SHEAR FORCE CAPACITY OF EXISTING CONCRETE SLABS

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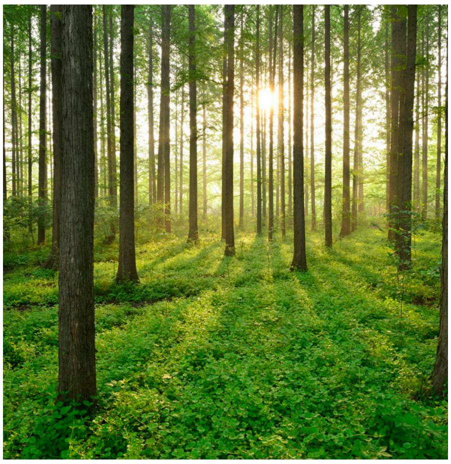

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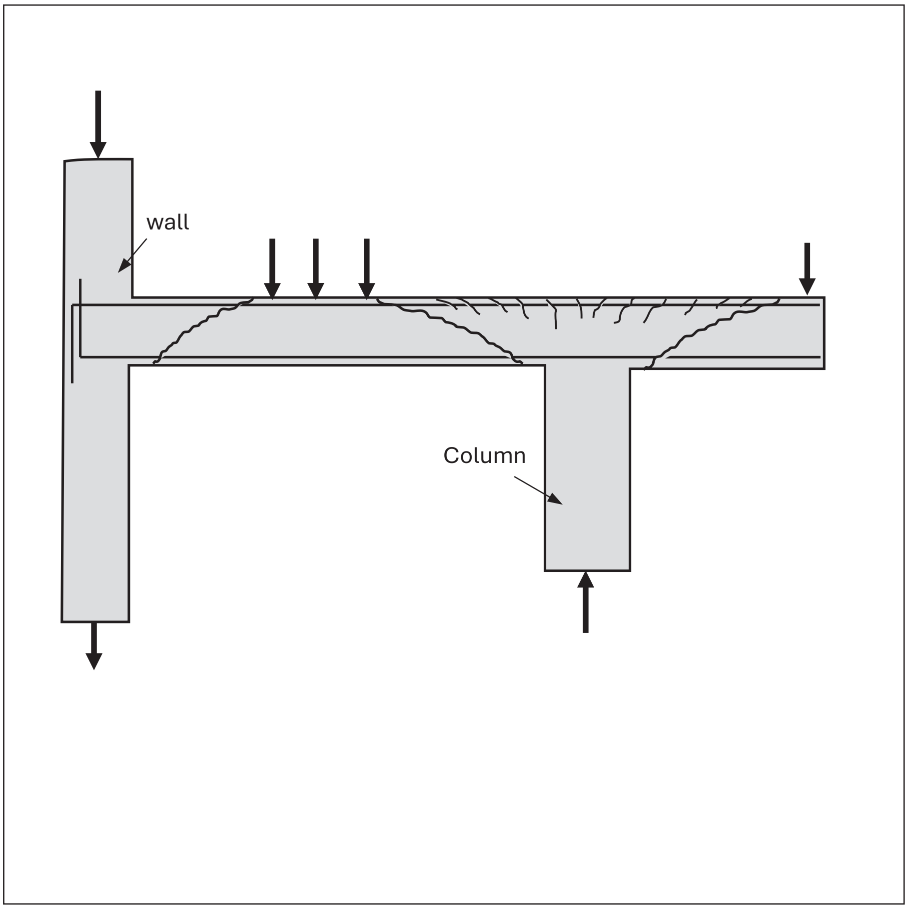

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SHEAR FORCE CAPACITY OF EXISTING CONCRETE SLABS

Study of shear force capacity of reinforced concrete slabs over the years

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Foreword

This report forms the results of a project performed within the Energiforsk Nuclear Power Concrete Program

The Energiforsk Nuclear Power Concrete Program aims to increase the knowledge of aspects affecting safety, maintenance and development of concrete structures in the Nordic nuclear power plants. A part of this is to investigate possibilities to facilitate and simplify the work that is performed in the nuclear business.

Shear force capacity in concrete slabs is crucial for structural integrity, especially in nuclear power plants where safety and reliability are paramount. In several cases low or sometimes insufficient shear force capacities have been observed in structural analyses of existing concrete slabs using modern codes. Understanding shear force capacity and how it has been treated in earlier codes helps ensure durability and safety, preventing structural failures.

This study aims to analyze shear force capacity in reinforced concrete slabs, comparing different design codes and identifying key parameters influencing shear capacity. Results show variations in shear capacity predictions across codes, highlighting the reasons for the observed relatively low shear capacities in existing concrete slabs.

The study was carried out by Lamis Ahmed, Vattenfall Power Solutions. The study was performed within the Energiforsk Nuclear Power Concrete Program, which is financed by Vattenfall, Uniper, Fortum, TVO, Skellefteå Kraft, Karlstads Energi, the Swedish Radiation Safety Authority and SKB.

These are the results and conclusions of a project, which is part of a research Program run by Energiforsk. The author/authors are responsible for the content.

Summary

This study presents a comparative analysis of shear force and punching shear capacity calculations in reinforced concrete slabs without shear reinforcement according to different design codes, including Eurocode 2 Boverkets Handbok om Betongkonstruktioner (BBK), Svensk Bygg Norm (B7), and Betonghandbok (BHB). The study highlights the strengths and limitations of each code and compares their shear force and punching shear capacity calculations separately.

Regarding shear force capacity, the results show the BBK04 method overestimates shear capacity, particularly for slabs with low longitudinal reinforcement ratios, while B7 consistently overestimates capacity, especially for slabs with moderate thickness.

Regarding punching shear capacity, the results show that BBK79 have good safety margin compared to EC2, due to its shorter effective perimeter calculation. B7 demonstrates alignment with EC2 for thicker slabs, but slightly overestimates capacity for thinner slabs. BHB aligns well with EC2 across most slab thicknesses, but deviations occur at greater depths, where BHB becomes less conservative.

The article also examines the behaviour of the compression strut angle θ in reinforced concrete beams and its influence on shear behaviour. The findings underscore the importance of considering geometric parameters and material properties in the design process and ensuring that θ remains within the prescribed range for accurate and safe shear capacity predictions.

Keywords

Shear force capacity. Punching shear capacity. Reinforced concrete slabs without shear reinforcement. Swedish design codes, BBK, B7, BHB. Effective perimeter calculation for old Swedish codes. Safety margin of old Swedish cods compared to Eurocode 2.

Sammanfattning

I föreliggande rapport presenteras en jämförande analys av beräkningar av tvärkraftskapacitet och bärförmåga avseende genomstansning i armerade betongplattor utan tvärkraftsarmering enligt olika konstruktionsnormer. De normer som jämförs är Eurokod 2 (EC2), Boverkets Handbok om Betongkonstruktioner (BBK), Svensk Byggnorm (B7) och Betonghandbok (BHB). I rapporten presenteras förutsättningarna och begränsningarna hos respektive norm samt de olika beräkningsmetoderna för tvärkraftkapacitet och genomstansning.

När det gäller beräkning av tvärkraftskapacitet visar resultaten att BBK04 överskattar kapaciteten, särskilt för plattor med låg mängd längsgående armering jämfört med metoden i EC2. Vidare visar jämförelsen att BBK04 med säkerhetsklass 3 har en bra marginal på 20%. I säkerhetsklass 1 erhålls liknande resultat med EC2 förutsatt att tryckhållfasthet understiger 18 MPa. SBN80 visar en överskattning av tvärkraftskapaciteten på 60% jämfört med EC2.

För bärförmåga avseende genomstansning visar resultaten att BBK79 har en god säkerhetsmarginal jämfört med EC2, vilket beror på antagandet om en mindre effektiv omkrets vid beräkning. B7 stämmer väl överens med EC2 för tjockare plattor men överskattar bärförmågan något för tunnare plattor. BHB överensstämmer generellt med EC2 för de flesta plattjocklekar, men avvikelserna ökar med den effektiva höjden och beräkningarna bli därför mindre konservativa.

I rapporten undersöktes även beräkning av bärförmåga för befintliga betongbalkar med tvärkraftsarmering och valet av vinkel på trycksträvan (θ) i beräkningsmodellen i EC2. Resultaten understryker vikten av att beakta geometriska parametrar och materialegenskaper i dimensioneringsprocessen. Resultaten visar även på vikten att vinkeln på trycksträvan är inom det föreskrivna intervallet (22.8° - 45°) för att säkerställa tillförlitliga beräkningar av tvärkraftskapaciteten.

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1 Introduction

1.1 INTRODUCTION

Normally, existing concrete slabs in nuclear facilities are designed without special shear reinforcement. However, at columns, where concentrated loads result in higher shear stresses, punching shear reinforcement may be provided to prevent punching failure. Experience from previous calculations has shown that it is often difficult to verify a concrete slabs shear capacity, and a contributing factor may be that earlier norms did not strictly handle shear in slabs. Other possible reasons include different models/formulas for calculating shear capacity, smaller safety margins on both capacity and load side, etc. For example, in the Swedish Concrete Association's handbook for Eurocode 2, shear from distributed load is checked for a three-sided supported slab. In the Swedish Building Regulations ((Boverkets Handbok om Betongkonstruktioner, 2004)), shear capacity for concrete slabs is described in relation to punching shear and shear at concentrated loads, but nothing is mentioned about distributed load.

It can be difficult to verify the shear capacity of a reinforced concrete slab due to various factors such as the slab's geometry, load distribution, and material properties. Furthermore, shear capacity is often influenced by complex failure mechanisms, which can be challenging to predict and analyze accurately. In D-regions, strut-and-tie models are commonly used as a design and analysis tool to account for these mechanisms.

As Swedish nuclear power plants are facing longer operating times, the risk of damage to the concrete slabs supporting the facilities increases. Due to degradation mechanisms, this can affect the slabs' ability to handle loads and, in turn, affect the shear capacity of the concrete slabs. Monitoring the condition of structural elements becomes more important, and the safety margins that exist in the original design may need to be utilized. Even changes to the facility, such as power increases, may require that the current norm (Eurocode 2) be considered applicable, which may mean that the structural integrity of slabs regarding shear cannot be demonstrated with conventional methods. In that case, either more sophisticated methods need to be used or costly reinforcements need to be carried out.

1.2 BACKGROUND

At the beginning of 2011, the European Standard Eurocode became mandatory for the design of supporting structures in Sweden, replacing the previous Swedish handbook BBK 04 (Boverkets Handbok om Betongkonstruktioner, 2004) . Eurocode 2 (EC2), which deals specifically with concrete structures, is the current standard in Sweden. It provides rules and guidelines in short sentences and equations, complemented by informative figures to assist users. According to the Swedish standards institute (SIS), the goal of establishing a unified standard is to facilitate cooperation between structural engineers from different countries all over Europe. A common technical language will increase the opportunity for the exchange of knowledge and services between countries. This is particularly important in the building industry, where differences in standards and practices can create barriers to cross-border collaboration.

In Sweden, Boverket and its predecessors in the construction sector, Byggnadsstyrelsen and Planverket, have issued various regulations over the years. These regulations have taken on different forms, including binding rules, general advice, instructions, and recommendations, as summarized in. Some regulations have specified strict requirements, detailing exactly what must be achieved, while others have set functional requirements, allowing for multiple approaches to meet the desired outcomes. When new rules are introduced, transitional provisions are typically provided, permitting continued use of the old rules for a specified period. The new regulations clearly define the applicable transitional provisions and their duration.

The evolution of building regulations in Sweden began with the 1947 Building Code, BABS 46, issued by Kungl. Byggnadsstyrelsens in 1946. This was followed by the 1960 Building Code, which introduced uniform building regulations across the country, abolishing local ordinances. Consequently, the Royal Building Board issued new Instructions for the Building Code, BABS 1960. In 1968, the Swedish Building Norm 67, along with the associated BABS 1967, replaced BABS 1960. These regulations were designed to emphasize functional requirements and coordinate all house-building regulations. Over time, several amendments and additions were made to these regulations.

In 1987, the Plan- och bygglagen (PBL) came into force, replacing the older Byggnadslagen och Byggnadsstadgan. Simultaneously, the Plan- och byggförordningen (PBF) was introduced. In 1989, Boverkets nybyggnadsregler (NR 1, BFS 1988:18) replaced the older Planverkets föreskrifter (PFS 1987:1). These new building regulations were structured around regulations and general advice.

On January 1, 1994, Boverkets byggregler (BBR 1, BFS 1993:57) and Boverkets konstruktionsregler (BKR 1, BFS 1993:58) came into effect, replacing the previous building regulations. The BBR introduced functional requirements, allowing for flexibility in achieving specific outcomes. Both BBR and BKR underwent numerous changes over the years, ranging from minor updates to extensive revisions. A continuous review process was eventually established to ensure the regulations remained up to date. On January 1, 2011, BKR was repealed and fully replaced by BFS 2010:28 (EKS 7). On May 2, 2011, new versions of BBR and EKS—namely BFS 2011:6 (BBR 18) and BFS 2011:10 (EKS 8)—were introduced.

As of January 1, 2011, the European construction standards, the Eurocodes, along with national provisions outlined in Boverkets föreskriftsserie (EKS), constitute the regulatory system that completely replaced the BKR. This shift marks a significant step toward harmonizing construction practices across Europe.

1.3 AIM

The purpose of the study is to increase knowledge about shear in reinforced concrete slabs, its capacity over the years. In addition to distributed load, the study will include punching shear. Another important part of the study is to investigate whether the problem of insufficient shear capacity for reinforced concrete slabs (without special shear reinforcement) has been addressed in previous Swedish studies. By analysing past research, primarily master's theses from various Swedish universities, the study identifies key parameters influencing shear capacity. This analysis helps explain why existing slabs may not meet modern Eurocode requirements and highlights the differences in design philosophy over time. The goal of the study is to shed light on the causes behind the insufficient shear capacity of existing slabs when using current norms and propose further studies to address this issue.

Table 1 Building codes in Sweden from 1947 until today (Boverket, Byggregler – en historisk översikt, 2022)

Standard		Published - valid to
BABS	Byggnadsstyrelsens anvisningar till byggnadsstadgan *Byggnadsstyrelsen's instructions for the Building Code*	1947 - 1950 1950 - 1960 1960 - 1968
SBN	Svensk Bygg Norm *Swedish Building Code*	1968 – 1976 1976 – 1982 1982 – 1987
PBL	Plan- och bygglagen *The Planning and Building Act*	1987 - 2011 2011 –1989
NR	Boverkets nybyggnadsregler *Boverket's Rules for New Construction*	BFS 1988:18 NR 1. 1989-1993
BBR [a)]	Boverkets byggregler *Boverket's Building Rules*	BFS 1993:57 BBR 1. 1994 - 2011 BFS 2011:6 BBR 18. 2011 - 2015
BKR	Boverkets konstruktionsregler *Boverket's Designing Rules*	BFS 1993:58 BKR 1 1994 -2010
EKS [a)]	Europeiska konstruktionsstandarderna, Eurokoder, EK *European Standards, Eurocodes, EC*	BFS 2008:8 EKS 1. 2008 - 2011 BFS 2011:10 EKS 8. 2011 - 2013

[a)]Valid

1.4 METHOD

The method for the study consists of the following:

- Search for other studies/dissertations that address the issue, perhaps related to other industries.
- Comparison of design codes of shear force control according to current and older building codes. The relevant sections in various standards that detail the calculation methods for shear force capacity and punching shear in reinforced concrete slabs without shear reinforcement were presented in Table 2.

Compilation of parameters that affect the dimensioning shear force and shear force capacity, such as strength, thickness, reinforcement content. As a first step to define the concrete class, a compilation of the different classification of concrete strength that are equivalent to the EC2 was presented in

- Table 3.
- Comparative calculations for some cases conducted manually according to different norms.

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Table 2 List of standards that referred to shear force capacity for reinforced concrete slab without shear reinforcement and punching shear.

Publications	Shear force capacity	Punching shear
Eurocode 2	Section 6.2	Section 6.4
BKR_BBK04	Section 3.7, there is a limit to tensile strength (f_{ct}) of 2.7 MPa	Section 3.12
BKR_BBk79	Section 3.7.2 Same as in BBK04 without any limit regarding f_{ct}	Section 6.5.4
SBN 80	B7:1968 page 23	B7 8:27
SBN 75	B7:1968 page 23	B7 8:27
SBN67	SOU 1957:25 Recommended to see B7:1968 page 23	B7 8:27

Table 3 Characteristic strength for different codes in Sweden that may equivalent to EC 2

Classification in Swedish codes	f_{cck_B7}# (MPa)	f_{ctk_B7} (MPa)	τ_{bo}¤ (MPa)	Concrete class⁺ (BBK04, 2004)	f_{cck_BBK04} (MPa)	f_{ctk_BBK04} (MPa)	Concrete class (SS-EN1992-1-1:2005, 2004)	$f_{ctk,\,0.05}$ (MPa)
K8*	5.50	0.75	-	-	-	-	-	-
K12*	8.50	0.90	-	-	-	-	-	-
K16 / K160	11.50	1.05	0.294	C12/15	11.50	1.05	C12/15	1.10
K20 / K200	14.50	1.20	0.343	C16/20	15.50	1.25	C16/20	1.30
K25 / K250	18.00	1.40	0.392	C20/25	19.00	1.45	C20/25	1.50
K30	21.50	1.60	0.422	-	-	-	-	-
K35 / K350	25.00	1.80	0.461	C25/30	24.40	1.70	C25/30	1.80
K40 / K400	28.50	1.95	0.491	C28/35	27.00	1.80	-	-
-	-	-	-	C30/37	29.00	1.9	-	-
K45 / K450	32.00	2.10	0.520	C32/40	30.50	2.00	C30/37	2.00
K50 / K500	35.50	2.25	0.550	C35/45	33.50	2.10	C35/45	2.20
K55 / K550	39.00	2.40	0.569	C40/50	38.00	2.40	C40/50	2.50
K60 / K600	42.50	2.50	0.590	C45/55	43.00	2.55	C45/55	2.70
K70	49.50	2.50	-	C50/60	47.50	2.75	C50/60	2.90
-	-	-	-	C54/65	51.50	2.80	C55/67	3.00
-	-	-	-	C55/67	52.00	2.85	-	-
-	-	-	-	C58/70	55.00	2.90	-	-
K80	56.60	2.50	-	C60/75	57.00	2.95	C60/75	3.10

* For lightweight concrete (BBK79, 1988), p.23.

The characteristic (5%) cylinder strength (150 mm x 300 mm) , 20 ± 2 °C temperature, and water-cure conditions, according to CEP-FIP Model Code for Concrete Structures (BBK79, 1988), p.23

⁺ Different characteristic compressive strength are used for lightweight concrete, Table 7.221b (BBK04, 2004), p.33

¤ Concrete shear stresses, Table 2:261 (1 MPa equals 0.0981 kp/cm^2) in (B7, 1968), p. 23.

1.5 OTHER DESCRIPTIONS OF THE PROBLEM

This study aims to examine the older Swedish norms for structural design and their differences from more recent standards such as the Eurocode. In order to achieve this goal, the study relies on literature primarily consisting of master's theses from various universities in Sweden. These theses provide valuable insights into the critical parameters that need to be considered in this study. By analyzing these differences and their implications for structural design, the study aims to contribute to a better understanding of the evolution of Swedish norms.

Different structural codes are based on varying historical approaches to safety and load treatment, reflecting different design philosophies. Sweden transitioned to a reliability-based design with the introduction of partial factors in BKR, aligning with probabilistic methods to account for uncertainties in loads and material properties. In contrast, the older deterministic approach used in SBN relied on fixed safety margins without explicitly incorporating statistical variations in load combinations. The shift to Eurocode and BKR introduced partial safety factors for both material strength and applied loads, ensuring a uniform safety level across different structures. This approach contrasts with SBN's simpler deterministic methodology, which did not distinguish between different sources of uncertainty. Although both EC2 and BKR follow reliability-based design principles, their partial factors differ due to national calibrations. Eurocode employs standardized safety factors across Europe, whereas BKR was adapted to Sweden-specific conditions and risk assessments. While both frameworks aim for the same probability of failure, variations arise due to historical load assumptions, material properties, and construction practices.

1.5.1 Shear force

Hellberg and Eryd (2018) highlights how loads are addressed in different standards and it is presented in Table 4. In this table, a comparison of the suggested values for distributed imposed loads in EC2, BKR, and SBN for two specific load categories are presented.

Table 4 Comparison between values for distributed imposed loads [kN/m²] in Eurocode, BKR and SBN. Values from SBN are presented with "normal occurrence" (Hellberg & Eryd, 2018)

Specific Use	Eurocode	BKR		SBN	
		Free*	Fixed*	Free*	Fixed*
Areas for domestic and residential activities	1.5 – 2.0	1.5	0.5	1.5	0.5
Office areas	2.0 – 3.0	1.5	1.0	1.5	1.0

* A fixed action always affects the same part of a structure in the same way. On the other hand, a free action can be moved or changed. Usually, actions have two parts - one that is fixed and one that is free.

Notably, the values show a good level of agreement across different codes, indicating consistency in the treatment of imposed loads despite variations in how safety margins are incorporated. The key difference in load treatment between Eurocode, BKR, and SBN lies in the use of partial factors for combining loads.

As shown in Table 5, Eurocode and BKR employ different multiplication factors for permanent and variable loads in the ultimate limit state (ULS), reflecting differences in national calibrations. While both codes are based on reliability principles, their specific partial factors differ due to regional considerations, historical safety levels, and variations in material properties and construction practices. Despite these differences, the underlying concept remains that both codes should lead to the same probability of failure, ensuring a consistent safety level across structural designs.

Table 5 Examples of partial factors for load combination in ultimate limit state according to Eurocode and BKR (Hellberg & Eryd, 2018).

Type of load	Eurocode	BKR
Permanent load	1.35	1.0
Variable load	1.5	1.3

In addition, both Eurocode and BKR account for three safety classes based on the potential extent of personal injury in the event of structural collapse. Safety class 1 corresponds to the lowest risk, while safety class 3 represents the highest. Multiplication factors are applied depending on the assigned safety class, as shown in Table 6

Table 6 Different safety classes and their corresponding partial factors (Hellberg & Eryd, 2018).

Category	Eurocode	BKR
Safety class 1	0.83	1.0
Safety class 2	0.91	1.1
Safety class 3	1.0	1.2

When combining the partial factors for load type and safety class, the resulting factors for Eurocode and BKR are relatively close. For instance, in safety class 3, the combined factor for variable loads is 1.5 under Eurocode and 1.56 under BKR.

Table 7 Combined partial factors for load combinations and safety classes for Eurocode and BKR (Hellberg & Eryd, 2018).

Category	Eurocode	BKR	Eurocode	BKR
	Permanent	Permanent	Variable	Variable
Safety class 1	1.12	1.0	1.25	1.3
Safety class 2	1.22	1.1	1.36	1.43
Safety class 3	1.35	1.2	1.5	1.56

This analysis highlights how the approaches in Eurocode and BKR differ in detail but lead to comparable levels of safety, while SBN adopts a distinct methodology that lacks the partial factors. As a result, this study will not consider the load effects, as the analysis above demonstrates that there is no significant difference in load treatment between the standards that would impact the outcomes.

The design of shear reinforcement has long been a critical area of research and engineering practice. Various national and international design codes, including EC2, AASHTO, BS, and DIN, provide guidelines for calculating the shear capacity of such sections. These codes differ in their approaches, underlying assumptions, and the factors they consider, leading to variations in predicted capacities. A study by Westerberg (2002) has been performed aiming to analyze and compare the shear capacity predictions of these codes in addition to BBK under various conditions, shedding light on their similarities and differences. Several examples has been presented with and without shear reinforcement for beams. For beam without shear reinforcement the shear strength are presented in Figure 1. In this example, the shear capacity is expressed in MPa because the cross-section maintains the same rectangular shape in all cases. This means the comparison between different codes is not affected because the factors that convert shear strength (MPa) to shear force (N) – the width (*b*) and effective depth (*d*) – remain the same, which does not change the comparison.

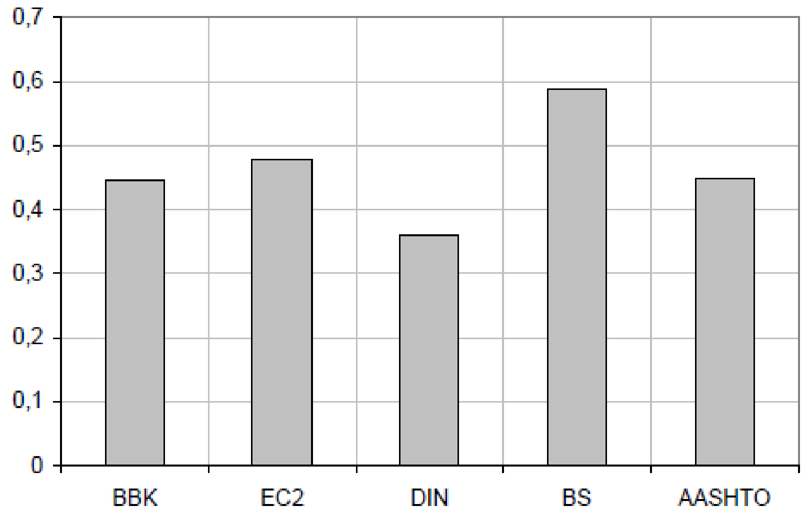


Figure 1 Shear strength (vertical axis) in MPa for different codes, from (Westerberg, 2002)

BBK, EC2, and AASHTO show comparable results, while BS predicts higher, and DIN predicts lower bearing capacities.

1.5.2 Punching shear

A comprehensive study on the punching failure of concrete slabs has been conducted based on literature, particularly by (Kölfors, 1993). The study provides a detailed examination of seven European building standards, including the Swedish standard BBK 79. It reviews documented research findings, failure modes, and the fundamental models underlying these standards. Furthermore, it includes a comparative analysis of the differences between the standards, supported by manual calculations to assess their dependence on various parameters. The standards and methods analyzed in this study include BBK 79, the more refined method described in the Swedish Concrete Handbook, DS, BS, CEB-FIP Model Code, B7, and DIN.

A key distinction among these standards is the adopted approach to punching shear capacity, which is primarily based on either the control surface model or the Kinnunen-Nylander model. The Kinnunen-Nylander model, developed from theoretical and experimental studies, describes punching failure by considering a radial shear stress distribution and the slab's rotational capacity, with load transfer occurring through inclined compression struts. This model is used in older Swedish codes, such as BBK 79, where the control perimeter is placed closer to the column.

In contrast, the control surface model assumes a uniform shear stress distribution along a defined control perimeter, which is located at a fixed distance from the column face.

This method is employed in CEB-FIP Model Code, BS, and DIN, leading to an increased allowable punching shear capacity as the column cross-section grows. For standards that follow this approach, the permissible load tends to increase more rapidly with the column's side length compared to the Kinnunen-Nylander model.

Among the studied standards, BBK 79 produces the lowest permissible load values, particularly for small column cross-sections. However, due to its control perimeter being positioned closer to the column, BBK 79 exhibits a steeper increase in allowable load with increasing column size. This is because the relative growth in control surface area is greater when the perimeter is nearer to the column.

Based on the comparison of these standards and methods, column dimensions emerge as a critical factor in evaluating punching shear capacity. The size and geometry of the column cross-section significantly influence the control surface area and, consequently, the permissible load. Notably, standards such as BBK 79 demonstrate a greater sensitivity to changes in column side length.

Additionally, the placement of the control surface relative to the column perimeter plays a crucial role in punching shear capacity calculations. For example, standards with control surfaces positioned closer to the column perimeter (e.g., BBK 79) produce different results than those where the control surface is placed further away.

Apart from geometric considerations, material properties—especially concrete compressive strength—are fundamental in determining punching shear capacity and are consistently emphasized across all standards. Furthermore, the amount of bending reinforcement significantly affects punching strength, highlighting its importance in design calculations.

In this study, these key parameters—column dimensions, control surface placement, concrete compressive strength, and bending reinforcement amount—will be systematically analyzed and compared across different design standards.

2 Shear force capacity

This section describes differences in the design codes previously mentioned in Table 2. This table summarizes the relevant sections in various standards that detail the calculation methods for shear force capacity and punching shear in reinforced concrete slabs without shear reinforcement. EC 2 specifies these calculations in Sections 6.2 and 6.4, respectively. The methods used in BKR_BBK04 and BKR_BBK79 for calculating shear force capacity are identical, except that BBK04 imposes a limit on f_{ct} of 2.7 MPa, which is absent in BBK79. Both standards also employ the same approach for punching shear calculations. For SBN standards (SBN80, SBN75, and SBN67), the calculation methods for both shear force capacity and punching shear rely on the recommendations outlined in B7 (1968), with referenced sections clearly indicated.

In this study, the focus is on comparing the methods outlined in EC2, BBK04, and B7 for the calculation of shear force capacity and punching shear in reinforced concrete slabs without shear reinforcement. These standards represent significant milestones in the evolution of structural design norms in Sweden.

Historically, the B7 method, introduced in 1968, was the primary standard referenced for shear force capacity and punching shear calculations in Sweden until 1980. Subsequent updates led to the adoption of BBK04, which remained in use until 2011. Since January 1, 2011, Sweden has transitioned to using EC2 as the mandatory standard for structural design, reflecting a shift towards a unified European approach.

2.1.1 Eurocode 2

According to (SS-EN1992-1-1:2005, 2004), section 6.2.2, the shear capacity for section without shear reinforcement is:

$$V_{Rd,c\_S} = \left(C_{Rd,c}\, k \sqrt[3]{100\,\rho\, f_{ck}} + k_1\, \sigma_{cp}\right) b_w d \qquad (1)$$

With a minimum of

$$V_{Rd,c} = \left(v_{min} + k_1\, \sigma_{cp}\right) b_w d \qquad (2)$$

$$v_{min} = 0.035\ \sqrt{k^3\, f_{ck}} \qquad (3)$$

where

$C_{Rd,c} = \frac{0.18}{\gamma_c}$

γ_c is partial factor for concrete that takes into account safety class; 1.5 for persistent and transient and 1.2 for accidental.

$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0$ (d in mm)

$\rho = \frac{A_s}{b_w d} \leq 0.02$ reinforcement ratio for longitudinal reinforcement.

f_{ck} is characteristic compressive strength of concrete in MPa.

k_1 = 0.15

σ_{cp} is mean compressive stress in uncracked cross-section caused by tensile force or normal force, in this work this is set to zero.

A_s is cross sectional area of reinforcement.

b_w is the smallest width of the cross-section in the tensile area.

d is effective depth.

2.1.2 Boverket BBK-04

Method 1

According to Boverket (BBK04, 2004), section 3.7.3, the shear force capacity for member without shear reinforcement:

$V_{Sd} \leq V_{c\_BBK\_S} + V_i$ (4) (equation 3.7.3.1a) in BBK04.

where

V_{Sd} is shear force of design load. Where confusion is excluded, the notation can be like Eurocode 2.

$V_{c\_BBK\_S}$ is concrete shear capacity according to section 3.7.3.2-5 in BBK04.

V_i is the effect of variable effective height according to Section 3.7.3.6 in BBK04. In this work the effect will not be considered since rectangular beams are discussed.

According to equation 3.7.3.2a in BBK04, concrete shear force capacity can be calculated as following:

$V_{c\_BBK\_S} = b_w \, d \, f_v$ (5)

where

f_v is concrete shear strength.

$f_v = 0.3\,\xi(1+50\rho)\,f_{ct}$ (6)

where

$$\xi = \begin{cases} 1.4 & \text{for} \quad d \leq 0.2 \text{ m} \\ 1.6 - d & \text{for} \quad 0.2 \text{ m} < d \leq 0.5 \text{ m} \\ 1.3 - 0.4d & \text{for} \quad 0.5 \text{ m} < d \leq 1.0 \text{ m} \\ 0.9 & \text{for} \quad 1.0 \text{ m} < d \end{cases}$$

$$f_{ct} = \frac{f_{ctk}}{\eta \gamma_m \gamma_n} \qquad (7)$$

f_{ctk} **is characteristic tensile strength in**

Table 3. f_{ct} is limited to the corresponding value f_{ctk} = 2.7 MPa.

For ultimate limit state, the term $\eta\,\gamma_m$ is equal to 1.5 for concrete strength and 1.5 for modulus of elasticity, according to Section 2.3.1 (BBK04, 2004) .

γ_n is partial coefficient for safety class according to Section 2:115 (BBK04, 2004):

- safety class 1, partial coefficient γ_n = 1.0
- safety class 2, partial coefficient γ_n = 1.1
- safety class 3, partial coefficient γ_n = 1.2

$v_{min} = 0.25 f_{cc} b_w d$ (8) from equation 3.7.4.1b

Method 2

BBK and Eurocode 2 present roughly the same methods to calculate the shear force capacity. According to Boverket (BBK04, 2004), section 3.7.3.7, the shear force capacity for member without shear reinforcement:

$$V_{Rd,c\_BBK} = \left(\frac{0.18\,k}{1.5\,\gamma_n}\sqrt[3]{100\,\rho\,f_{ck}} + 0.15\,\sigma_{cm}\right) b_w d \quad (9)$$

With a minimum of

$$V_{Rd,c} = (v_{min} + 0.15\,\sigma_{cm})\, b_w d \quad (10)$$

$$v_{min} = \frac{0.035}{\gamma_n}\sqrt{k^3\, f_{ck}} \quad (11)$$

Where

σ_{cm} is mean compressive stress in uncracked cross-section caused by tensile force or normal force, in this work this is set to zero.

BKK method 1 is the calculation method that differs from the other two Eurocode 2 and BBK04 method 2. The biggest difference is that BKR method 1 uses the tensile strength of concrete and not the compressive strength of concrete, which both Eurocode and BKR method 2 use. Another difference is the fact that BKR method 1 uses the design tensile strength while the other two methods use the characteristic compressive strength. This means that even BKR method 1 takes into account which safety class the construction part belongs to.

2.1.3 Staten Betong kommitte', B7

According to (B7, 1968) Section 2:261, the resistance may either be assumed to A) consist of the resistance coming from the shear reinforcement or it may B) be assumed to consist of both the resistance coming from the shear reinforcement and contributing concrete. Assumption B) is used here without shear reinforcement. Thus, equation (9) in (B7, 1968) was used in this study:

$$V_{c\_B7\_S} = \tau_{bo} b_w d \quad (12)$$

τ_{bo} **is shear strength in**

Table 3.

Thus, the shear stress in a non-shear reinforced concrete slab is controlled against a resistance, calculated from a base value of shear strength tabulated for different values of concrete strength classes.

Table 3 presents the tabulated values, with concrete strength classes of that time.

2.2 PUNCHING SHEAR CAPACITY

2.2.1 Eurocode 2

According to (SS-EN1992-1-1:2005, 2004), section 6.4.4, the punching shear capacity for section without shear reinforcement is:

$v_{Rd,c\_P} = C_{Rd,c}\, k \sqrt[3]{100\, \rho_1\, f_{ck}} + k_1\, \sigma_{cp} \geq \left(v_{min} + k_1\, \sigma_{cp}\right)$ (13)

where

$C_{Rd,c}$, k and v_{min} is given in Section 2.4.1 which is the same as for shear force capacity and that for k_1 is 0.1. σ_{cp} is also set to zero.

$\rho_1 = \sqrt{\rho_{1y} . \rho_{1z}} \leq 0.02$ reinforcement ratio for longitudinal reinforcement in y- and z-directions respectively.

The values ρ_{1y} and ρ_{1z} should be calculated as mean values taking into account a slab width equal to the column width plus 3*d* each side.

The basic control perimeter u_1 may normally be taken to be at a distance 2*d* from the loaded area and should be constructed so as to minimize its length, see Figure 2. The effective depth of the slab is assumed constant and may normally be taken as:

$\boldsymbol{d_{eff}} = \frac{(\boldsymbol{d_y} + \boldsymbol{d_z})}{\boldsymbol{2}}$ 14

where d_y and d_z are the effective depths of the reinforcement in two orthogonal directions.

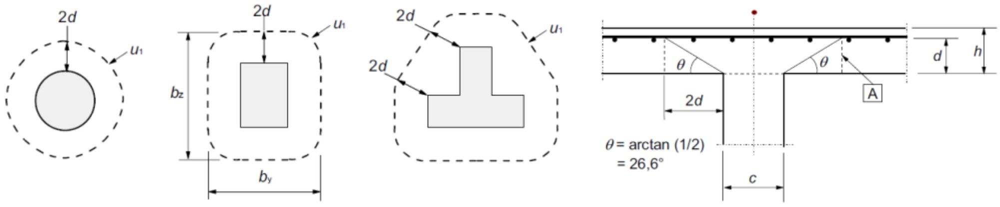


Figure 2 Typical basic control perimeters around loaded areas (SS-EN1992-1-1:2005, 2004).

In this study, a constant depth for slab was assumed. Thus, the shear resistance should be checked at the face of the column and at the basic control perimeter u_1.

$V_{Ed,c\_P} = v_{Ed,c\_P} . u_1 . d_{eff}$ (15)

2.2.2 Boverket BBK-79

For punching shear capacity the Boverket (BBK79, 1988), in section 6.5.4 permits a greater formal shear strength due to the occurrence of a multiaxial state of stress during punching. Thus, concrete punching strength becomes:

$f_{v1} = 0.45\, \xi (1+50\rho_2)\, f_{ct}$ (16)

where

$$\xi = \begin{cases} 1.4 & \text{for} \quad d \le 0.2 \text{ m} \\ 1.6 - d & \text{for} \quad 0.2 \text{ m} < d \le 0.5 \text{ m} \\ 1.3 - 0.4d & \text{for} \quad 0.5 \text{ m} < d \le 1.0 \text{ m} \\ 0.9 & \text{for} \quad 1.0 \text{ m} < d \end{cases}$$

$\rho_2 = \sqrt{\rho_{2y}.\rho_{2z}} \le 0.01$ reinforcement ratio for longitudinal reinforcement in y- and z-directions respectively. ρ_2 must be at least 0.003.

In BBK 79, it is assumed that the fracture surfaces of the truncated cone are inclined at 45° to the plane of the plate. As a result, the critical section is defined as a section perpendicular to the plane of the plate at a distance of $d/2$ from the corner of the column, see Figure 3 . Alternatively, the section should be drawn with the shortest possible length, denoted by u.

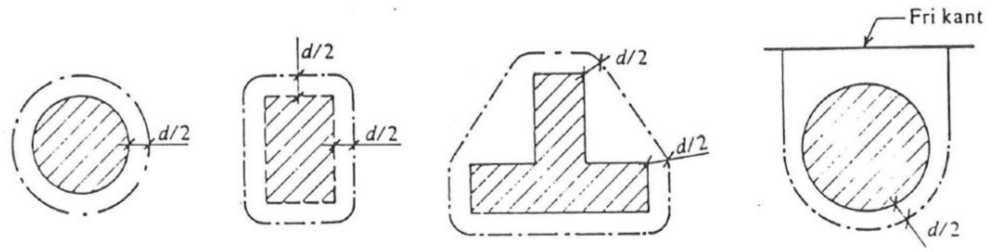


Figure 3 Example of drawing the critical section according to BBK 79 (Kölfors, 1993).

$$V_{c\_BBK\_P} = \mu \, . f_{v1} \, .u. d_{eff} \qquad (17)$$

where

μ is eccentricity factor and has a value of 1.0 since the eccentricity factor was not taken into consideration in this study.

2.2.3 Staten Betong kommitte', B7

B7 states that slabs subjected to concentrated loads should be designed based on the guidelines provided by the Statens Betongkommittes kommentardel Kl. In the comments section, it is mentioned that the method is based on Kinnunen/Nylander's theory and is suitable for circular loaded areas.

If the cross-section of the loaded area is not circular, it needs to be recalculated to a circular shape of a diameter of B using this method. However, for rectangular column cross-sections, different calculations may be required.

$$B = \frac{2}{\pi} (d_1 + d_2) \qquad (18)$$

Where d_1 and d_2 are the sides of the column cross section. However, the long side may be counted as a maximum of 1.5 times longer than the short side.

For a slab with B/h ≤ 3.5 and as shown in Figure 4 the nominal shear stress for a cylinder surface at the distance h/2 from the edge of the column is calculated according to:

$$\tau_{nom} = \frac{p}{\pi h(B+h)} \quad (19)$$

$$\tau_1 = \tau_0 \frac{15}{10+\frac{c}{2h}} \quad (20)$$

Where

$\tau_{nom} = 0.65\ \tau_1$ assume that no further control is necessary.
$u_{\_B7} = \pi(B+h)$ is the critical section
$\boldsymbol{\tau_o}$ the shear strength capacity from Table 8
c is the extent of the support arm over the column.

$$V_{c\_B7\_P} = 0.65\ \tau_1\ u_{\_B7}\ h \quad (21)$$

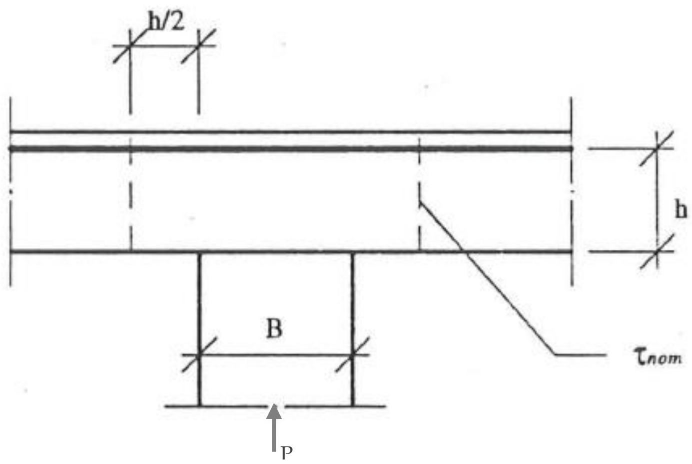


Figure 4 The critical section according to K1 (Kölfors, 1993).

Table 8 Concrete shear stresses (Kölfors, 1993)

Concrete class	K200	K250	K300	K350	K400
τ_o MPa*	1.03	1.13	1.23	1.28	1.32

*(1 MPa equals 0.0981 kp/cm^2).

2.2.4 Concrete Handbook

Kinnunen and Nylander's model is one of the first models to describe the interactions between forces at the failure. It was carried out in 1960 by casting 61 samples of mechanical punching in circular concrete slabs laid on circular columns. This process allowed for the acquisition of important information regarding the orientation of cracks, how forces affecting the plate are directed, and how the forces affect the concrete and reinforcement. The plate is divided into sector elements to make it easier to orient force directions, and by studying how the deformations look on the element, the directions can be obtained. The cracks that form can go in two directions, either radially (starting from the center and then going horizontally outwards) or tangentially (going vertically), see . The sector elements are delimited by radial cracks on the sides and tangential cracks at the front. The starting point of the cracks is at the cross-sectional surface of the column, and then they spread outwards in a cone-like shape, causing the elements to twist around that surface.

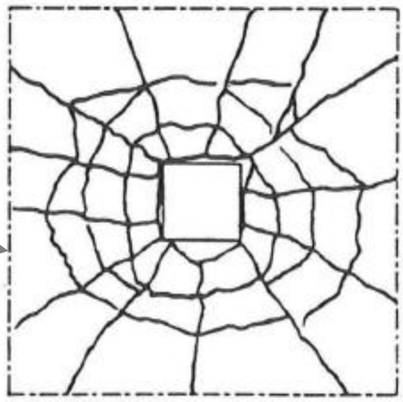


Figure 5 Crack pattern in the slab around the loaded area (Kölfors, 1993).

In the Concrete Handbook (BHB), the B7 design method has been modified in connection with BBK 79, to apply Kinnunen and Nylander's model. This model has been simplified with some approximations, so that it has become easier to apply. When this model was introduced, it was not very common for designers to have access to powerful computers that could be programmed to make iterative calculations, or in a simple way process logarithmic functions. Therefore, for example, the concrete handbook published in 1983 did not contain so many formulas, but mainly diagrams that were to be read. The method takes into account cross-sectional height, the ratio of distance from the center of the column to the moment zero and the bending reinforcement content. Figure 6 shows the load case with a circular plate supported by a circular column that is loaded with a line in the outer area.

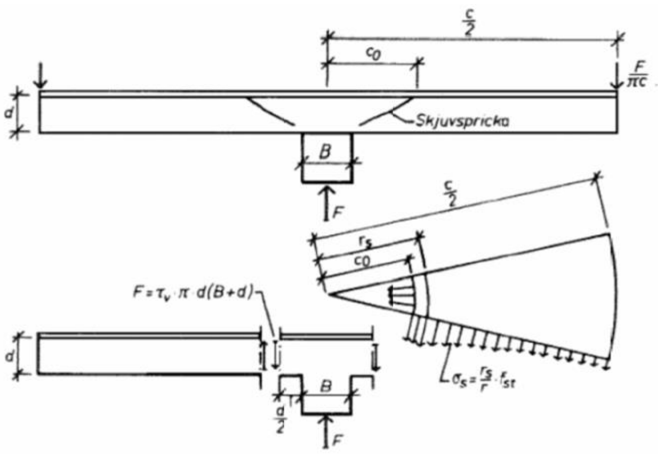


Figure 6 The load case with a circular plate supported by a circular column. *c_0* distance from the column to top of shear crack (skjuspricka), *c* diameter of the slab over the column where the radial bending moment is zero, *F* is design shear force, *r_s* radius within which the flexural reinforcement has reached yield strength (Kölfors, 1993).

The plate radius ($c/2$) is set equal to the distance from the center of the column to the moment zero point in the real plate. The yield strength f_y is reached in the reinforcement within an area of the slab with radius (r_s) starting from the column center. When $r_s > c/2$ implies that r_s extends beyond half the $c/2$. This signifies that the reinforcement has yielded over a larger area, leading to bending failure. Conversely, when $r_s < c/2$ the plastic radius is smaller than half the plate radius. This indicates localized stress and a more concentrated failure mechanism, resulting in punching failure. In other words, bending failure occurs when $c < 2\,r_s$, and punching failure occurs when $c > 2r_s$ (Kölfors, 1993).

In this study, an assumption was made to estimate the value of c that provides the maximum punching shear capacity, where c is set equal to $2r_s$.

$$\tau_{v,nom} = \frac{F}{\pi d(B+d)} \leq f_{v1} \quad (22)$$

f_{v1} shear strength without shear reinforcement.

$$f_{v1} = 0.9 \cdot 0.6 \cdot \xi \cdot \alpha \cdot f_{v1,id} \quad (23)$$

where

0.9 is a reduction factor that takes into account that the punching load occurs suddenly, and due to low values of the partial coefficients for permanent load.

ξ is a coefficient that takes into account the influence of the plate thickness.

α is a coefficient that depends on the ratios B/d, c/d and f_{st}, f_{cc}.

$f_{v1,id}$ is the value of when all reinforcement within the plate part c reaches the yield point.

When B/d<3.5 and d/c <0.3, $f_{v1,id}$ is determine as

$$f_{v1,id} = \frac{2\,\rho\, f_{st}}{\left(1+\frac{B}{d}\right)\cdot\left(1-\frac{B}{d}\frac{d}{c}\right)} \cdot \frac{z}{d} \quad (24)$$

If B/c > 0.3, the value of $\frac{B}{d} \cdot \frac{d}{c}$ is equal to 0.3.

Where

$$\frac{z}{d} = \frac{3 + 200\,k\rho}{3 + 300\,k\rho}$$

$$k = \frac{1}{0.57 + 0.43\,\frac{f_{cc}}{13}\sqrt{\frac{400}{f_{st}}}}$$

However, there is a limitation with regard to the shear strength, if $f_{v1} > f_{vo}$ so, the design should continue to be carried out with f_{vo} , alternatively, the slab should be shear reinforced.

$$f_{vo} = 0.28\,\xi_1 \sqrt{f_{cc}}\,\frac{15}{\left(10+\frac{1}{2}\frac{c}{d}\right)} \quad (25)$$

Where

$$\xi_1 = \begin{cases} \xi_{min} & \text{for } \frac{2\,r_s}{c} \leq 0.3 \text{ m} \\ \xi_{min} + \frac{\frac{2\,r_s}{c} - 0.3}{2} & \text{for } 0.3 \leq \frac{2\,r_s}{c} \leq 0.6 \\ \xi_{min} + 0.15 & \text{for } 0.6 \leq \frac{2\,r_s}{c} \end{cases}$$

$\xi_{min} = \xi$ according to BBK04

$$\frac{r_s}{d} = \frac{380}{f_{st}} \cdot K_1$$

$$K_1 = \frac{3.8 + 0.4 \cdot \frac{d}{B} \cdot \frac{1}{100\,k\,\rho}}{100\,k\,\rho + \frac{d}{B}} \cdot K_2$$

$$K_2 = \begin{cases} 1 & \text{For } \frac{B}{d} \leq 2 \\ 0.7 + 0.15 \cdot \frac{d}{B} & \text{For } \frac{B}{d} > 2 \end{cases}$$

$$\alpha = \begin{cases} \frac{r_s}{d} \cdot 2 \cdot \frac{d}{c}\left[1 + \ln\left(\frac{c}{2d} \cdot \frac{d}{r_s}\right)\right] & \text{for } \frac{c_0}{d} \leq \frac{r_s}{d} \leq \frac{c}{2d} \\ \frac{r_s}{d} \cdot 2 \cdot \frac{d}{c}\left[1 + \ln\left(\frac{c}{2d} \cdot \frac{d}{c_0}\right)\right] & \text{for } \frac{r_s}{d} \leq \frac{c_0}{d} \\ 1 & \text{for } \frac{r_s}{d} \geq \frac{c}{2d} \end{cases}$$

3 COMPARISON OF THE DIFFERENT CALCULATION METHODS

3.1 COMPARISON OF THE FORMULAS FOR SHEAR FOCE CAPACITY OF SLAB

This section of the work outlines the different methods of calculating the shear force capacity in a way that makes them look more similar, making the comparison easier.

- **Eurocode 2**

$$V_{Rd,c} = \left(\frac{0.18\ k}{\gamma_c}\ \sqrt[3]{100\ \rho\ f_{ck}}\right) b_w d$$

- **BBK 04 Method 1**

$$V_{Rd,c\_BBK04} = (0.3\ \xi(1+50\rho)\ f_{ct})\ b_w\ d$$

BBK 04 Method 2

$$V_{Rd,c\_BBK04} = \left(\frac{0.18\ k}{1.5\ \gamma_n}\ \sqrt[3]{100\ \rho\ f_{cck}}\right) b_w d$$

- **B7**

$$V_{Rd,c\_B7} = \tau_{bo} b_w d$$

The comparison between BBK04 method 2 and EC 2 shows that these methods are very similar, with the only difference being that BBK04 method 2 has a partial coefficient γ_n that has no equivalent in the method described in EC 2. The partial coefficient depends on which safety class the construction part belongs to. In the lowest safety class (class 1), which means that there is no difference between BBK04 method 2 and the method in EC 2. The difference arises in safety classes 2 and 3, where in BBK04 it is 1.1 and 1.2 respectively. In both safety class 2 and 3, BBK04 method 2 will result in a lower shear force capacity than EC 2. It should be noted that Eurocode 2 also has safety classes, with partial coefficients of 1.5 for persistent and transient loads and 1.2 for accidental loads but they are used in determining the design load and material.

It can be seen also from the expressions above that BBK04 method 2 and EC 2 both contain the factor k that has the same function as the factor ξ in BBK04 method 1. These factors take into consideration the influence of effective depth d. Figure 7 shows the results of comparing these factors obtained from Eurocode 2 and BBK04 for various slab thicknesses. From the figure it can be observed that the values obtained from EC 2 are generally higher than those obtained from BBK04, but the two functions are similar with a difference of only 0.6.

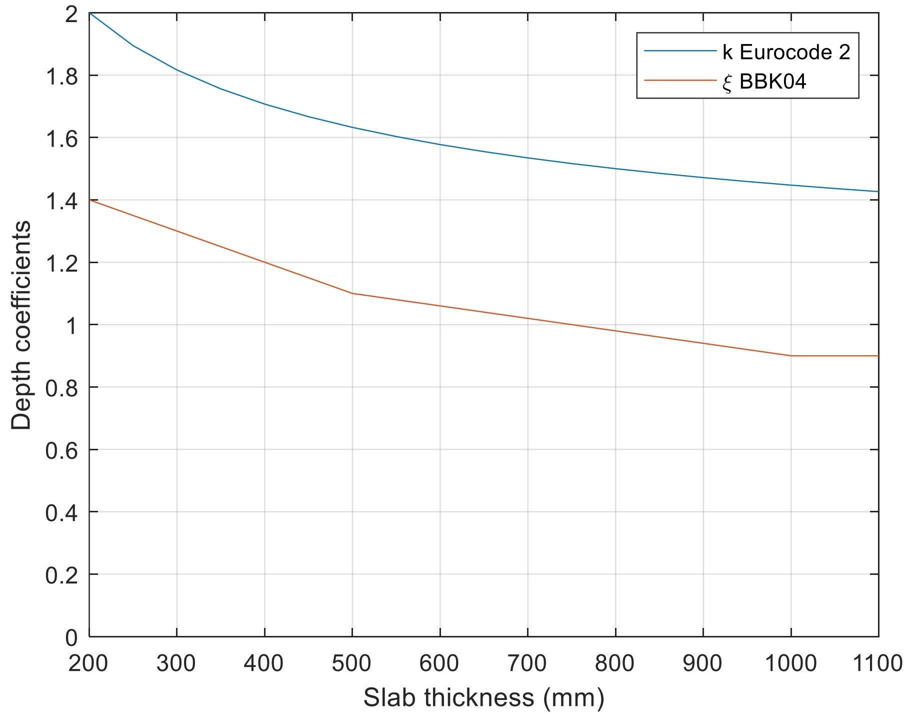


Figure 7 Comparison of factors ξ and k assume the slab cover equals to 30 mm.

In the following, the comparison was made between the calculated shear force capacities of BBK method 1 and B7, with EC2 as the reference standard. The comparison the shear force capacity are compared for various slab thickness concrete strength changed. All three methods used the same amount of longitudinal reinforcement.

To determine the practical limits of the longitudinal reinforcement ratio in reinforced concrete slab design, a study was conducted in using MATLAB and presented in Appendix A:. The study established the maximum and minimum values of the reinforcement ratio that can be practically applied and plotted the results as reinforcement ratio against the spacing c/c between the longitudinal reinforcement. The study assumed the use of ϕ 20 and 16 steel bars with c/c spacing of 100 mm and 500 mm for each. The findings provide valuable insights into the practical limitations of longitudinal reinforcement ratios in reinforced concrete slab design for calculating shear force capacity according to design codes and standards.

3.1.1 BBK04 Method 1

A comparison between the calculated shear force capacity as a ratio of BBK04 method 1 to EC 2 are presented in Figure 8 and Figure 9. The BBK04 method 1 was used to calculate the shear force capacity for safety class 1 and 3 by applying γ_n equals to 1 and 1.2 in Equation (6), respectively. The results are plotted against slab thickness for various arranged patterns of longitudinal bars. The obtained results were generated through calculations using a MATLAB script for various compressive strength. In Figure 8, the BBK04 shear force capacity was calculated by considering safety factor equals to 1. The results shows that the BBK04 shear force capacity was overestimated when compared to EC2, despite the small safety margins observed for high amounts of longitudinal reinforcement, as shown in Figure 8 (b) and (c). However, it is clear that the overestimate factors; i.e. $V_{c\_BBK04}/V_{Rd,c}$ ratio greater than 1, rise with an increase in the compressive strength of concrete higher than 15 MPa. It can been seen also the similarity in the functions of the two design methods for varying slab thicknesses eliminates the discrepancy of 0.6 observed in Figure 7. However, a slight variation ranging between 5 – 10 % can be observed for moderate reinforcement ratios, as shown in Figure 8 (b) and (c).

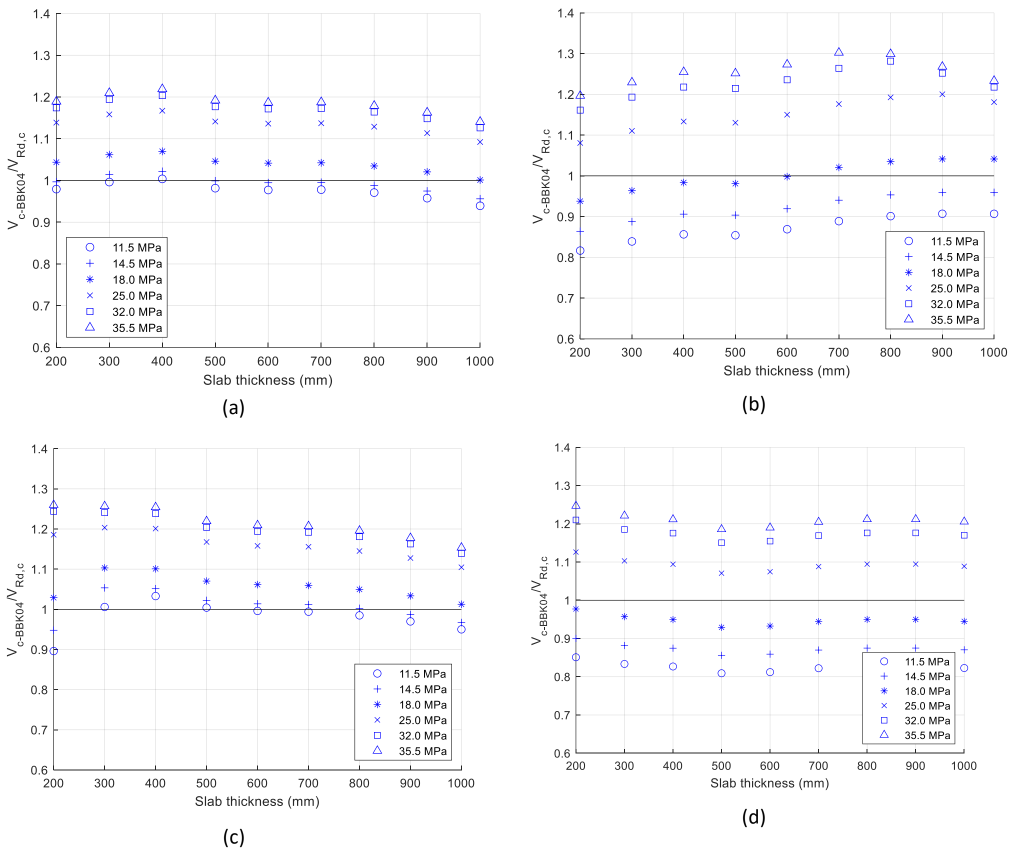


Figure 8 Ratio between shear force capacity of BBK04 method 1 and EC2. For BBK04 the safety factor =1 was applied in the calculation. $V_{c\_BBK04}/V_{Rd,c}$ plotted against slab thickness for various reinforcement amount; (a) ϕ 16 mm, c/c s 500 mm, (b) ϕ 16 mm, c/c s 100 mm, (c) ϕ 20 mm c/c, s 500 mm and (c) ϕ 20 mm, c/c s 100 mm.

The same MATLAB script utilized in the previous paragraph was used to determine the BBK04 shear force capacity for the safety case 3 by applying γ_n equals to 1.2 instead of 1 in Equation (7). The results are presented in Figure 9 which show that EC 2 provides a lower shear capacity compared to BBK04 corresponding to safety case 3. This is due to the fact that the contribution of safety factor is not taken into account when calculating the shear force capacity according to EC 2. The obtained results indicate that the shear capacity according to BBK04 ranges from 80% to 100% of Eurocode 2 for low longitudinal reinforcement, depending on the compressive strength, as shown in Figure 9 (a) and (d). Similarly, for high longitudinal reinforcement, the range is around 60 % to 110%, as shown in Figure 9 (c) and (d). It's important to note that the results are still subject to sensitivity based on the amount of longitudinal reinforcement.

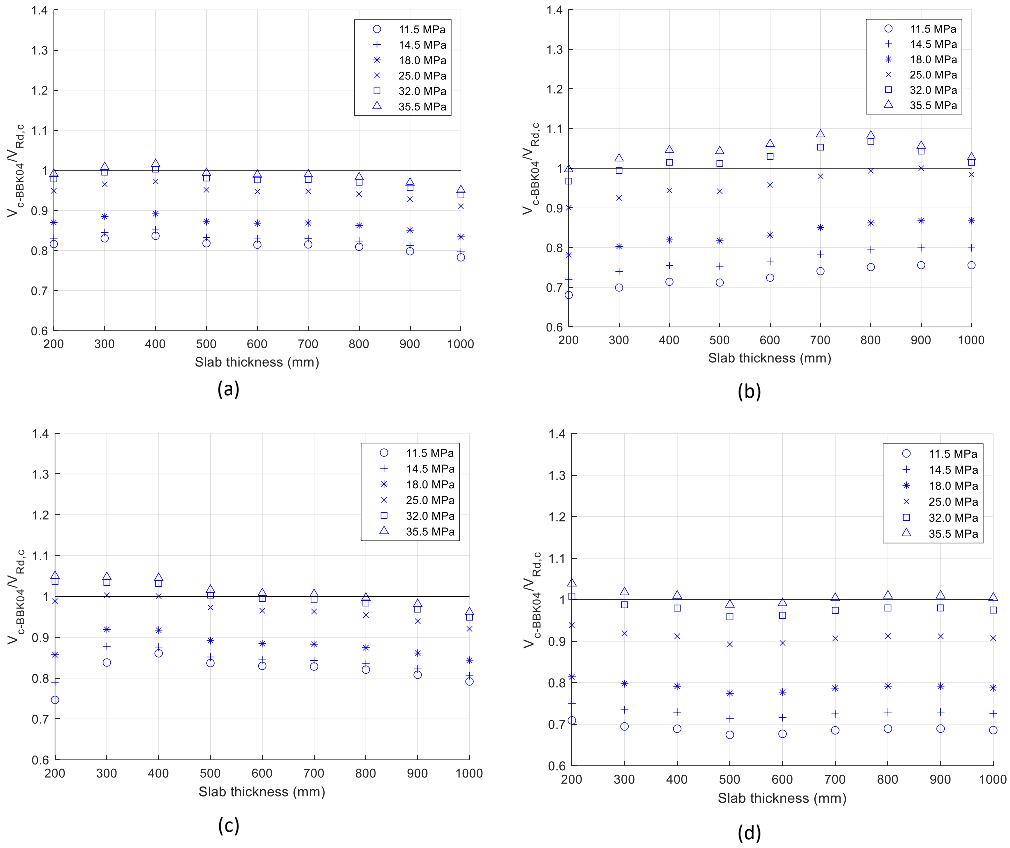


Figure 9 Ratio between shear force capacity of BBK04 method 1 and EC2. For BBK04 the safety factor =1.2 was applied in the calculation. Vc_BBK04/$V_{Rd,c}$ plotted against slab thickness for various reinforcement amount; (a) ϕ 16 mm, c/c s 500 mm, (b) ϕ 16 mm, c/c s 100 mm, (c) ϕ 20 mm c/c, s 500 mm and (c) ϕ 20 mm, c/c s 100 mm.

3.1.2 B7

The capacity was determined according to SBN using the MATLAB script. The results are presented in Figure 10 as a ratio of B7 to EC2. The results show that the B7 shear force capacity was overestimated when compared to EC 2, despite the a significant underestimate margins observed for high amounts of longitudinal reinforcement within slab thickness between 200 to 400 mm, as shown in Figure 10 (b) and (c). The slab thickness has a significant effect due to the fact that in B7 method the resistance to shear stress in a non-shear reinforced concrete slab is determined by calculating it against a base value of shear strength that is listed for various concrete strength classes without any contribution reinforcement.

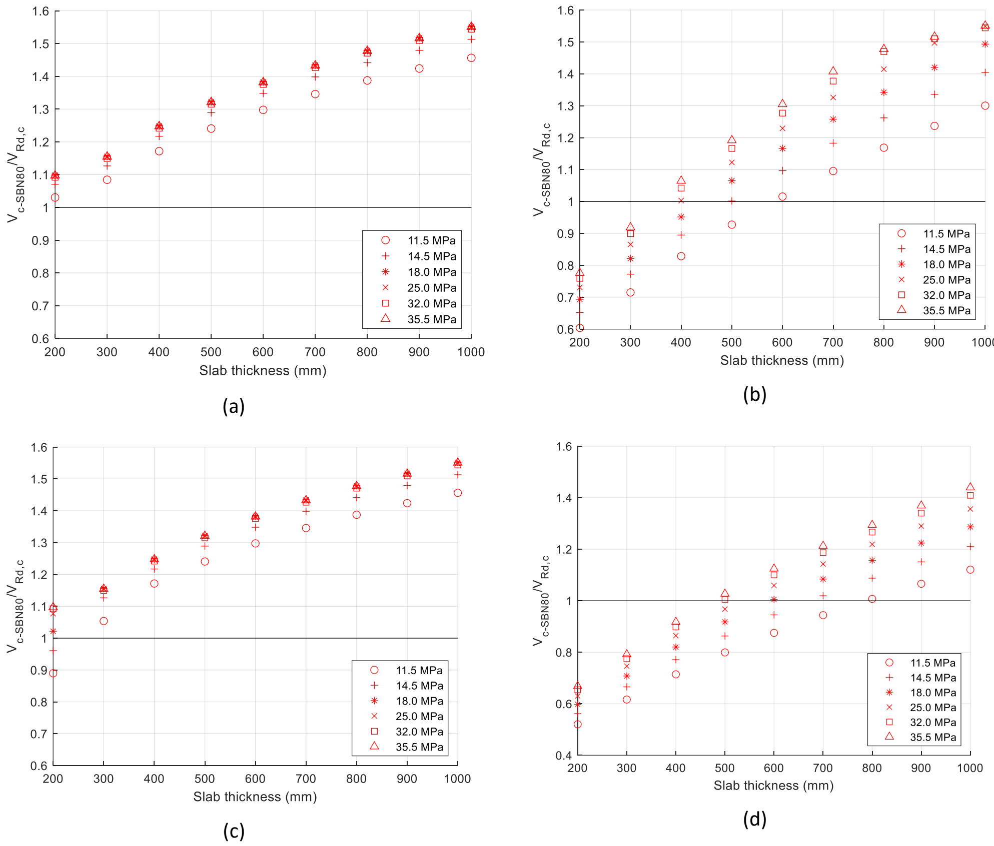


Figure 10 Ratio between shear force capacity of B7 and EC2. Vc_SBN80/VRd,c plotted against slab thickness for various reinforcement amount; (a) ϕ 16 mm, c/c s 500 mm, (b) ϕ 16 mm, c/c s 100 mm, (c) ϕ 20 mm c/c, s 500 mm and (c) ϕ 20 mm, c/c s 100 mm.

3.2 COMPARISON OF THE FORMULAS FOR PUNCHING SHEAR CAPACITY

In this section, a comparison was made between three different methods - EC 2, BBk79, B7 and BHB- for calculating the punching shear capacity of slabs with varying thicknesses. The section presents the punching shear capacity ratio of these methods when the concrete strength changes. The concrete strength considered are 11.5 MPa and 35.5 MP, as specified in

Table 3. The reinforcement content has been in a range of 0.3% to1%, which corresponds to the lower and upper limit in BBK79. However, in the Eurocode, the longitudinal reinforcement ratio is up to 2%.

The comparison was made between the calculated punching shear force capacities of BBK79, B7, and BHB, with EC 2 as the reference standard. The results are presented as the ratio of the punching shear force capacity of each method to that of Eurocode 2, plotted against the effective depth for circular columns with diameter of 500 mm and 1000 mm. The analysis considers longitudinal bars with diameters of ϕ16 mm and 20 mm at a center-to-center spacing of 100 mm, as shown in Figure 11 through Figure 13. The results were generated through calculations using a MATLAB script for compressive strengths of 14.5 MPa and 35.5 MPa.

The results show a good margin for the three methods, despite the small margin observed for low amounts of longitudinal reinforcement in methods B7 and BHB, as shown in Figure 12(a) and Figure 13(a). The column diameter has a small effect on the results, as the total punching shear depends primarily on the perimeter, which in turn is related to the column diameter.

In Figure 11, it can be seen that the effective depth has an insignificant impact on the BBK79 method due to the similarity in how the effect of the slab thickness is considered that eliminates the difference of this. Although the factor increases from 0.3 in Eq. 6 to 0.45 in Eq. 16, the margin is approximately 35% for the two concrete classes and longitudinal reinforcement. This is because of the total punching force calculated by considering a distance of 2d in Eurocode, compared to $d/2$ used in determining the effective perimeter.

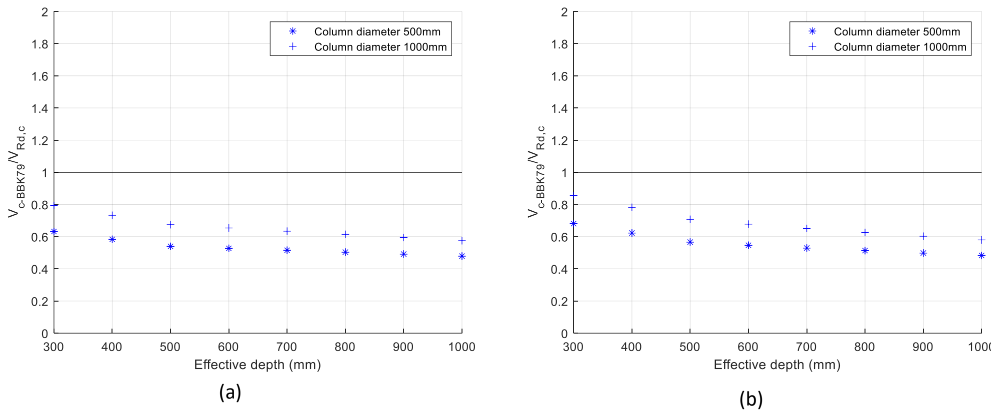
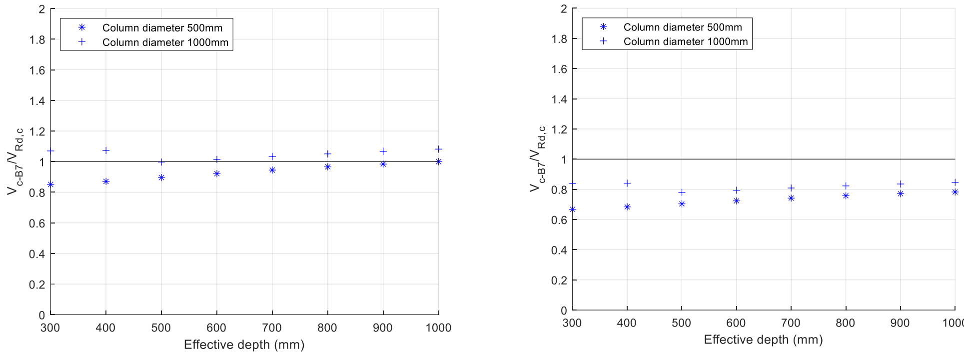


Figure 11 Ratio between shear force capacity of BBK 79 and EC2. Vc_BBK79/$V_{Rd,c}$ plotted against slab thickness for various concrete strength and reinforcement amount; (a) Concrete strength f_{ck}= 14.5 MPa, reinforcement ϕ 16 mm, c/c s 100 mm, (b) Concrete strength f_{ck}= 35.5 MPa, reinforcement ϕ 20 mm, c/c s 100 mm.

The B7 method depends on the value of c, which is determined by the calculation of r_s. The value of r_s is influenced by the compressive strength of the concrete and the reinforcement strength. As shown in Figure 12, the effective depth has a significant impact on the results. This is because the effective depth affects the total punching force, which is directly related to the perimeter. As d (and corresponding h) increases, u_2 also increases in a direct and linear manner.

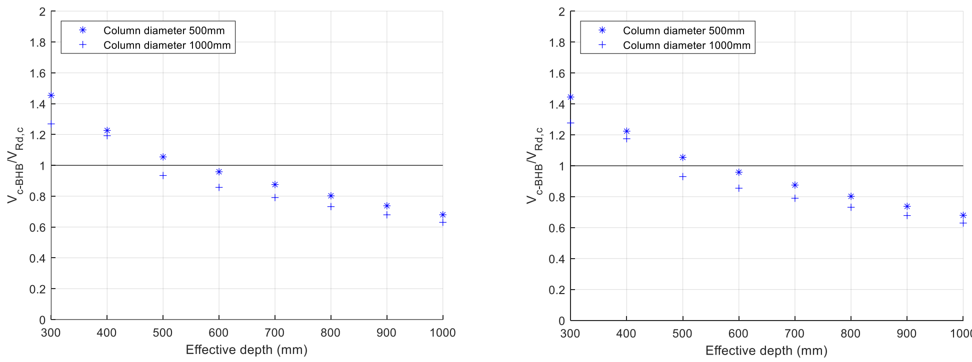
Despite a slight overestimation of capacity observed in Figure 12 (b) for case with low reinforcement ratios and low concrete compressive strength, a good margin of accuracy can be achieved for effective depths ranging from 300 to 500 mm. For cases with high concrete strength and reinforcement amounts, the method demonstrates reliable margins across a broader range of effective depths. The slight overestimation is attributed to the total punching force capacity being linked to the constant value t_{bo}, whereas Eurocode relies on the reinforcement ratio for its calculations.



(a) (b)

Figure 12 Ratio between shear force capacity of B7 and EC2. Vc_B7/$V_{Rd,c}$ plotted against slab thickness for various concrete strength and reinforcement amount; (a) Concrete strength f_{ck}= 14.5 MPa, reinforcement ϕ 16 mm, c/c s 100 mm, (b) Concrete strength f_{ck}= 35.5 MPa, reinforcement ϕ 20 mm, c/c s 100 mm.

In Figure 13, the ratio of BHB/Eurocode decreases as the effective depth increases, regardless of the reinforcement content. This suggests that BHB becomes less conservative compared to EC 2 as the depth increases. At smaller depths (300–500 mm), the ratios of BHB/EC2 are greater than 1, indicating that BHB predicts higher punching shear capacities compared to Eurocode. Beyond 500 mm, the ratio falls below 1, showing that BHB becomes less aggressive and predicts lower capacities than Eurocode. It can be seen that the ratios are similar for both low and high reinforcement cases implies that the relationship between reinforcement content and punching shear capacity is consistent across both methods. In other words, if reinforcement is increased, both methods increase the predicted punching capacity by a relatively similar proportion.



(a) (b)

Figure 13 Ratio between shear force capacity of BHB and EC2. Vc_BHB/$V_{Rd,c}$ plotted against slab thickness for various concrete strength and reinforcement amount; (a) Concrete strength f_{ck}= 14.5 MPa, reinforcement ϕ 16 mm, c/c s 100 mm, (b) Concrete strength f_{ck}= 35.5 MPa, reinforcement ϕ 20 mm, c/c s 100 mm.

4 Example on cot θ in Eurocode 2 for beam

In older structures, the engineers did not explicitly choose a cot (θ) value during the design, as these were not standard considerations in pre-Eurocode norms. Shear failure in reinforced concrete beams occurs when the applied shear forces exceed the capacity of the concrete and shear reinforcement, leading to the formation of diagonal cracks. To model this behavior, EC2 adopts the truss-strut model, where the shear force is resisted by an inclined concrete strut and transverse reinforcement (stirrups). This model assumes that after a shear crack forms, the forces are redistributed between the compression strut and the shear reinforcement. Modern assessments require determining θ (compression strut angle).

In EC2, the design shear resistance is determined using two key equations:

1. Shear resistance of the concrete strut:

$$V_{Rd,max} = 0.9\ \alpha_{cw}\ b_w\ d\ \nu_1 f_{cwd}/(\cot\theta + \tan\theta) \qquad (26)$$

This equation limits the shear force that can be transferred through the concrete strut before crushing occurs.

2. Shear resistance of the web reinforcement:

$$V_{Rd,s} = A_{sw}\ z\ f_{ywd} \cot\theta/s \qquad (27)$$

This equation governs the shear force that can be resisted by the transverse reinforcement.

The compression strut angle must be between 21.8° and 45° according to EC2. This restriction ensures that the structure can undergo sufficient plastic redistribution before failure. If θ is too small (steep strut), the tensile force in the longitudinal reinforcement increases significantly, leading to higher risk of brittle failure. If θ is too large, the compression strut becomes too shallow, reducing the effectiveness of shear reinforcement.

In an example, θ is calculated iteratively by balancing $V_{Rd,max}$ and $V_{Rd,s}$ in the EC2 equations resulting in a calculated θ = 8.1°, which is much lower than the minimum allowed value of 21.8°. This is illustrated in Figure 14, where the intersection of the two curves shows the governing θ value. The reason for this very low θ is that the concrete's shear resistance is dominant, while the transverse reinforcement must bear additional tensile forces. This finding highlights why EC2 enforces a minimum θ to ensure a realistic force distribution and prevent excessive strain in the shear reinforcement.

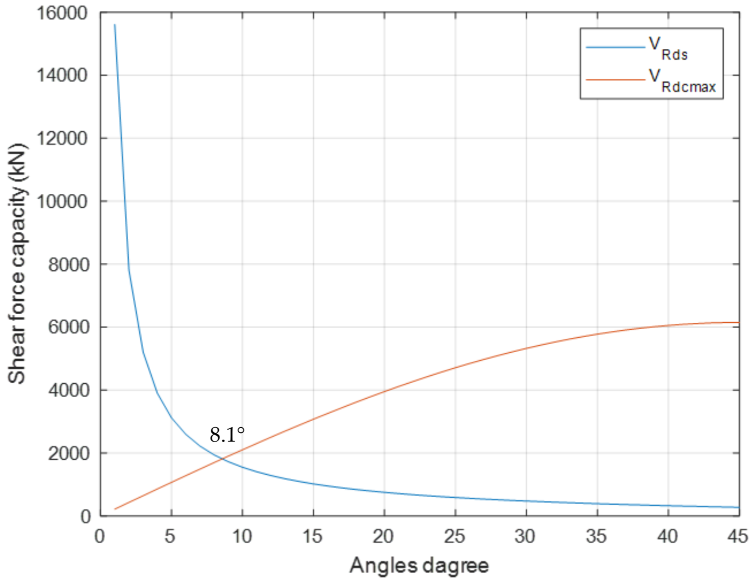


Figure 14 The shear resistance of concrete ($V_{Rd,max}$) and of the web reinforcement ($V_{Rd,s}$) for the given example. The beam dimensions are w x h = 1000 mm x 1835 mm, concrete C25/30, α_{cc} = 1.0), stirrups T12-300, A_{sw} = 226 mm²/m, f_{ywd} = 220/1.15 = 204 MPa (very old reinforcement, not used today!), s= 300 mm, α = 90 degrees, v = 0.6*(1-25/250) = 0.54.

However. the resulting θ value depends on the amount and spacing of shear reinforcement, rather than just beam width or bar diameter. In older structures, engineers did not explicitly choose cot(θ), as pre-Eurocode norms did not include this requirement. In modern assessments, θ is derived using EC2 equations (6.8 and 6.9).

4.1 STRUCTURAL REDISTRIBUTION AND STRUT ANGLE

The ability of a beam to redistribute plastic strains after shear cracking governs the final θ value. This redistribution is primarily affected by:

- Spacing of shear reinforcement – Closely spaced stirrups allow for better redistribution, leading to a larger allowable θ.
- Amount of shear reinforcement – Higher reinforcement ratios increase shear capacity and reduce the need for extreme redistribution.

Thus, θ is not purely a function of geometry (beam width, stirrup spacing, reinforcement diameter) but rather a result of strain redistribution after cracking.

4.2 EXTRA TENSILE FORCE IN LONGITUDINAL REINFORCEMENT

When θ decreases, the compression strut becomes steeper, increasing the horizontal component of force. This results in:

- Higher tensile forces in the longitudinal reinforcement, requiring additional reinforcement to prevent excessive cracking.
- Increased bending moment demand, influencing flexural capacity.

This interplay between θ, shear reinforcement, and longitudinal reinforcement must be considered in the design and assessment of existing structures.

In determining the shear capacity of existing structures, the first step is verifying that the spacing of shear reinforcement meets EC2 criteria. If this condition is satisfied, any strut angle between 22.8° and 45° may be selected. However, for concentrated loads near supports, the strut angle must be chosen to ensure that shear cracks form between the beam edge and the applied load, preventing early failure. The redistribution of plastic strain after shear crack formation is a key aspect of the truss model, governed by the shear reinforcement's ability to accommodate these deformations. A lower θ results in a steeper compression strut, increasing tensile forces in the longitudinal reinforcement, which must be accounted for in design assessments.

5 Discussion and Conclusions

Specific attention to shear in slabs is not given to a greater extent than that it should be avoided. Thus, a conclusion can be made that specific shear design in reinforced slabs before BBK became the acting regulation was a rare occurrence. SBN offers very limited design methods other than a simple method for checking the shear resistance and providing general advice to avoid situations where shear becomes the designing failure mode.

In general, the shear capacity of a reinforced concrete slab without web reinforcement is significantly influenced by the longitudinal reinforcement ratio. An increase in the ratio of longitudinal reinforcement results in stronger dowel action between the concrete and the reinforcement, which in turn provides better restraint to the development of cracks. This also enhances the transfer of shear strength between the interfacial interaction of inclined cracks and increases the depth of the concrete shear zone. Consequently, the specimen's shear capacity is correspondingly improved.

The earlier discussion about θ being influenced by reinforcement placement, spacing, and shear force redistribution applies directly here.

5.1 SHEAR FORCE CAPACITY COMPARISON

The comparison of shear force capacity between EC2, BBK04, and B7 highlights significant variations in the underlying design philosophies and their implications for practical applications. BBK04's Method 1 consistently overestimates shear force capacity compared to EC2, particularly for slabs with low longitudinal reinforcement ratios. This overestimation can be attributed to BBK04's reliance on a tensile strength factor, f_{ct}, which is less uncertain than EC2's compressive strength-based approach. For high reinforcement levels, the results converge, suggesting that BBK04's method compensates for increased reinforcement by incorporating additional safety margins. The role of the partial coefficient (γ_n) in calculation of f_{ct} (Eq.7) introduces a significant difference from EC2. While this coefficient ensures safety class adaptability, it results in aligning BBK04 results closely with those of EC2 in such cases.

Similarly, the B7 standard demonstrates considerable overestimations for shear capacity, especially for slabs with moderate thicknesses (200–400 mm). The lack of a clear contribution from reinforcement in B7's calculation method underscores its limitations in accounting for modern design practices. While B7's approach relies on base shear strength values, it accounts for slab thickness, which significantly influences shear capacity. However, the absence of a dynamic consideration of reinforcement effects limits its adaptability to present requirements.

EC2 provides a more balanced and uniform framework for shear force capacity calculations, especially when compared to BBK04's overestimations and B7's outdated assumptions. Specifically:

- BBK04 Method 1 overestimates shear capacity by up to 25% compared to EC2 for low reinforcement scenarios but aligns closely for higher reinforcement levels. However, this overestimation becomes more pronounced as concrete compressive strength increases for $f_{ck} > 15$ MPa. This suggests that BBK04 may not effectively capture the diminishing returns of increased concrete strength on shear capacity.

- BBK04 method 1 predicts higher shear capacities than Eurocode 2 for safety class 3, primarily due to the inclusion of the safety factor (γ_n)=1.2 in BBK04's calculations, which is not explicitly accounted for in EC2. The results indicate that BBK04 shear capacity ranges between 80% and 100% of EC2 for low longitudinal reinforcement, with the difference increasing alongside compressive strength. For high longitudinal reinforcement, BBK04 shear capacity ranges from approximately 60% to 110% of EC2, emphasizing the influence of reinforcement content on the results. These findings highlight the sensitivity of the results to both reinforcement levels and compressive strength, underscoring the importance of careful parameter consideration when comparing BBK04 with EC2. For higher compressive strengths and reinforcement levels, the BBK04's predictions exceed EC2, reflecting distinct design assumptions in each standard. In general introducing a partial coefficient (γ_n) for safety classes, ensures adaptability but resulting in lower capacity predictions for higher safety classes (10–15% lower than EC2 for safety class 3).
- B7 consistently overestimates capacity, particularly for slabs with moderate thickness (200–400 mm), B7 predicts shear capacities that are up to 25% higher than EC2, due to its reliance on base shear strength values without considering modern reinforcement contributions.
- These differences show how design standards have evolved. EC2 focuses on consistency and flexibility for various scenarios, while BBK04 and B7, though suitable for older designs, show clear limitations compared to EC2's more advanced approach.

5.2 PUNCHING SHEAR CAPACITY COMPARISON

Section 3.2 presents a comparative analysis of punching shear capacity calculations across four methods: EC2, BBK79, B7, and BHB. The results show important differences in how these standards address punching shear, particularly concerning slab thickness, reinforcement, and control perimeter definitions.

The BBK79 method shows consistently lower punching shear capacities compared to EC2, with safety margin of up to 40%. This difference arises from BBK79's shorter effective perimeter calculation (d/2 from the column face) versus EC2's 2d approach. This difference in perimeter length significantly affects the predicted capacity, especially for thinner slabs, where BBK79's reduced perimeter inflates the resistance value.

The B7 method incorporates empirical adjustments based on reinforcement and concrete strength, producing results closer to EC2 but with a notable dependency on slab depth. For thin slabs, B7 slightly overestimates capacity due to a fixed resistance factor (τ_{bo}) that doesn't dynamically adapt to reinforcement content. However, for thicker slabs, B7 aligns more closely with EC2, indicating that its approach to perimeter and depth effects offsets for its empirical simplifications.

The Concrete Handbook (BHB), based on the Kinnunen-Nylander model, aligns well with EC2 across most slab thicknesses. However, deviations occur at greater depths, where BHB becomes less conservative. This decrease in conservatism is attributed to BHB's reliance on diagrams and simplified approximations that underestimate depth effects.

The analysis of punching shear capacity across the methods highlights the strengths and limitations of each approach:

- BBK79’s punching shear capacity show a significant safety margin by approximately 35% compared to EC2 due to its shorter effective perimeter calculation.
- B7 demonstrates a better alignment with EC2 for thicker slabs but slightly overestimates capacity for thinner slabs. The calculation of *c* in the B7 method introduces an essential link between reinforcement and punching shear capacity, partially addressing the influence of reinforcement. However, the method relies heavily on empirical base shear strength values, limiting its accuracy for modern designs with advanced materials and configurations. While this approach improves alignment with reinforcement behavior, B7 still overestimates punching shear capacity compared to EC2, particularly for higher reinforcement ratios or slab thicknesses.
- BHB results for thinner slabs show that the overestimation ratio ranges from 10% to 35%, depending on the reinforcement ratio and concrete strength. The effect diminishes as the slab depth increases. As slab depth increases beyond 500 mm, BHB becomes more conservative, with the ratio dropping below 1.0, i.e. the safety margin increase linearly to up to 40%.

5.3 EXAMPLE ON COT(Θ) IN EUROCODE 2 FOR BEAM

Chapter 4 examines the behavior of the compression strut angle θ in reinforced concrete beams, using EC2 as the governing standard. The upper and lower limits for cot(θ), 1 and 2.5, respectively, act as boundaries to ensure realistic and safe designs. Variations below the minimum angle (corresponding to θ =21.8^{o}) or above the maximum angle (θ = 45^{0}) indicate potential overstressing of concrete or reinforcement. Narrow beams exhibit higher θ values compared to wider beams at identical stirrup spacings. This behavior is attributed to the geometric constraints imposed on the truss model, which affect force distributions and the effective angle of load transfer.

The role of the strut angle (θ) has been explored in relation to Eurocode 2's design methodology. The findings indicate that while geometric properties such as beam width, stirrup spacing, and reinforcement bar diameter influence the calculated strut angle, its primary governing factor is the ability of the structure to redistribute plastic strain after shear cracking. This redistribution is directly linked to the amount and spacing of shear reinforcement. Furthermore, the selection of an appropriate strut angle is crucial in ensuring that additional tensile forces in the longitudinal reinforcement are properly accounted for.

For existing structures, the assessment process begins with verifying whether the shear reinforcement spacing meets EC2 requirements. If this criterion is satisfied, the strut angle can be chosen within the allowable range of 22.8° to 45°. If the spacing between the shear reinforcement exceeds 0.75*d* then the strut angle is 45°. However, for beams subjected to concentrated loads near supports, the angle must be carefully selected to control the formation of shear cracks in a predictable manner.

5.4 GENERAL CONCLUSIONS

Table 9 presents a comparative analysis of different shear design approaches—BBK04 / BBK79, B7, and BHB—based on key criteria related to shear force capacity, punching shear, and general observations. The comparison highlights differences in prediction accuracy, the effect of concrete strength, safety factors, and adaptability to slab thickness. It also assesses how well these methods align with EC2 standards. The findings indicate that some methods tend to overestimate shear capacity, particularly for moderate slab thicknesses, while others provide conservative estimates, particularly in punching shear calculations.

Table 9 Comparison of Shear Design Approaches for Reinforced Concrete Slabs

Comparison Criteria	BBK04 / BBK79	B7	BHB
1. Shear Force Capacity	Overestimates (up to 25%) for low reinforcement; aligns with EC2 for high reinforcement.	Overestimates (up to 25% for slabs of 200–400 mm thickness)	
- Effect of Strength	Overestimate with an increase in the compressive strength of concrete higher than 15 MPa.	Base shear strength approach does not adjust for compressive strength	
- Safety Factor	Using safety factor (γn = 1.2) leading to lower predictions (10–15% lower than EC2 for safety class 3)	No explicit safety classes adjustments	
- Slab Thickness	Predictions depend on reinforcement levels more than slab thickness. However, a slight variation ranging between 5 – 10 % can be observed for moderate reinforcement ratios.	Overestimates capacity for moderate slab thicknesses (200–400 mm) up to 25% higher than EC2.	
2. Punching Shear	Underestimates by ~35-40% due to a shorter control perimeter (d/2 from the column face)	Closer to EC2 for thicker slabs but overestimates for thinner slabs	Matches EC2 for most slabs but underestimates for deep slabs (>500 mm)
- Reinforcement Impact	Conservative estimates, lacks reinforcement adaptability	Uses experiential factors but still overestimates	Overestimation reduces as depth increases
3. General Observations	Not fully align with EC2 for shear force. Good margins for punching shear.	Lacks reinforcement adaptability and overestimates in moderate slab thicknesses for both shear and punching	Generally, matches EC2 for deep slabs in punching shear

5.5 FURTHER STUDIES

- Investigate the impact of material deterioration on shear capacity over time, considering factors such as aging, environmental exposure, and sustained loading effects.
- Analyze real-world case studies of structural failures to validate theoretical models and assess differences between predicted and actual shear capacities.
- Evaluate the effectiveness of retrofit strategies, such as FRP strengthening, additional shear reinforcement, or alternative strengthening techniques, to enhance shear performance in older slabs and extend their service life.

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Appendix A: The practical limitations of longitudinal reinforcement ratios in reinforced concrete slab design

It was highlighted the significance of slab thickness i.e. the effective depth d, in influencing the shear capacity of the reinforced concrete slab without web reinforcement. The effective depth is also a crucial variable in calculating the longitudinal reinforcement ratio ρ. Thus, a study was conducted to determine the practical limit of the longitudinal reinforcement ratio in reinforced concrete slab design. Using a MATLAB script, the study established the maximum and minimum values of the reinforcement ratio that can be practically applied. The results were plotted as reinforcement ratio ρ against the spacing c/c between the longitudinal reinforcement and presented in Figure A1 for slab thicknesses of 200 mm and 1000 mm. Table A1shows the practical minimum and maximum longitudinal reinforcement ρ used in the comparison for each analysis. As a results, the study assumed ϕ 20 and 16 steel bars were used in the reinforced concrete, with c/c spacing of 100 mm and 500 mm for each. The findings of this study provide valuable insights into the practical limitations of longitudinal reinforcement ratios in reinforced concrete slab design which can be used to calculate the shear force capacity according to design codes and standards.

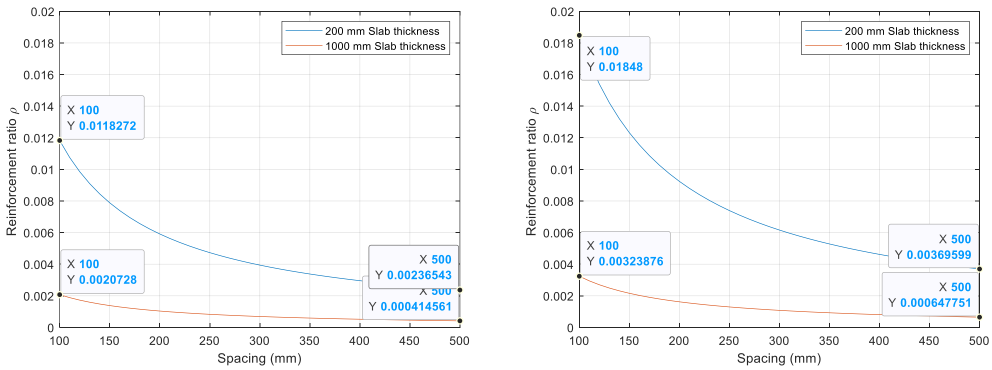

Figure A1 longitudinal reinforcement ratio *ρ* steel bar diameter (a) 16 mm and (b) 20 mm.

Table A1 Practical maximum and minimum *ρ* used in the analysis

Bars arrangment	ρ max at s 100 mm	ρ min at s 500 mm	Comments
Φ 16, h 1000	0.0021	0.0004	Assume cover = 30 mm
Φ 16, h 200	0.0118	0.0027	
Φ 20, h 1000	0.0032	0.0006	
Φ 20, h 200	0.0185	0.0037	

SHEAR FORCE CAPACITY OF EXISTING CONCRETE SLABS

Specific attention to shear in slabs is not given to a greater extent than that it should be avoided. The specific shear design in reinforced slabs before BBK became the acting regulation was a rare occurrence. SBN offers very limited design methods other than a simple method for checking the shear resistance and providing general advice to avoid situations where shear becomes the designing failure mode. A comparison between BBK04 and EC2 shows that BBK04 with safety class 3 has a good margin of 20%, while with safety class 1 you have similar results as with EC2 for concrete with a compressive strength less than 18 MPa. SBN80 shows an overestimation of 60% compared to EC2.

For the punching shear capacity, the results show that BBK79 have good safety margin compared to EC2, due to its shorter effective perimeter calculation. B7 demonstrates alignment with EC2 for thicker slabs, but slightly overestimates capacity for thinner slabs. BHB aligns well with EC2 across most slab thicknesses, but deviations occur at greater depths, where BHB becomes more conservative.

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