the three tools the following tasks can be performed, which is shown in figure 8:

Simulation: Theoretical Modelling & Numerical Analysis
 Validation: Numerical Analysis & Experimental Analysis
 Identification: Experimental Analysis & Theoretical Modelling

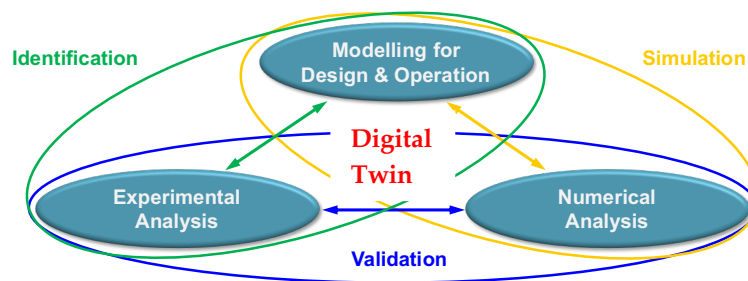


Figure 8: Tools and Tasks to solve Vibration Problems in Mechanical Engineering, based on [Source: D. J. Ewins. *Exciting vibrations: the role of testing in an era of supercomputers and uncertainties*. *Meccanica - An International Journal of Theoretical and Applied Mechanics*. DOI 10.1007/s11012-016-0576-y]

In the next two chapters 2.3 and 2.4 it will be shown, that for the development of the Digital Twin for torsional vibrations of turbogenerators all three tasks simulation, validation and identification are necessary to achieve the requirements for the Digital Twin. The concept of the Digital Twin will be presented in chapter 2.3, followed by the presentation of the components and their interaction in chapter 2.4.

2.3 CONCEPT OF THE DIGITAL TWIN FOR TORSIONAL VIBRATIONS

The concept of the Digital Twin for torsional vibrations of turbogenerators is presented in figure 9. The real system of the turbogenerator (black frame) is excited by air gap torques leading to real torsional vibrations along the shaft train. Some of these vibrations are measured by sensors at defined locations. These sensor signals are input data for the Digital Twin of the turbogenerator (blue frame). They are transferred to the internal difference location.

An important part of the Digital Twin is a model, in this case a Finite Element Model: 1. With this FE Model, natural vibrations as well as forced torsional vibrations due to the excitation can be calculated at each location along the shaft line. The part of the calculated torsional vibrations corresponding to the sensor positions is also transferred to the difference location: 2.

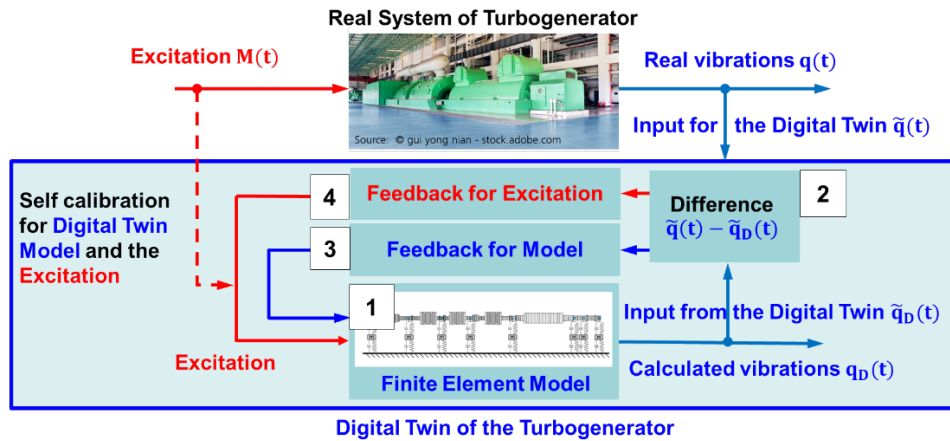


Figure 9: Concept of the Digital Twin for Torsional Vibrations of Turbogenerators

The differences between the measured and the calculated vibrations are introduced via feedback into the calibration component of the Digital Twin. The feedback and the calibration are subdivided into two parts, one for calibrating the system parameters of the model (3) and one for the excitation (4). This is necessary since the torsional vibrations as a system output can be influenced either by the excitation (system input) or by possible changes of the system parameters. If the Digital Twin works without problems in the way of the described concept, the state of the torsional vibrations of the turbogenerator can be determined at any time of operation and at any location of the FE model. Besides the torsional vibration, other derived quantities like stresses, torques etc can be determined as well. In figure 9 the four components of the Digital Twin are marked by their numbers: 1. Finite Element Model of the turbogenerator, 2. Comparison point of the Digital Twin (Difference of measured and calculated vibrations), 3. Calibration of the mass and stiffness matrices and calibration of the modal damping and 4. Calibration of the air gap torque. These components are described in more detail in chapters 3 to 7.

3 Finite Element Model of the Digital Twin (Component 1)

For the development of the Digital Twin, the first component has been built up as a Finite Element Model of the turbogenerators shaft train (see Figure 9). The Energiforsk steering group decided, that as a test case in this research project the first Digital Twin should be developed for the unit OL3 Turbogenerator in Olkiluoto.

3.1 THE FINITE ELEMENT MODEL FOR TORSIONAL VIBRATIONS

In chapter 1.2 it has already been discussed that the torsional vibrations can be described by a model with mass- damping- and stiffness-matrices $\mathbf{M}$, $\mathbf{D}$, $\mathbf{K}$ and an excitation vector $\mathbf{M}_{el}$ for the air gap torque. The equations of motion (2) are repeated here as equation (3.1):

$$\mathbf{M} \ddot{\mathbf{q}}(t) + \mathbf{D} \dot{\mathbf{q}}(t) + \mathbf{K} \mathbf{q}(t) = \mathbf{M}_{el}(t) \quad (3.1)$$

The elements to set up the equations of motion are mainly the cylindrical elements of the shaft train (chapter 3.2) and the attached turbine blades (chapter 3.3). The cylindrical shaft train elements as shown in figure 10 for a general turbogenerator, influence the dynamic behaviour with their moments of inertia in the mass matrix $\mathbf{M}$ and with their torsional stiffness in the stiffness matrix $\mathbf{K}$.

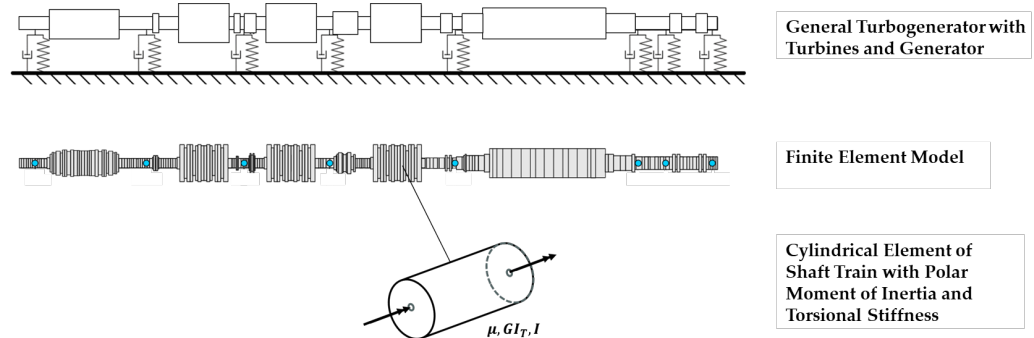


Figure 10: General Turbogenerator and Finite Element Model with Cylindrical Elements

The original model was first set up in MADYN2000, which provides all necessary basic modeling functionalities. A disadvantage is however that the system matrices (stiffness, mass and damping properties) are not available and cannot be changed programmatically. This provides a major restriction for the Digital Twins adaptability. Therefore, the modeling is changed to an implementation by Fraunhofer LBF, in this case realized in MATLAB, where all the geometry data of

the MADYN model are used. The process of reading MADYN data is automated, so it can be repeated for other models.

In MATLAB, a much better insight in the model is possible, so every parameter can be changed and automated procedures with changing parameters in programming loops are possible. Especially the system matrices are available and can be changed. Part of the global system matrices are the local system matrices – the so-called substructures. These can also be changed independently, which is a big advantage when implementing fault scenarios at certain parts of the rotor. Furthermore, there are much better possibilities to share the data for other users.

The MATLAB finite element model is composed of a coupling of torsional spring-damper elements with mass properties at the degrees of freedom. The Finite Element for the single substructures is derived from literature. Each substructure has two torsional degrees of freedom that are coupled with one another. The coupling of the single substructures is done using an addition of the single substructures according to the global degrees of freedom in the rotor. In this way, the global matrices (Mass, stiffness) are obtained.

The model is transformed to a modally reduced form using the well-known approach of modal transformation and truncation. In this way, the model's degree of freedom could be reduced, so it becomes more efficient in simulation. This can become relevant for larger models but in this case also the full model is fast to compute since it does not have a lot of degree of freedom.

Finally, a good agreement with the Madyn model (FRF-comparison) was observed underlining the approach being reasonable.

3.2 MASS AND STIFFNESS OF THE TURBOGENERATOR SHAFT TRAIN OL3

The mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ of the overall shaft train consist of the matrices of the cylindrical elements of the shaft and the mass and stiffness contributions from the blades (chapter 3.3). In a first step we consider only the element matrices $\mathbf{K}_n$ and $\mathbf{M}_n$ of the cylindrical shaft elements ($n = 1, 2, \dots, N$). They are also known as local stiffness and mass matrices for each Finite Element. Due to the fact that each shaft element has two torsional degrees of freedom (each on one side), $\mathbf{K}_n$ and $\mathbf{M}_n$ are simple 2×2 matrices. They can be derived for each of the elements with approach functions via the principle of virtual work:

$$\mathbf{K}_n = (G I_T / l)_n \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (3.2) \quad \mathbf{M}_n = (\rho I)_n \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \quad (3.3)$$

The necessary geometry and material data for each cylindrical element n are the following:

- l [m] Shaft length of element
- I_T [m⁴] Torsional moment of inertia ($G I_T / l$ Torsional stiffness)

$\hat{\mu}$ [kg m] Polar moment of inertia per unit length (mass effect)

G [N/m²] Shear modulus of the material

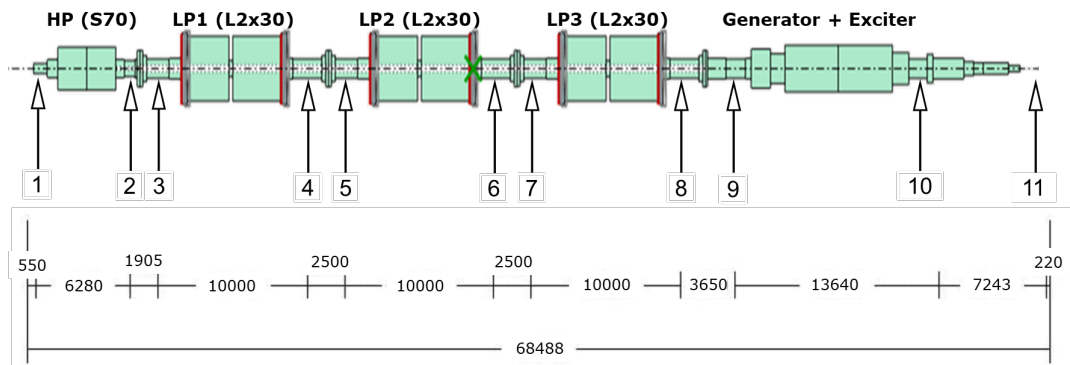


Figure 11 Geometrical Data for the OL3 Turbogenerator

The geometrical data in terms of length and diameter (Figure 11) and the material data for the local matrices $\mathbf{K}_n$ and $\mathbf{M}_n$ were delivered by TVO. The inner and outer diameters of the shaft elements were described in a separate table, which is not shown here. These diameters are needed to calculate the torsional moments of inertia I_T and the polar moments of inertia $\hat{\mu}$.

By superposition of all shaft elements, we obtain a first part of the two global matrices $\mathbf{K}$ and $\mathbf{M}$ of the Turbogenerator shaft train. Due to the chain arrangement of the shaft elements, the two matrices have a band-structured shape, which has advantages for the numerical calculations.

3.3 MODELLING OF THE TURBOGENERATOR WITH LAST STAGE BLADES

The turbine blades with large moments of inertia contribute mainly to the polar moment of inertia terms in the mass matrix $\mathbf{M}$. However, the last stage blades in the Low-Pressure Turbines (LPT) have also to be considered in the stiffness matrix $\mathbf{K}$ as flexible beam elements in the frequency range between 30 Hz to 120 Hz. In this range the blades form a dynamic rotor-blade interaction system with the shaft. Furthermore, the dynamic behaviour of the blades may depend on the rotational speed of the shaft train as well due to stiffening effects by centrifugal forces.

Unfortunately, it was not possible to obtain the exact geometrical data for the last stage blades of the LPT's to model the dynamic rotor-blade interaction. This is especially a disadvantage for modelling the stiffness behaviour of the blades. The only data we could receive for the last stage blades are related to the inertia terms:

- Mass of one blade including the root part m_{bl} 272 kg
- Radius of the center of gravity of one blade R_{CG} 2.156 m
- Number of blades in the LPT last stage row 44

The part of the polar moment of inertia for the flexible last stage blade row is therefore:

$$\Theta_{LSB} = m_{bl} \times R_{cG} \times 44 = 55631 \text{ kg m}^2$$

This inertia term has to be considered for each last stage blade row, when the dynamic rotor blade interaction is modelled with stiffness and inertia effects.

In Table 2 the total polar moments of inertia are presented for the different rotor parts of the OL3 shaft train. These values include all inertia terms of the shaft train and the blades as well. When creating the mass matrix M for the complete system it has to be considered, how the inertia terms are distributed to the parts of the shaft and the blades.

To consider the dynamic rotor blade interaction of the six last stage blade rows of the LPT's in an optimal way, we developed a simplified model (see figure 12), based on the following assumptions: When the blades are excited by the torsional motion of the shaft, the blades will vibrate mainly in a bending mode in tangential direction. All blades will vibrate with the same phase. If all blades vibrate only in their first bending mode, the simplest model for the complete blade row would be a ring with a polar moment of inertia, which is connected to the rotor by one torsional spring.

Rotor	Length l [mm]	Mass m [kg]	Polar Moment of Inertia Θ_p [kgm ²]
HP	7685	96670	37601
LP1	12500	322852	427799
LP2	12500	322852	427799
LP3	12500	322852	427799
Generator	16775	250678	100017
Exciter	6528	25303	3965

Table 1: Length, Mass and Polar Moment of Inertia of the Rotors of the OL3 Shaft Train

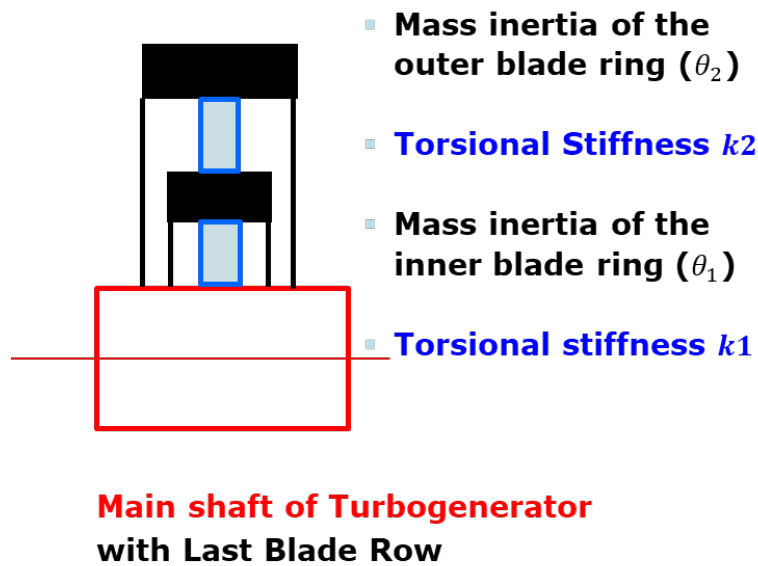


Figure 12 Modelling of the LPT Last Stage Blade Row by a simplified TDOF Model

Experience showed that this model was not sufficient to lead to good results. The model has therefore been extended to a Two Degree of Freedom (TDOF) model, consisting of two rings with polar moments of inertia Θ_1 and Θ_2 and two torsional springs with torsional stiffness values k_1 and k_2 . This model is shown in figure 12 together with the last stage blade row of the turbogenerator OL3.

To obtain an optimal FE-Model for the turbogenerator with the above described effects (e.g. rotor-blade interaction, etc.) numerical calculations for the torsional natural frequencies and the corresponding mode shapes were performed and compared with existing results from the manufacturer of the turbogenerator. Later on, the calculated results have also been compared with measured results. The process of the model development was performed by a Sensitivity analysis (see chapter 5.1), in which the parameters of the model could be identified and adjusted by comparison of the calculated natural frequencies and the corresponding reference values from the manufacturer. During the identification procedure the mass- and stiffness-parameters of the shaft train did not change very much. The parameters of the TDOF model for the attached last stage blades had finally the following values for the adjusted model:

Polar Moment of Inertia		Torsional Stiffness	
$\Theta_1 =$	37086 kgm ²	$k_1 =$	6,4 e+09 Nm/rad
$\Theta_2 =$	18543 kgm ²	$k_2 =$	3,2 e+09 Nm/rad

Table 2: Polar Moment of Inertia and Torsional Stiffness of Blade Model

With the identified matrices **M** and **K**, we have a validated FE-Model, which describes very well the inertia and stiffness characteristics of the OL3 turbogenerator at normal operation. With this **Base Model** of the Digital Twin, we can already determine very well the torsional natural frequencies and mode shapes of the turbogenerator shaft train including the interaction with the blades at

normal operating conditions. Damping in the turbogenerator is very small and can therefore be neglected when calculating the natural frequencies and mode shapes. However, we will come back to mechanical and electrical damping later, when damping becomes the resistance to excitation.

3.4 NATURAL FREQUENCIES AND MODE SHAPES OF THE BASE MODEL

The developed FE-Model for torsional vibrations of the OL3 Turbogenerator is shown in figure 13. As described in chapter 3.3 it considers the inertia and stiffness effects of the shaft train and the turbine blades. As a special feature the rotor-blade interaction has been considered in the model at the six last stage blade rows of the three LPT's. The dynamic characteristic of this **Base Model** is determined by the stiffness matrix **K** and the mass matrix **M** for **normal operation** conditions.

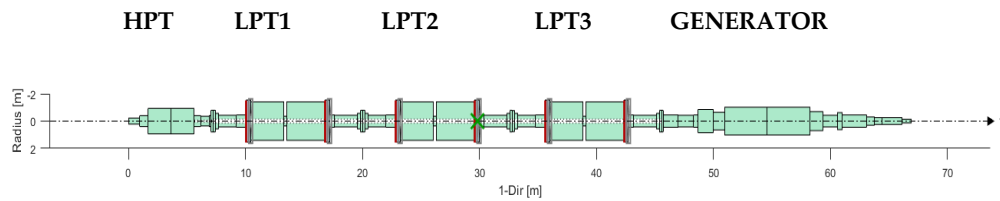


Figure 13 FE-Model of the OL3 Turbogenerator with Turbines and Generator

As already shown in chapter 1.3 we can solve the Eigenvalue problem (3.4)

$$(\mathbf{K} - \omega^2 \cdot \mathbf{M}) \cdot \boldsymbol{\varphi} = \mathbf{0} \quad (3.4)$$

to obtain the circular torsional natural frequencies ω_j [1/s] and the corresponding torsional mode shapes $\boldsymbol{\varphi}_j$. The torsional natural frequencies in Hz can be determined by

$$f_j = \omega_j / 2\pi \text{ [Hz]}. \quad (3.5)$$

3.5 COMPARISON WITH RESULTS FROM THE MANUFACTURER

Figure 14 shows the 18 first torsional natural frequencies in Hz up to the frequency of 80 Hz, which were calculated with the **Digital Twin Base Model**. The natural frequencies have been compared with the design natural frequencies of the manufacturer (Reference). The frequency values of the Digital Twin FE-Model are in good correlation with the reference values. The error in [%] is less than 2 % in the whole frequency range up to 80 Hz. This promises a good performance of the Digital Twin in the required frequency range.

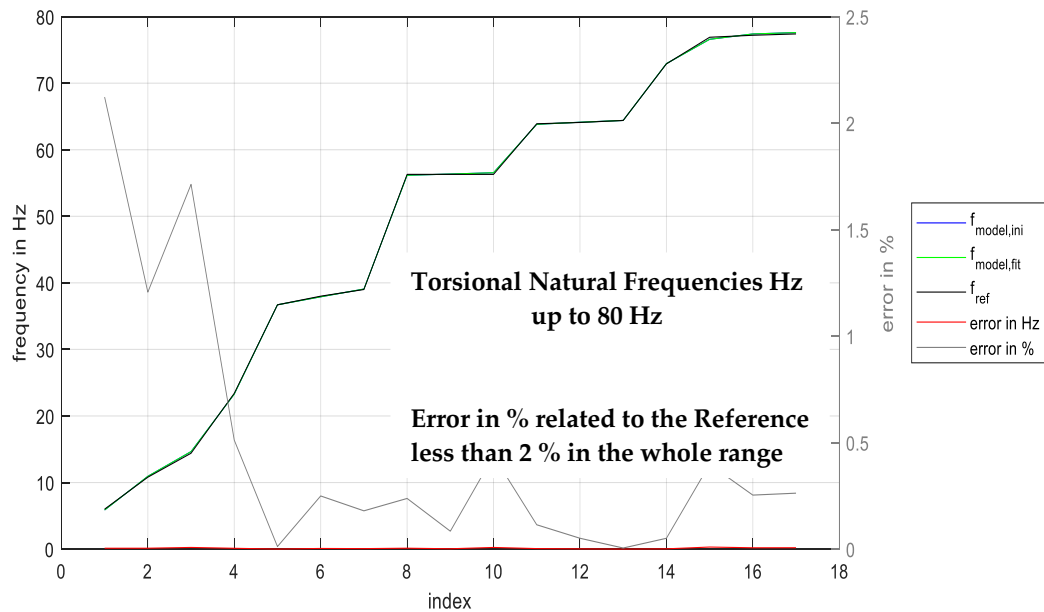


Figure 14 Torsional Natural Frequencies f_j [Hz] of Turbogenerator OL3 and Error [%]

3.6 COMPARISON WITH RESULTS FROM MEASUREMENTS

During commissioning of OL3, first torsional natural frequency measurements were taken at the turbogenerator at normal operation. In Table 3 these measured natural frequencies are compared with calculated values from the **FE-Base-Model**.

No.	FE-Model Digital Twin	Measurement at OL3	Error [%]
1	5,90 Hz	5,67 Hz	+ 4,0 %
2	10,90 Hz	10,62 Hz	+ 2,6 %
3	14,60 Hz	14,37 Hz	+ 1,6 %
4	24,00 Hz	23,24 Hz	+ 1,0 %
5	38,28 Hz	35,61 Hz	+ 10,3 %
6	39,10 Hz	39,28 Hz	- 0,4 %
7	40,52 Hz	40,24 Hz	+ 0,7 %
8	57,65 Hz	55,88 Hz	+ 3,2 %
9	58,55 Hz	60,59 Hz	- 3,4 %
10	65,88 Hz	64,90 Hz	+ 1,5 %

Table 3 Comparison of Measured and Calculated Torsional Natural Frequencies f_j [Hz], [Source of Measurement eigenfrequencies: TVO]

As can be seen in the frequency spectrum from 0 to 70 Hz at normal operation (figure 15 and 16) the four first torsional natural frequencies could be identified very well. And the comparison with the calculated natural frequencies from the FE-Base Model is also quite good. At higher frequencies (30 – 70 Hz) the peaks could still be identified by measurements, although the signal to noise ratio is not optimal. With the exception of the calculated natural frequency 38,28 Hz with an error of 10 % all other frequencies can well be predicted by means of the FE-Base Model with deviations in the range of 1% to 3 %. As a conclusion, the FE-Base

Model predicts very well the torsional natural frequencies of the turbogenerator OL3 in the required frequency range. This statement can also be confirmed by another comparison of calculated and measured torsional natural frequencies in figure 16. In comparison to measurements figure 16 shows a clear and accurate presentation of the dynamic behaviour of the turbogenerator by means of the FE-Base Model.

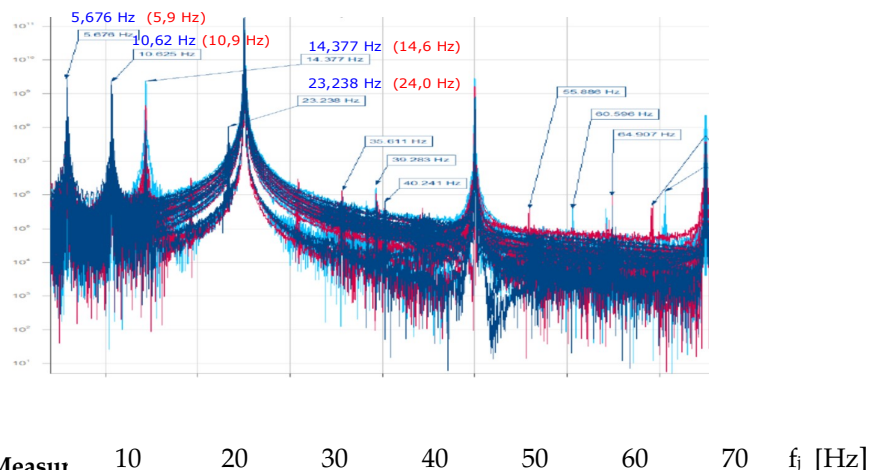


Figure 15: Measured torsional natural frequencies of the turbogenerator OL3 in the frequency range up to 100 Hz [Source: VTT / TVO]

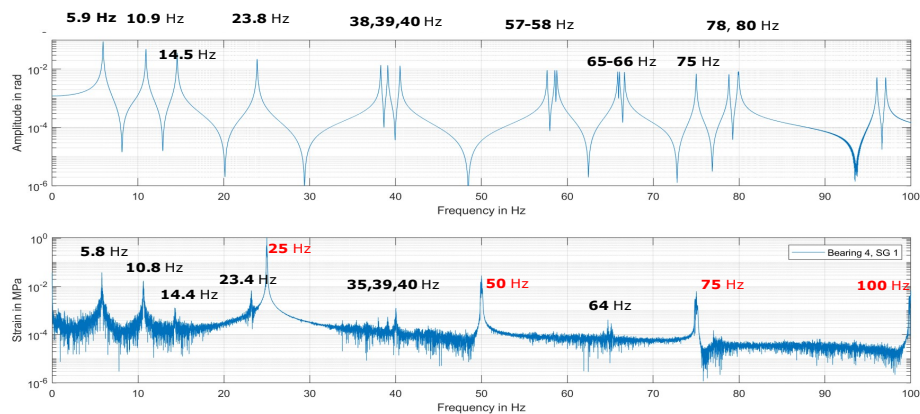


Figure 16: Comparison of Measured and Calculated Torsional Natural Frequencies f_j [Hz] of the Turbogenerator OL3 in the frequency range up to 100 Hz [Source: VTT]

3.7 DAMPING OF THE BASE MODEL

From chapter 3.4 we learned, that the torsional natural frequencies ω_j and the corresponding mode shapes φ_j can be calculated very accurately, if only the mass matrix $\mathbf{M}$ and the stiffness matrix $\mathbf{K}$ are known. The damping matrix $\mathbf{D}$ is not needed for the determination of ω_j and φ_j , because the damping of turbogenerators is very small and has nearly no influence on the modal parameters ω_j and φ_j . However, damping becomes important when torsional vibrations of a turbogenerator due to air gap torque excitations have to be investigated. In case of

resonances, particularly in the case of Sub Synchronous Resonance (SSR), positive damping of any size helps to stabilize the vibrations. Damping should therefore in any case be considered, when forced torsional vibrations are investigated.

The damping of the combined mechanical-electrical vibration system consists of two parts: the **mechanical damping** of the turbogenerator shaft train and the **electrical damping** from the generator and the grid. It is well known that the mechanical damping (material, steam, blade joints) of the shaft train is small compared to the electrical damping. It has also to be considered, that the mechanical damping as well as the electrical damping depend on the electrical load of the turbogenerator.

In chapter 3.8 it will be shown, how forced torsional vibrations due to time dependent air gap torques can be calculated by means of the numerical Modal Analysis. The Modal Analysis procedure decouples the originally coupled Multi Degree Of Freedom (MDOF) equation system (8) of order N into N Single Degree Of Freedom (SDOF) systems, each consisting of a modal mass, a modal stiffness a modal damping and a modal excitation vector. While modal mass and modal stiffness can easily be determined by means of the orthogonality relations of the mode shapes ϕ_j (chapter 3.8), the determination of modal damping needs experimental support via measurements. This can be done either in the time domain, considering the decay rate of transient natural vibrations or in the frequency domain by the method of the Half Power Bandwidth (HPB).

In figures 15 and 16 Frequency Response Functions (FRFs) can be seen for the turbogenerator between two positions, e.g. between the air gap (input) and one of the SSR sensors (output, see also chapter 4.1). If the resonances or the torsional natural frequencies in the FRFs are not too close to each other, each resonance peak can approximately be considered as a peak of a SDOF-system, and we can apply the Half-Power Bandwidth method for the identification of the modal damping D. The procedure for a SDOF vibration system is explained in figure 17. If a Frequency Response Function $V = \hat{q} / \hat{F}$ of a general SDOF system has been measured, we can determine the resonance frequency ω_j and the maximum value $V_{\max}$ at this frequency. Based on these values the frequency difference $\Delta\omega$ can be determined at the amplitude of the FRF: $V_{\max}/\sqrt{2}$. By some theoretical derivatives it follows, that the modal damping D can be determined by the simple equation:

$$D = \Delta\omega / 2\omega_j \quad (3.6)$$

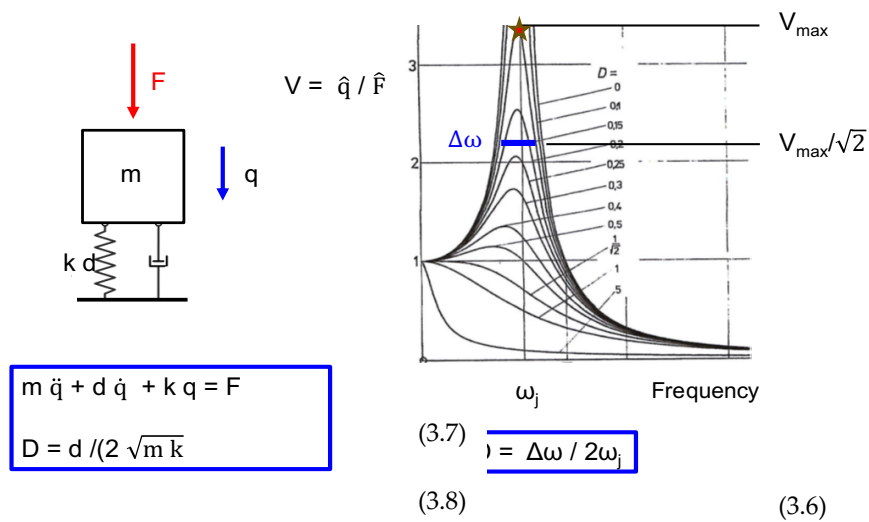


Figure 17: Half Power Bandwidth Method for the Determination of the Modal Damping D
[Source: R. Nordmann, Machine Dynamics]

The described procedure of the HPB-method has been applied for the four lowest torsional natural frequencies $f_1 - f_4$ of different frequency spectra, measured by VTT at the sensor locations of the OL3 turbogenerator (see chapter 4.1). The data were taken for the test cases Turbine trip 0 MW, Generator load 850 MW, Power ramp down 1040 MW and Power ramp up 1510 MW.

The measured values of the modal damping as function of the power of the turbogenerator are presented in figure 18 and table 3 for the four lowest torsional natural frequencies $f_1 - f_4$ in Hz. They are in the range of $D = 4,9 \text{ e-4}$ to $27,9 \text{ e-4}$ or in percentage $D [\%] = 0,049$ to $0,279$ %. These values are very low. The diagram shows an increase of the modal damping with power. However, a decrease of the damping values at around 1000 MW was not expected and could not be explained up to now. This needs additional investigations. However, at this stage the damping values in figure 18 and table 3 were used for the further analysis with the base model.

Since the quality of the measured frequency spectra data was not very good in the higher frequency range, the evaluation of modal damping values was not satisfying. Therefore, for all higher modal damping values, an estimated value of $D = 0,001$ or in percentage $D [\%] = 0,1$ % was assumed for the Base Model.

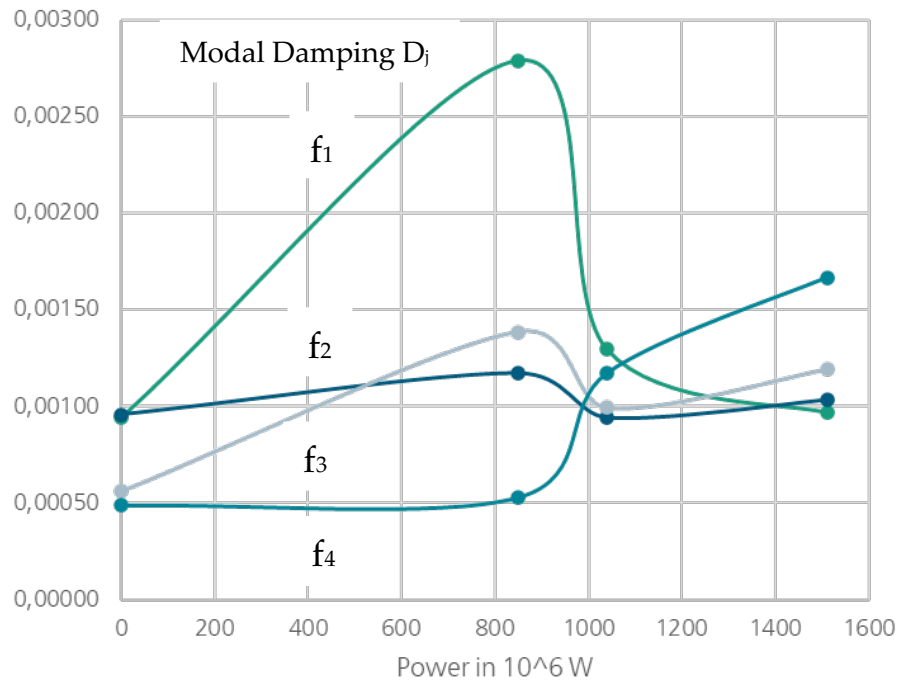


Figure 18: Modal Damping D versus Power for the Torsional Frequencies f1 -f4

$f_1 = 5,67 \text{ Hz}$ $f_2 = 10,62 \text{ Hz}$ $f_3 = 14,37 \text{ Hz}$ $f_4 = 23,24 \text{ Hz}$

Power				
0	0,00094	0,00096	0,00056	0,00049
850	0,00279	0,00117	0,00139	0,00053
1040	0,00130	0,00094	0,00099	0,00117
1510	0,00097	0,00103	0,00119	0,00167

Table 3: Modal Damping D versus Power for the Torsional Frequencies f1 – f4

3.8 FORCED TORSIONAL VIBRATIONS WITH THE BASE MODEL

In this chapter, it will be shown, how forced torsional vibrations due to time dependent air gap torques can be calculated by means of the numerical Modal Analysis method. We start with the coupled equations of motion (3.9) for the turbogenerator, which have already been described in chapters 1 and 3.

$$\mathbf{M} \ddot{\mathbf{q}}(t) + \mathbf{D} \dot{\mathbf{q}}(t) + \mathbf{K} \mathbf{q}(t) = \mathbf{M}_{el}(t) \quad (3.9)$$

The Modal Analysis procedure decouples the **Multi Degree Of Freedom** (MDOF) equation system (3.9) of order N (N number of degrees of freedom) into N **Single Degree Of Freedom** (SDOF) systems, each one consisting of a modal mass, a modal stiffness, a modal damping and a modal excitation vector. This decoupling of the equations of motion can be achieved by a development of the torsional displacement vector $\mathbf{q}(t)$ in terms of the eigenvectors $\boldsymbol{\varphi}_n$ of the system without damping (see chapters 1.3 and 3.4 and equation (3.10)).

$$\mathbf{q}(t) = \sum_{n=1}^N p_n(t) \boldsymbol{\varphi}_n \quad (3.10)$$

We introduce equation (3.10) into equation (3.9), multiply from the left side with a transposed eigenvector $\boldsymbol{\varphi}_k^T$ and obtain the following equation (3.11)

$$\sum_{n=1}^N \{ \boldsymbol{\varphi}_k^T \mathbf{M} \boldsymbol{\varphi}_n \ddot{p}_n(t) + \boldsymbol{\varphi}_k^T \mathbf{D} \boldsymbol{\varphi}_n \dot{p}_n(t) + \boldsymbol{\varphi}_k^T \mathbf{K} \boldsymbol{\varphi}_n p_n(t) \} = \boldsymbol{\varphi}_k^T \mathbf{M}_{el}(t) \quad (3.11)$$

In the theory of vibrations, it is shown that due to the orthogonality character of the eigenvectors $\boldsymbol{\varphi}_n$ the following eigenvector products are defined

$$\boldsymbol{\varphi}_k^T \mathbf{M} \boldsymbol{\varphi}_n = m_k \text{ for } k = n \quad \text{and} \quad \boldsymbol{\varphi}_k^T \mathbf{M} \boldsymbol{\varphi}_n = 0 \text{ for } k \neq n \quad (3.12)$$

$$\boldsymbol{\varphi}_k^T \mathbf{K} \boldsymbol{\varphi}_n = k_k \text{ for } k = n \quad \text{and} \quad \boldsymbol{\varphi}_k^T \mathbf{K} \boldsymbol{\varphi}_n = 0 \text{ for } k \neq n \quad (3.13)$$

where m_k is the so called modal mass and k_k is the modal stiffness, both of them belong to the natural torsional eigenfrequency f_k or ω_k (subscript k). If the assumption is made, that the damping matrix $\mathbf{D}$ is proportional to $\mathbf{K}$ and $\mathbf{M}$ (Rayleigh-Damping), then the following relations are also true for the damping

$$\boldsymbol{\varphi}_k^T \mathbf{D} \boldsymbol{\varphi}_n = d_k \text{ for } k = n \quad \text{and} \quad \boldsymbol{\varphi}_k^T \mathbf{D} \boldsymbol{\varphi}_n = 0 \text{ for } k \neq n \quad (3.14)$$

With the introduced modal quantities m_k , d_k , k_k we can now write down the N decoupled SDOF equations, which belong to the different natural frequencies ω_k

$$m_k \ddot{p}_k(t) + d_k \dot{p}_k(t) + k_k p_k(t) = \boldsymbol{\varphi}_k^T \mathbf{M}_{el}(t) \quad (k = 1, 2, 3, \dots) \quad (3.15)$$

On the right-hand side of each SDOF equation the scalar vector product $\boldsymbol{\varphi}_k^T \mathbf{M}_{el}(t)$ describes the time dependent excitation. By means of this vector product it can be

found out how strong the torsional vibrations can be excited in a resonance at ω_k . The other important factor in a resonance is the damping. For the damping, we can introduce the modal damping values from chapter 3.7. The relation between the modal damping D_k and the damping d_k from equation 3.15 is

$$D_k = d_k / (2 m_k \omega_k) = d_k / 2 (\sqrt{k_k m_k}) \quad (3.16)$$

When the equations 3.15 as SDOF equations have been solved, the results $p_k(t)$ can be introduced in equation 3.10 and the solution for the complete vector $\mathbf{q}(t)$ with the torsional displacements can be obtained.

As an example, in figure 19 the torsional response amplitudes in rad are shown for the location of the sensor position 8 between the LPT3 and the generator (see figure 23 in chapter 4.2), when the OL3 turbogenerator is excited by a harmonic unit air gap torque in the frequency range between 0 and 100 Hz. In figure 19, we clearly see the different modal contributions of the different natural frequencies and modes. The resonance peaks are relatively sharp due to the very low damping values.

With component 1, the Finite Element Model of the turbogenerator, the most important component of the Digital Twin has been developed and can be used.

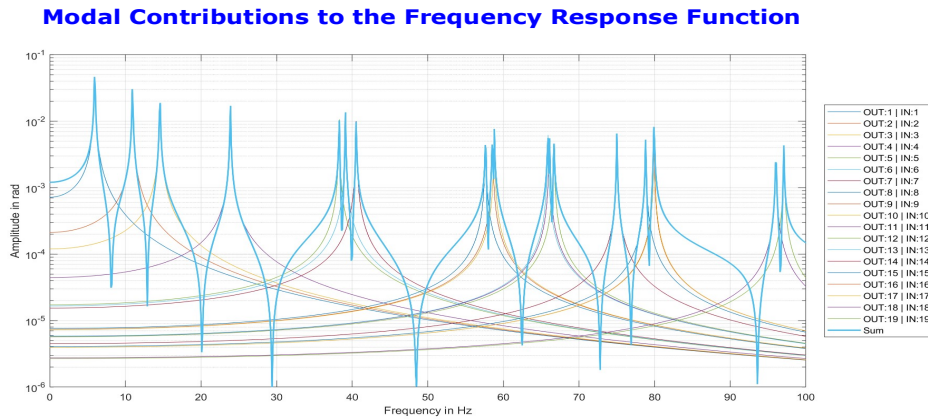


Figure 19: Torsional Response Amplitudes [rad] due to Harmonic Unit Air Gap Torque

4 The Comparison Point of the Digital Twin (Component 2)

The second component of the Digital Twin is the Comparison Point 2. The comparison point has two inputs. The first input contains measured data from the real system of the turbogenerator and the second input delivers corresponding data from the Finite Element Model. The difference of the two inputs is the output of the comparison point. This difference will be transferred to the components 3 and 4 for calibration (Figure 20) of the Finite Element Model (Model parameters and excitation).

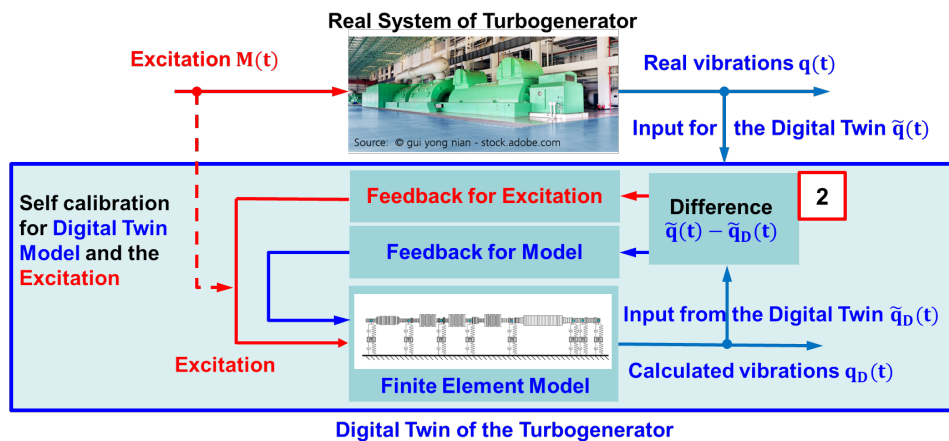


Figure 20: Comparison Point of the Digital Twin for measured and calculated vibrations

4.1 THE MEASURED TORSIONAL VIBRATIONS OF THE SHAFT TRAIN

The measurement of torsional vibrations at the real turbogenerator system is performed outside of the Digital Twin (Figure 20). It is therefore not a part of this research project. However, some comments are presented here to understand, how the Digital Twin could work in a real Nuclear Power Plant environment including the measurement part.

For the measurement part outside the Digital Twin, a Data Logging system can be used. It is an automated tool to collect, store and analyse data. The key components of a Data Logging system are:

Sensors: To detect and measure physical properties such as torsional vibrations of the turbogenerator shaft train or stresses of the material.

Data Logger: A device that collects data from sensors and stores these data. Data Loggers can be integrated into other systems.

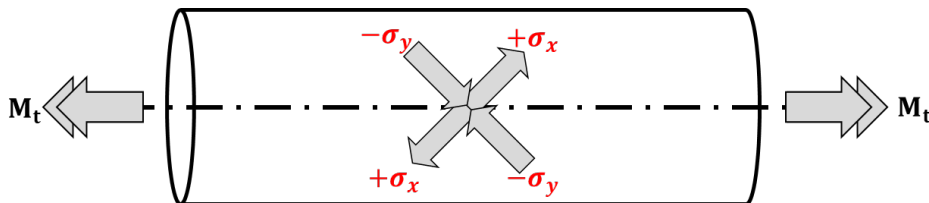
Storage: The data can be stored internally in the logger or transferred to external storage like a computer.

Software: Used for the configuration the data logger, visualizing and analysing the collected data.

A Data Logging system is therefore an important tool to monitor the vibration data of a turbogenerator, particularly also for the very sensitive torsional vibrations. In the context of the Digital Twin for turbogenerators, the Data Logging system is a very important tool in collecting real time torsional vibration data from the sensors. This data will be transferred to the comparison point of the Digital Twin and will be used to adjust the Digital Twin. Continuous data collection ensures that the Digital Twin accurately reflects the current state of the real turbogenerator.

The Data Logging system is not a part of this project. Therefore, we will not describe the system in more detail. However, in different reports from VTT, more information about the Data Logging system for the OL3 unit can be found.

During the commissioning phase of the turbogenerator OL3, the permanently installed SSR sensors from Siemens were used to measure torsional vibrations. In Figure 21, some details are explained about this sensor type. It uses the anisotropic magnetostrictive measuring principle, where stresses at the material surface can be observed in a change of magnetic permeability. High stresses and frequencies up to around 200 Hz can be measured in this way. In addition, strain gauges were installed by VTT for the commissioning phase only. They are no longer available for future measurements.



The **Touchless Torque Sensor** measures the static and dynamic torques (stresses) at the rotating turbogenerator shaft train. It is based on the influence of **mechanical stresses** on the **magnetic permeability** of ferromagnetic materials.

Figure 21: Monitoring of Torsional Vibrations with the SSR Torque Sensors – Siemens
[Source: Ingo Balkowski, Siemens: Direct Touchless sensing of torsional vibration stresses in Power Plants, EF Vibration Seminar 2023]

The location of the two sensor types during the commissioning phase are shown in figure 23. The strain gauges (blue arrows) have been installed at the locations 4, 6 and 8 and the SSR sensors (green arrows) at the locations 3, 5 and 8. The probably most important sensor point is 8, which is between the LPT3 and the Generator. Examples of measured torsional vibrations have been presented in the frequency domain in figures 15 and 16 (see chapter 3.6).

4.2 THE CALCULATED TORSIONAL VIBRATIONS OF THE SHAFT TRAIN

Calculated torsional vibrations at the corresponding measurement points (sensor locations) are the second input for the comparison point. How the torsional vibrations of the turbogenerator shaft train can be determined by calculations with the Finite Element Model has been demonstrated in detail in chapter 3. This is possible for all locations of the Finite Element Model. In Figure 23, the essential calculation possibilities are highlighted again.

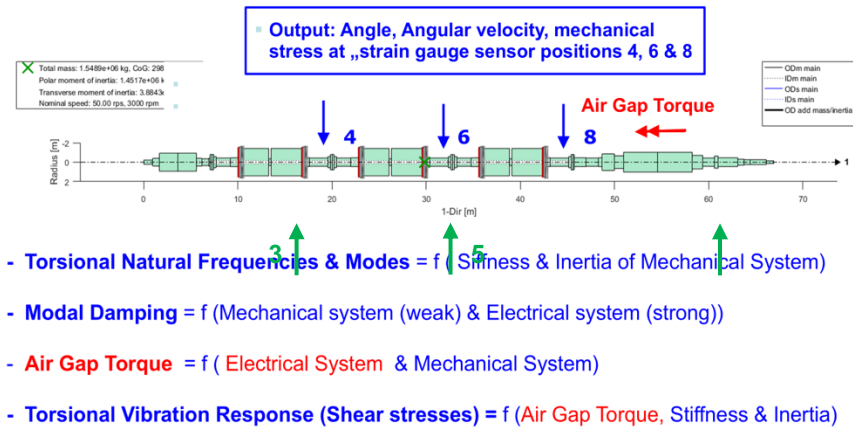


Figure 22: Calculation of Torsional Vibrations by means of the Digital Twin FE-Mode

4.3 DIFFERENCE OF MEASURED AND CALCULATED VIBRATIONS

As has been shown in figure 20 at the comparison point 2 of the Digital Twin (Comparison point), the measured and calculated torsional vibrations can be compared for locations of the shaft train, where vibration sensors are installed (see figures 22 and 23). For this comparison, measured and calculated values should have the same physical quantity, e.g. torsional displacements, strains or stresses of the shaft train. Due to the fact, that the installed sensors measure shaft stresses or strains, relations have to be known between the different physical quantities.

The conversion from torsional strain ε to shear stress τ and the shaft torque M_t can be calculated with the following equations:

$$\text{Shear (Torsional) Stress} \quad \tau = M_t / W_p \quad (4.1)$$

$$\text{Torsional Moment of Resistance} \quad W_p = \pi D^3 / 16 \quad (4.2)$$

$$\text{Torsional Strain} \quad \varepsilon = (1 + \nu) M_t / (E W_p) \quad (4.3)$$

ν , E and D are the poisson's ratio, the Young's modulus and the diameter of the shaft.

5 The Calibration of the Mass and Stiffness Matrices of the Shaft Train (Component 3)

The calibration of mass and stiffness parameters of the turbogenerator is based on measured and calculated torsional vibrations. The differences between the calculated and the measured torsional natural frequencies indicate possible changes of masses or stiffness values. A procedure is described, how these frequency differences are used for the calibration.

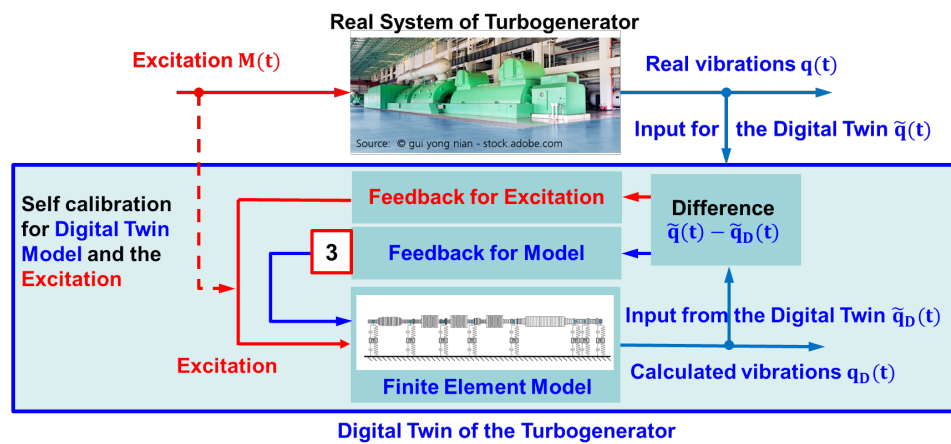


Figure 23 Component 3 of the Digital Twin Calibration of Mass and Stiffness

5.1 SENSITIVITY ANALYSIS FOR THE TORSIONAL NATURAL FREQUENCIES

In chapter 1.3 it has been shown, that the torsional natural frequencies ω_i depend on the mass matrix $\mathbf{M}$ and on the stiffness matrix $\mathbf{K}$. From this it can be concluded that a change of the natural frequencies can be considered as some change of mass and/or stiffness parameters. For the introduced Base FE Model at normal operating conditions, the system matrices $\mathbf{M}$ and $\mathbf{K}$, the natural frequencies ω_i and the mode shapes $\boldsymbol{\varphi}_i$ are known from the design, from measurements during the commissioning phase and from latest measurements during normal operation. Therefore, the question arises how possible parameter changes $\Delta\mathbf{M}$ and /or $\Delta\mathbf{K}$ can be determined from measured deviations of the torsional natural frequencies related to the Base Model for normal operation. This calibration task is performed in the component 3 of the Digital Twin (see figure 24). It is based on a Sensitivity Analysis for the torsional natural frequencies. The idea of the calibration is, that the torsional natural frequencies will change more or less, when the system parameters like mass and stiffness values will change.

The eigenvalue problem from chapter 1 is repeated in (5.1) for the natural frequency ω_i and described with the substitution $\lambda_i = \omega_i^2$ in equation (5.2). This eigenvalue equation can be derived according to any system parameter p_k , where

p_k can be a mass parameter or a stiffness parameter as well. The result of this partial differential derivation is shown in formula (5.3).

$$(\mathbf{K} - \omega_i^2 \mathbf{M}) \cdot \boldsymbol{\varphi}_i = \mathbf{0} \quad (5.1)$$

$$(\mathbf{K} - \lambda_i \mathbf{M}) \cdot \boldsymbol{\varphi}_i = \mathbf{0} \quad (5.2)$$

$$\lambda_{i,k} = \boldsymbol{\varphi}_i^T (-\lambda_i \mathbf{M}_{,k} + \mathbf{K}_{,k}) \boldsymbol{\varphi}_i \quad (5.3)$$

As a sign for the partial derivative of a quantity (shown here as braces) according to p_k we have used the short form, shown in (5.4)

$$\partial(\) / \partial p_k = (\)_{,k} \quad (5.4)$$

In equation (5.3) it is also assumed, that the eigenvectors are normalized to 1 corresponding to equation (5.5)

$$\boldsymbol{\varphi}_i^T \mathbf{M} \boldsymbol{\varphi}_i = 1. \quad (5.5)$$

The partial derivation of an eigenvalue λ_i according to a system parameter p_k in (5.3) is called the Sensitivity, which can be expressed as

$$S_{ik} = \lambda_{i,k} = \partial(\lambda_i) / \partial p_k. \quad (5.6)$$

The change of an eigenvalue λ_i due to changes of a set of system parameters p_k can be expressed by a Taylor series. A good result can often be achieved with a linear approximation (5.7). The Δ quantities for the eigenvalues and the parameters indicate the deviations related to the normal operation

$$\Delta \lambda_i = \lambda_{i,1} \Delta p_1 + \lambda_{i,2} \Delta p_2 + \lambda_{i,3} \Delta p_3 + \dots + \lambda_{i,K} \Delta p_K \quad (5.7)$$

If we consider several eigenvalues ($i = 1, 2, \dots, I$) and several parameters ($k = 1, 2, \dots, K$) the relations between all changed eigenvalues and the changed parameters can be expressed by the two vectors $\Delta \boldsymbol{\lambda}$ for the I eigenvalues and $\Delta \mathbf{p}$ for the K system parameters. The $I \times K$ matrix $\mathbf{S}$ contains the sensitivities (see 5.3 and 5.6)

$$\Delta \boldsymbol{\lambda} = \mathbf{S} \Delta \mathbf{p} \quad (5.8)$$

From the measured eigenvalue changes $\Delta \boldsymbol{\lambda}$ and the known sensitivity matrix $\mathbf{S}$ for the normal operation condition, the parameter changes $\Delta \mathbf{p}$ can finally be determined by an inverse calculation.

5.2 THE CALBRATION PROCEDURE FOR MASS AND STIFFNESS

The calibration procedure, which has been implemented at the Fraunhofer Institute is shown in the Block diagram in figure 25. It describes the iterative procedure to find out mass and stiffness parameter changes in case of changes in the measured spectrum of the natural frequencies.

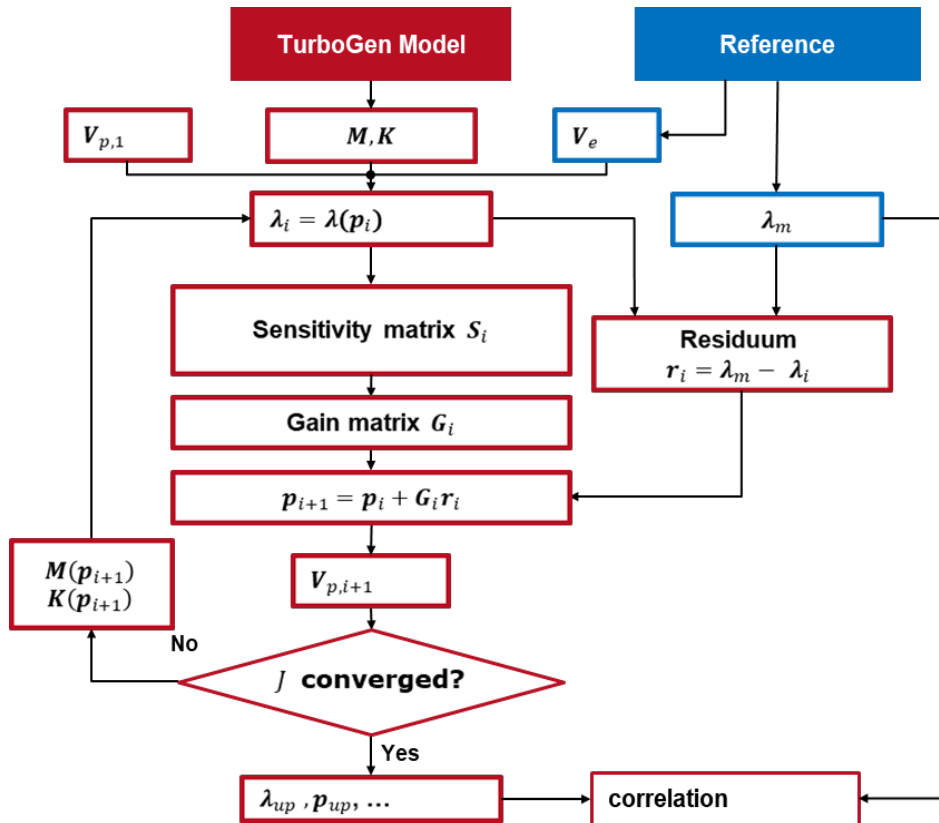


Figure 24: Calibration Procedure for Mass and Stiffness Parameters of the Turbogenerator

6 The Calibration of the Modal Damping (Component 3)

Besides the possible changes of mass and stiffness values during operation of the turbogenerator, the modal damping values may also show variations due to different effects: Power change, disturbances in the electrical system, SSR, etc. In such cases, the modal damping has to be calibrated as well during operation (Component 3 in Figure 24).

6.1 THE HALF-POWER-BANDWIDTH METHOD

The Half Power Bandwidth method to determine the modal damping has already been explained in chapter 3.7. The same method will also be applied, when the modal damping of the different mode shapes is changing during the operation compared to the Base Model (see figure 26)

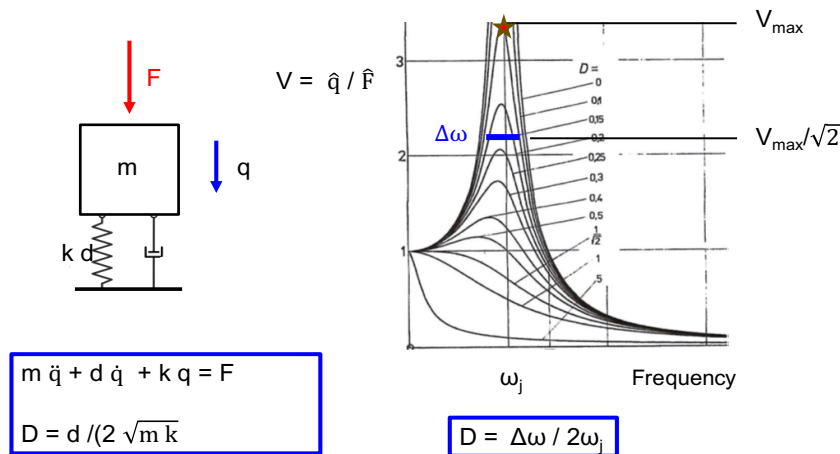


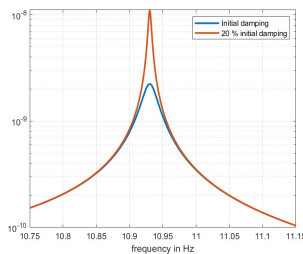
Figure 25: Half Power Bandwidth Method for the Determination of the Modal Damping D [Source: R. Nordmann]

6.2 THE CALIBRATION FOR THE MODAL DAMPING AT OPERATION

During operation of the turbogenerator the modal damping values can be checked continuously by using the Half Power Bandwidth method for the actual measured sensor signals, presented in the frequency spectra. It is important to note again, that the modal damping consists of two parts: mechanical damping and electrical damping with a dominance of the electrical part. The cause of changes of modal damping during operation of the turbogenerator will therefore in the most cases an electrical event. The variation of the Power is one example, where modal damping changes. Another very important case is an SSR-event, which may lead to a strong variation of the damping. In a worst case the modal damping may also become negative, which means instability for torsional vibrations of the shaft train.

As an example, for the calibration of the modal damping during operation the procedure has been tested with a simulated measurement signal, in which the modal damping of the 2nd torsional natural frequency at 10,9 Hz has been reduced to 20 % of the original damping $D = 0,001$ ($D [\%] = 0,1 \%$, figure 27).

- Modal damping of the torsional natural frequency at 10,9 Hz has been reduced in a simulated test signal (measurement) to 20 % of the original value $D = 0,001$ (Figure 18)
- Using halfpower-bandwidth method for determination of the changed modal damping
- Digital Twin calculates the correct change in damping for the simulated test signal



Bearing	Initial damping	Calculated damping (initial)	Changed damping	Calculated damping (reduced)
4	0,001	0,001	0,0002	0,00021
6		0,001		0,00021
8		0,001		0,00021

Figure 26: Calibration of the Modal Damping for the 2nd Torsional Natural Frequency

By applying the half power bandwidth method according to the generated test signal, the changed modal damping value could be identified very well, leading to the new value of $D = 0,00021$ ($D [\%] = 0,021 \%$).

7 Calibration of the Air Gap Torques (Component 4)

The change of the torsional vibration behaviour of the turbogenerator can be caused either by parameter changes ΔM , ΔK and ΔD or by a change the Excitation, e.g. by the Air Gap Torque. In this chapter we describe, how the identification of a change of the Air Gap Torque can be recognized, based on the measured sensor signals (Component 4 in Figure 27).

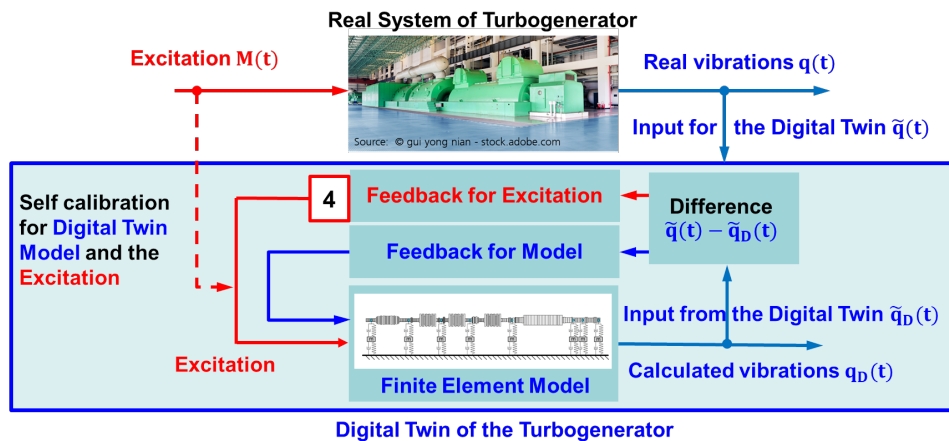


Figure 27 Component 4 of the Digital Twin: Calibration of Air Gap Excitation

7.1 COMPARISON OF CALCULATED AND MEASURED SHAFT STRESSES

It has to be mentioned again, that shaft stresses are measured by means of the SSR sensors. Therefore, at the Comparison Point the calculated values by means of the FE Model have also to be shaft stresses. Equations (4.1 – 4.3) in chapter 4.3 are used to calculate the shaft stresses with the FE-Model.

7.2 THE CALIBRATION PROCEDURE FOR THE AIR GAP TORQUE

One possibility to identify the actual Air Gap Torque during normal operation or in a special excitation event (Short circuit, negative sequence current, SSR, ...) could be via measurements of electrical quantities, which influence the Air Gap Torque (see equation 1.1 in in chapter 1.1). In the Digital Twin development another procedure was used, which works with the measured and calculated shaft stresses, which are compared at the Comparison Point 2 (see Figures 20 and 27).

The Calibration procedure for the Air Gap Torque is shown in figure 28. With the FE Base Model the dynamic shaft torques are calculated for each Finite Element of the turbogenerator in the frequency range between 0 and 100 Hz, when the Air Gap Torque excites the shaft train with a Unit Torque Amplitude of 1 kNm.

- Excitation Torque in the Air Gap of the OL3 Turbogenerator

Procedure: **Input:** Harmonic Torque with unit amplitude from 0 to 100 Hz

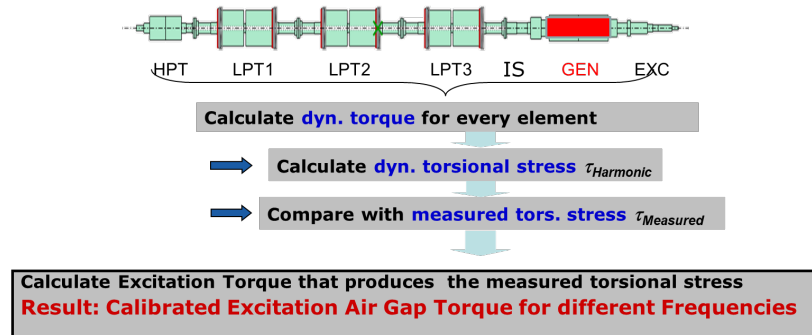


Figure 28 Component 4 of the Digital Twin: Calibration of the Air Gap Torque

From the calculated dynamic shaft torques the dynamic torsional stresses can be determined for each element by means of equation 4.1 and 4.2, especially for those elements, where the sensors are located. The calculated dynamic stresses are then compared with the measured stresses. Based on this comparison the Excitation Air Gap Torque will be adjusted corresponding to the measured torsional stress. As a result, the calibrated Air Gap Torque will be obtained for all frequencies in the range of 0 to 100 Hz.

In chapter 10 it will be shown, how the Air Gap Torque for the two cases of a Two-Phase Short Circuit and a Negative Sequence Current can be determined by the described procedure.

8 Digital Twin - a Continuous Monitoring System for the Turbogenerator

The developed Digital Twin for Torsional Vibrations of Turbogenerators can be used as an extended Monitoring System, that continuously observes the dynamic torsional behaviour during operation and at special events. The produced data of the FE-Model can be presented at all locations of the Turbogenerator system (Figure 29).

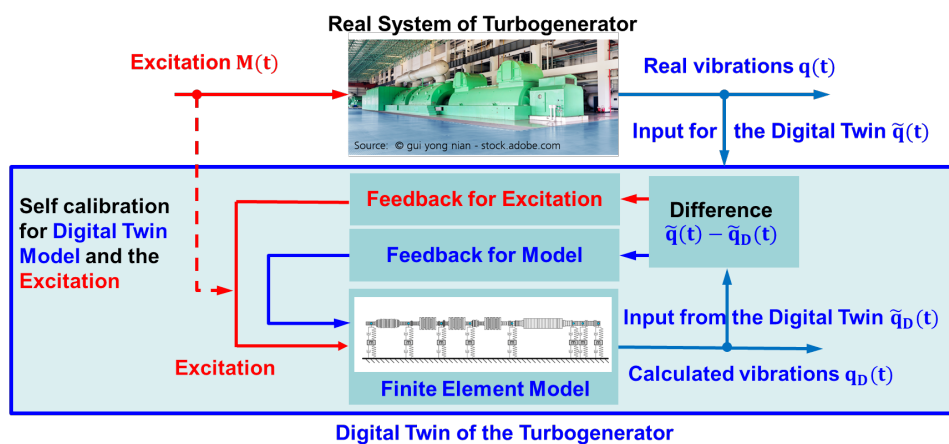


Figure 29: Digital Twin as a Continuous Monitoring System

8.1 VIBRATIONS AND STRESSES AT ALL LOCATIONS- VIRTUAL SENSORS

It has been shown in the chapters before, that the Digital Twin can be used as a continuously working Monitoring System, that delivers torsional vibrations, torsional shaft torques and torsional stresses at each location of the turbogenerator (Virtual Sensor) during normal operation and at special vibration events. Results can be presented in the time domain as well as in the frequency domain.

If the Digital Twin is not connected to the real system via the Comparison Point 2, it can also be used as a standalone system. In this mode of operation, the Digital Twin can for example investigate measures for improvements of the dynamic behaviour.

9 Identification of System Parameter Changes

In this chapter, two cases are discussed, how changes of system parameters (mass, damping, stiffness) can be identified by means of the Digital Twin. In the first example, modal damping is identified. The second example shows, how stiffness parameter changes can be determined. The less likely change of masses in turbogenerators is not considered in this chapter.

9.1 EXAMPLE 1: CHANGE OF ONE MODAL DAMPING, SSR-EVENT

In this example, a simulated measurement is shown in the time domain and in the frequency domain for a change of the modal damping in the 3rd mode with the torsional natural frequency of 14,6 Hz. The modal damping for normal operation has been identified as $D = 0,001$ (see figure 18) and it is then reduced to about 20 % of the original value. Such a reduction of the modal damping may for example occur in case of a Sub Synchronous Resonance (SSR). With the measurement signal from one of the SSR sensors the change of the modal damping can be identified by means of the Half Power Bandwidth method. The influence of the lower damping can then be used to investigate additional calculations with the FE-Model.

Figure 30 shows the system response for the unit amplitude excitation for the two cases of the damping values in the time domain and in the frequency domain. When the damping has been identified, the Digital Twin can permanently produce such responses, which may help to explain the development of the dynamic behaviour of the turbogenerator system.

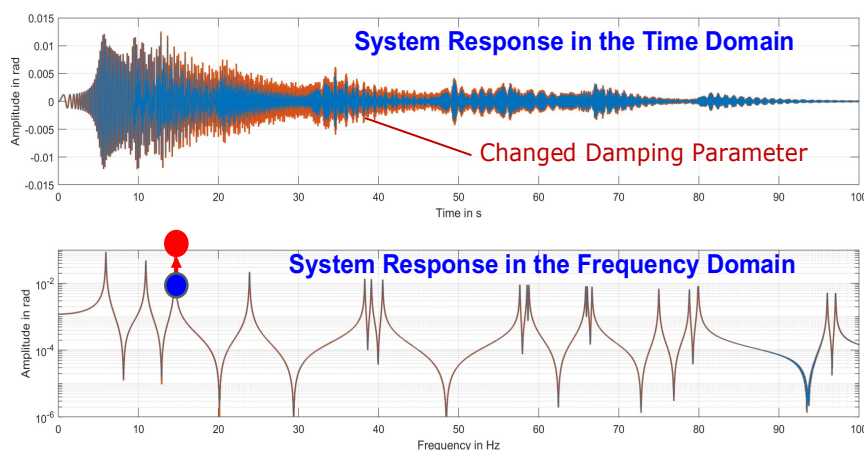
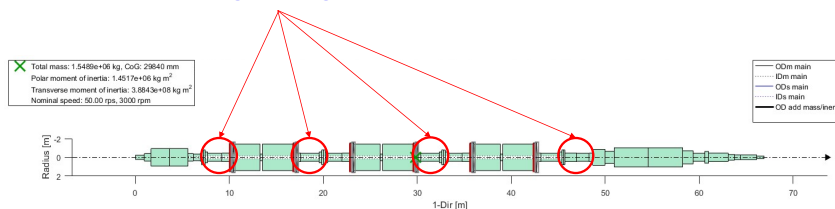


Figure 30: System Response in the Time and in the Frequency Domain for the Case of Changed Damping in the 3rd mode – Possible SSR Event

9.2 EXAMPLE 2: CHANGE OF COUPLING TORSIONAL STIFFNESS

In this second test case, the torsional stiffness of the four couplings is reduced successively by 10 %. For each of the four test cases, the change of the torsional natural frequencies of the turbogenerator is calculated with the FE Model. The changed frequency spectrum is then compared with the spectrum of the system with normal stiffness values of the couplings. This spectrum coincides with the natural frequency spectrum of the turbogenerator at normal operation (see figures 15 and 16) and can be considered as spectrum of the Base Model.

Locations for Coupling Damage



➤ Torsional Stiffness in Couplings has been reduced by 10 %

- Sensitivity based approach is used for Identification of Stiffness Change and Location
- In theory every Location can be observed for Parameter Changes

Figure 31: Locations of Couplings with reduced Torsional Stiffness

By means of the calibration procedure with the sensitivity analysis (chapter 5) the following results could be obtained for each of the four cases of stiffness changes in the coupling:

- The percentage change of the torsional natural frequencies in the frequency range between 0 and 100 Hz (see figures 32 and 33)
- The change of the coupling stiffness in percentage and the determination of the location of the stiffness change in the turbogenerator

The four test examples show that the calibration procedure for the system parameters based on the sensitivity analysis (chapter 5) works accurately and in real-time. Detailed results are presented in figures 32 and 33.

Digital Twin estimates Damage Location and Severity

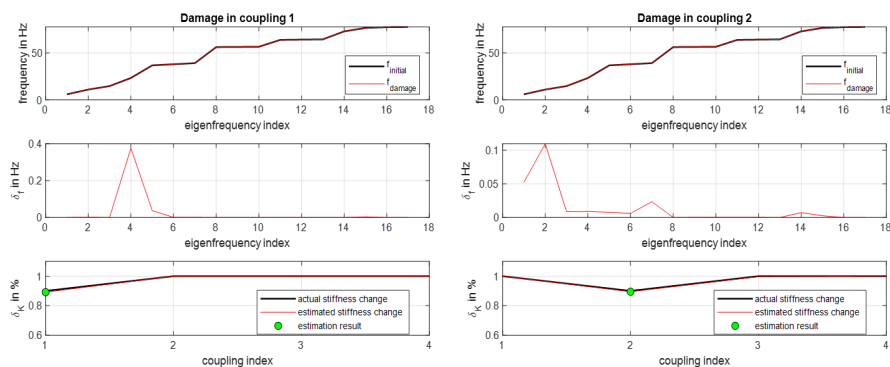


Figure 32: Deviations of Torsional Natural Frequencies, Estimated Changed Stiffness Values and location of Damage for the Couplings 1 and 2

Digital Twin estimates Damage Location and Severity

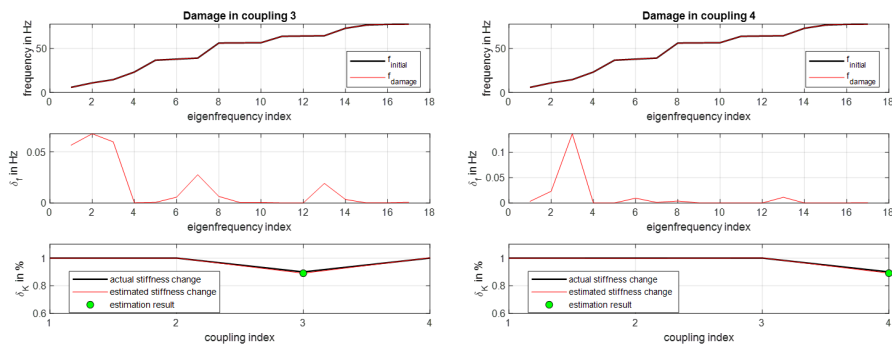


Figure 33: Deviations of Torsional Natural Frequencies, Estimated Changed Stiffness Values and location of Damage for the Couplings 3 and 4

10 Identification of the Air Gap Torque

In this last chapter, two cases are discussed, how changes of the Air Gap Torques can be identified by means of the Digital Twin. The general Calibration Procedure for the Air Gap Torque has been described in chapter 7. The first example considers the Air Gap Torque of a Two-Phase Short Circuit, the second example treats the case of the Negative Sequence Current (Unsymmetric Load). For both excitation cases no direct measurements of shaft stresses were available as input data at the Comparison Point. Due to this fact the Air Gap Torques for the two load cases were generated based on existing formula (see figures 4, 6, 33 and 35) and introduced as corresponding stresses at the input location of the Digital Twin (Comparison Point 2). In case of the Two-Phase Short Circuit the identification procedure has been tested for an input with and without noise (figures 33 and 34). The following figures 33 -35 show for the two load cases the generated Air Gap Torque as excitation time function (left side) and the identified Air Gap Torque versus frequency (0 – 100 Hz) with amplitudes in Nm (right side). The accuracy of the identified Air Gap Torques is very good (deviations 2 – 3%), when noise is not considered (Figures 33 and 35). There are some deviations in the range of 20 % when a signal to noise ratio of 10 db is considered (Figure 34).

10.1 EXAMPLE 3: CHANGE AIR GAP TORQUE DUE TO 2-PH. SHORT CIRCUIT

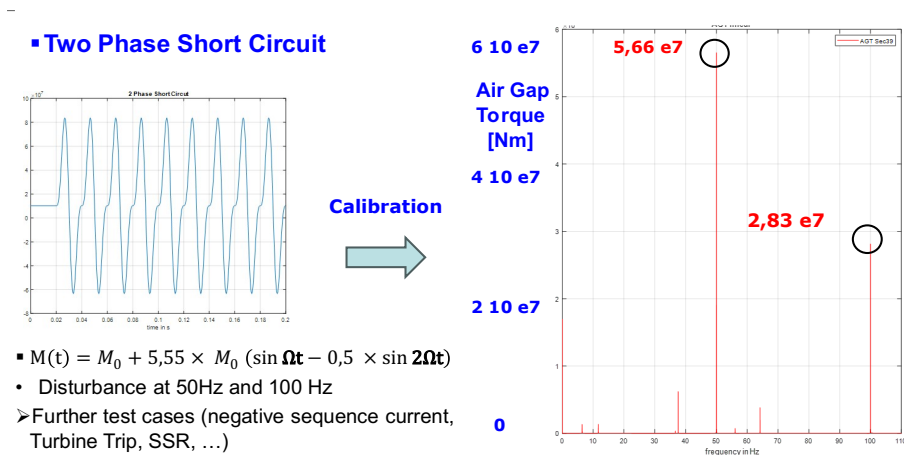
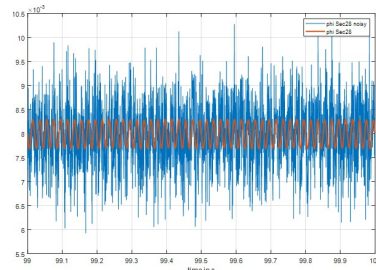


Figure 34: Calibration of Air Gap Torque in case of Two-Phase Short Circuit

Two phase Short circuit

- Excitation and results data
- with (artificial) noisy signal
- Signal to noise ratio: 10 dB



Air Gap Torque [Nm]

Calibration

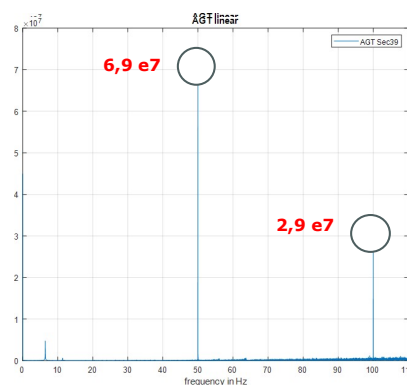
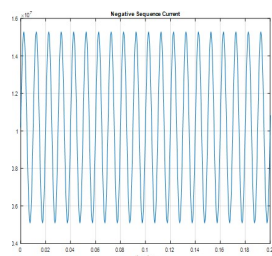


Figure 35: Calibration of Air Gap Torque in case of Two-Phase Short Circuit with Noise

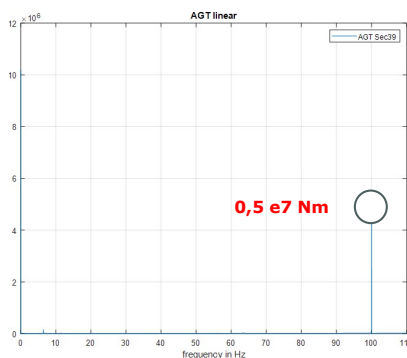
10.2 EXAMPLE 4: CHANGE AIR GAP TORQUE DUE TO UNSYMM. GRID

▪ Negative Sequence Current



Air Gap Torque [Nm]

Calibration



- $M(t) = M_0 + M_{NSC} \sin(2\Omega t)$
- $M_{NSC} = 0.5 M_0$
- Disturbance at 100 Hz

Figure 36: Calibration of Air Gap Torque in case of Negative Sequence Current

11 Conclusions and Outlook

11.1 CONCLUSIONS

This report describes the development of a Digital Twin for Torsional Vibrations of Turbogenerators in Nuclear Power Plants.

The Digital Twin works with a Finite Element Model (FEM), which can calculate torsional vibrations either as natural vibrations with natural frequencies and mode shapes or as forced vibrations due to the electro-mechanical air gap excitation. During operation the real torsional vibrations of the turbogenerator are determined at specified locations, where sensors measure torsional displacements or shear stresses. By comparison of the measured and the corresponding calculated torsional vibrations the vibration difference is introduced into a calibration loop of the Digital Twin, in which the actual system parameters of the turbogenerator as well as the actual air gap torques can be identified and adjusted.

The developed Digital Twin can be used as an online system, running permanently parallel to the real turbogenerator:

- as a monitoring system, which determines continuously the torsional vibration state of the Turbogenerator at each location of the shaft train in the time domain as well as in the frequency domain (Virtual Sensor)
- as a detection system for the identification and diagnosis in case of system failures, disturbances (Air Gap Torques) and parameter changes (mass, stiffness, damping)

The successful operation of the Digital Twin could be demonstrated by some test cases.

11.2 OUTLOOK

For the operation of the developed Digital Twin in a Nuclear Power Plant, a Data Logging system is needed to prepare the measured sensor signals as input for the Comparison Point (Component 2 of the Digital Twin). First trials to operate the Digital Twin in a plant could be started at the OL3 Turbogenerator, because OL3 is already operating with SSR-sensors and has a Data Logging system.

The developed Digital Twin can also be used in other Scandinavian Nuclear Power Plants. However, it needs an adjusted Component 1 of the Digital Twin: The FE-Modell for the respective turbogenerator of a selected plant.

DEVELOPMENT OF A DIGITAL TWIN FOR TORSIONAL VIBRATIONS OF TURBOGENERATORS

Turbogenerators, integral to nuclear power plants, comprise steam turbines that transform thermal energy into mechanical energy and generators that convert this mechanical energy into electrical energy. However, disturbances in the electrical generator-grid system can lead to significant transient torsional vibrations in the shaft train, necessitating careful monitoring for safe operation. This report outlines a Digital Twin designed to monitor and evaluate these torsional vibrations, employing theoretical modeling, numerical analysis, and experimental methods for successful simulation and validation. Utilizing a Finite Element Model (FEM), the Digital Twin accurately calculates and compares real-time torsional vibrations against theoretical predictions. This system can not only continuously monitor the vibrational state of the turbogenerator but also aid in diagnosing system failures and parameter changes. The development of this Digital Twin is particularly focused on the Olkiluoto turbogenerator OL3, with the potential for adaptation to other units through model adjustments.

A new step in energy research

The research company Energiforsk initiates, coordinates, and conducts energy research and analyses, as well as communicates knowledge in favor of a robust and sustainable energy system. We are a politically neutral limited company that reinvests our profit in more research. Our owners are industry organisations Swedenergy and the Swedish Gas Association, the Swedish TSO Svenska kraftnät, and the gas and energy company Nordion Energi.

